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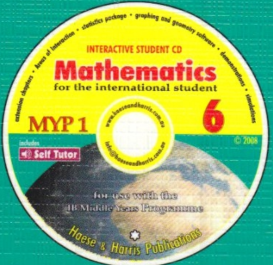

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includes
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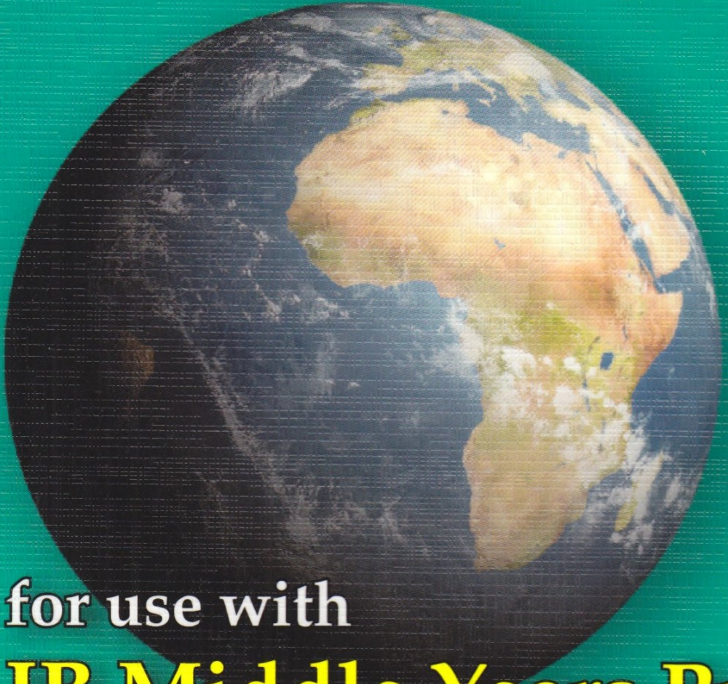

Pamela Vollmar
Michael Haese
Robert Haese
Sandra Haese
Mark Humphries

for use with

IB Middle Years Programme

HAESE & HARRIS PUBLICATIONS
Specialists in mathematics publishing

Mathematics
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IB Middle Years
Programme

MATHEMATICS FOR THE INTERNATIONAL STUDENT 6 (MYP 1)

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FOREWORD

This book may be used as a general textbook at about 6th Grade (or Year 6) level in classes where students are expected to complete a rigorous course in Mathematics. It is the first book in our Middle Years series 'Mathematics for the International Student'.

In terms of the IB Middle Years Programme (MYP), our series does not pretend to be a definitive course. In response to requests from teachers who use 'Mathematics for the International Student' at IB Diploma level, we have endeavoured to interpret their requirements, as expressed to us, for a series that would prepare students for the Mathematics courses at Diploma level. We have developed the series independently of the International Baccalaureate Organization (IBO) in consultation with experienced teachers of IB Mathematics. Neither the series nor this text is endorsed by the IBO.

In regard to this book, it is not our intention that each chapter be worked through in full. Time constraints will not allow for this. Teachers must select exercises carefully, according to the abilities and prior knowledge of their students, to make the most efficient use of time and give as thorough coverage of content as possible.

We understand the emphasis that the IB MYP places on the five Areas of Interaction and in response there are links on the CD to printable pages which offer ideas for projects and investigations to help busy teachers (see p. 5).

Frequent use of the interactive features on the CD should nurture a much deeper understanding and appreciation of mathematical concepts. The inclusion of our new **Self Tutor** software (see p. 4) is intended to help students who have been absent from classes or who experience difficulty understanding the material.

The book contains many problems to cater for a range of student abilities and interests, and efforts have been made to contextualise problems so that students can see the practical applications of the mathematics they are studying.

We welcome your feedback. Email: info@haeseandharris.com.au

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PV, PMH, RCH, SHH, MH

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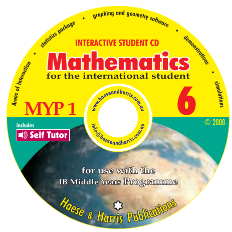
The publishers wish to make it clear that acknowledging these individuals does not imply any endorsement of this book by any of them, and all responsibility for the content rests with the authors and publishers.

USING THE INTERACTIVE CD



The interactive CD is ideal for independent study.

Students can revisit concepts taught in class and undertake their own revision and practice. The CD also has the text of the book, allowing students to leave the textbook at school and keep the CD at home.

By clicking on the relevant icon, a range of new interactive features can be accessed:

- Self Tutor
- Areas of Interaction links to printable pages
- Interactive Links – to spreadsheets, video clips, graphing and geometry software, computer demonstrations and simulations

SELF TUTOR is a new exciting feature of this book.

The **Self Tutor** icon on each worked example denotes an active link on the CD.

> Simply 'click' on the **Self Tutor** (or anywhere in the example box) to access the worked example, with a teacher's voice explaining each step necessary to reach the answer.
>
> Play any line as often as you like. See how the basic processes come alive using movement and colour on the screen.
>
> Ideal for students who have missed lessons or need extra help.

Example 9 **Self Tutor**

Find: **a** $\frac{3}{4} - \frac{1}{3}$ **b** $\frac{5}{6} - \frac{1}{3} - \frac{2}{9}$

a $\frac{3}{4} - \frac{1}{3}$ {LCD $= 12$}

$= \frac{3\times3}{4\times3} - \frac{1\times4}{3\times4}$ {converting to 12ths}

$= \frac{9}{12} - \frac{4}{12}$ {simplifying}

$= \frac{5}{12}$ {subtracting the numerators}

b $\frac{5}{6} - \frac{1}{3} - \frac{2}{9}$ {LCD $= 18$}

$= \frac{5\times3}{6\times3} - \frac{1\times6}{3\times6} - \frac{2\times2}{9\times2}$ {converting to 18ths}

$= \frac{15}{18} - \frac{6}{18} - \frac{4}{18}$ {simplifying}

$= \frac{5}{18}$ {subtracting the numerators}

See **Chapter 8**, **Fraction Operations**, p. 151

AREAS OF INTERACTION

The International Baccalaureate Middle Years Programme focuses teaching and learning through five Areas of Interaction:

- Approaches to learning
- Community and service
- Human ingenuity
- Environments
- Health and social education

The Areas of Interaction are intended as a focus for developing connections between different subject areas in the curriculum and to promote an understanding of the interrelatedness of different branches of knowledge and the coherence of knowledge as a whole.

Click on the heading to access a printable 'pop-up' version of the link.

In an effort to assist busy teachers, we offer the following printable pages of ideas for projects and investigations:

LINKS click here

TENNIS RANKINGS

Areas of interaction:
Human ingenuity, Approaches to learning

Links to printable pages of ideas for projects and investigations

Chapter 2: Operations with whole numbers p. 44	**TENNIS RANKINGS** Human ingenuity, Approaches to learning
Chapter 3: Points, lines and angles p. 64	**MAKING A PROTRACTOR** Human ingenuity
Chapter 5: Number properties p. 104	**CICADAS** Environments, Approaches to learning
Chapter 7: Polygons p. 142	**PROTECTING YOURSELF, THE OLD FASHIONED WAY** Human ingenuity
Chapter 11: Operations with decimals p. 213	**BODY MASS INDEX** Health and social education
Chapter 12: Measurement p. 232	**CALCULATING YOUR CARBON FOOTPRINT** Environments, Community and service
Chapter 15: Time and temperature p. 293	**HOW MANY STEPS DO YOU TAKE EACH DAY?** Environments, Health and social education
Chapter 19: Area, volume and capacity p. 370	**HOW MANY BRICKS ARE NEEDED TO BUILD A HOUSE?** Approaches to learning
Chapter 20: Equations p. 388	**HOW ARE DIVING SCORES CALCULATED?** Human ingenuity
Chapter 24: Solids and polyhedra p. 449	**PLATONIC SOLIDS** Human ingenuity, Approaches to learning

TABLE OF CONTENTS

Chapter 1

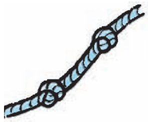
Number systems

Contents:

Archaeologists and **anthropologists** study ancient civilizations. They have helped us to understand how people long ago counted and recorded numbers. Their findings suggest that the first attempts at counting were to use a **tally**.

For example, in ancient times people used items to represent numbers:

scratches on a cave wall showed the number of new moons since the buffalo herd came through



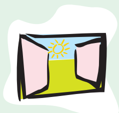
knots on a rope showed the rows of corn planted

pebbles on the sand showed the number of traps set for fish

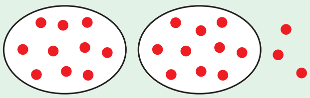
notches cut on a branch showed the number of new lambs born

In time, humans learned to write numbers more efficiently. They did this by developing **number systems**.

OPENING PROBLEM



The number system we use in this course is based on the **Hindu-Arabic system** which uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.



The number of dots shown here is twenty three. We write this as 23, which means '2 tens and 3 ones'.

How was the number 23 written by:

- ancient Egyptians
- ancient Greeks
- Romans
- Mayans
- Chinese and Japanese?

A DIFFERENT NUMBER SYSTEMS

The ancient Egyptians used tally strokes to record and count objects.

||||||||||||||||||||||| indicated there were 23 objects.

In time they replaced every 10 strokes with a different symbol. They chose ∩ to represent ||||||||||.

So, 23 was then written as ∩∩|||.

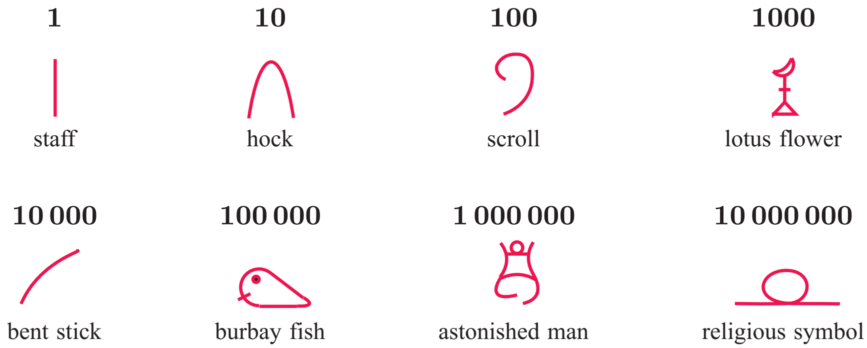
We still use tallies to help with counting. Instead of ||||| we now use 𝍸.

||||||||||||||||||||||| objects would be recorded as 𝍸 𝍸 𝍸 𝍸 |||.

THE EGYPTIAN NUMBER SYSTEM

There is archaeological evidence that as long ago as 3600 BC the Egyptians were using a detailed number system. The symbols used to represent numbers were pictures of everyday things. These symbols are called **hieroglyphics** which means sacred picture writings.

The Egyptians used a tally system based on the number ten. Ten of one symbol could be replaced by one of another symbol. We call this a **base ten system**.



The order in which the symbols were written down did not affect the value of the numerals. The value of the numerals could be found by adding the value of the symbols used.

So,  or  would still represent 35.

The **Egyptian system** did not have **place values**.

EXERCISE 1A.1

1 **a** In the Hindu-Arabic number system, 3 symbols are used to write the number 999. How many Egyptian symbols are needed to write the Hindu-Arabic 999?

b Write the Egyptian symbols for 728 and 234 124.

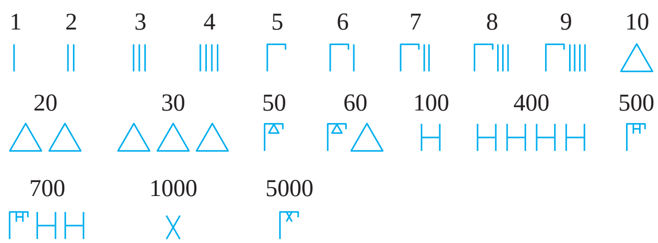
2 Convert these symbols to Hindu-Arabic numerals:

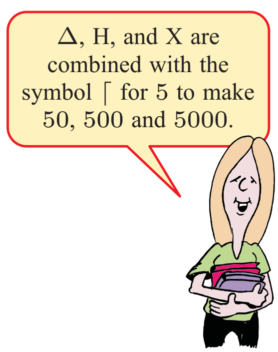
a  **b** 

THE ANCIENT GREEK OR ATTIC SYSTEM

The Ancient Greeks saw the need to include a symbol for 5. This symbol was combined with the symbols for 10, 100, and 1000 to make 50, 500, and 5000.

Some examples of Ancient Greek numbers are:





This number system depends on addition **and** multiplication.

Example 1 — Self Tutor

Change the following Ancient Greek numerals into a Hindu-Arabic number:

a ΧΗΗΗΔΔΙΙΙΙ **b** 𐅆Χ𐅅ΗΗ𐅄ΔΔΔΙ

a

Χ	1000
ΗΗΗ	300
ΔΔ	20
ΙΙΙΙ	+ 4
	1324

b

𐅆Χ	6000
𐅅ΗΗ	700
𐅄ΔΔΔ	80
Ι	+ 1
	6781

EXERCISE 1A.2

1 Change the following Ancient Greek numerals into Hindu-Arabic numbers:

a ΔΙΙΙ **b** ΔΔΓΙ **c** ΗΗΙΙΙΙ

d ΧΧΧΗΗΔΔΔΔ **e** 𐅅ΗΗΔΔΙΙΙ **f** 𐅆ΗΗ𐅄ΓΙΙΙΙ

2 Write the following Hindu-Arabic numbers as Ancient Greek numerals:

a 14 **b** 31 **c** 99 **d** 555 **e** 4082 **f** 5601

ROMAN NUMERALS

Like the Greeks, the Romans used a number for five.

The first four numbers could be represented by the fingers on one hand, so the V formed by the thumb and forefinger of an open hand represented 5.

Two Vs joined together  became two lots of 5, so ten was represented by X.

C represented one hundred, and half a  or L became 50.

One thousand was represented by an . With a little imagination you should see that an  split in half and turned on its side became , so D became half a thousand or 500.

1	2	3	4	5	6	7	8	9	10	
I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	
20	30	40	50	60	70	80	90	100	500	1000
XX	**XXX**	**XL**	**L**	**LX**	**LXX**	**LXXX**	**XC**	**C**	**D**	**M**

Unlike the Egyptian system, numbers written in the Roman system had to be written in order.

For example:

IV stands for 1 before 5	or 4	whereas VI stands for 1 after 5	or 6.
XC stands for 10 before 100	or 90	whereas CX stands for 10 after 100	or 110.

There were rules for the order in which symbols could be used:

- I could only appear before V or X.
- X could only appear before L or C.
- C could only appear before D or M.

One less than a thousand was therefore not written as IM but as CMXCIX.

Larger numerals were formed by placing a stroke above the symbol. This made the number 1000 times as large.

5000	10 000	50 000	100 000	500 000	1 000 000
$\overline{\text{V}}$	$\overline{\text{X}}$	$\overline{\text{L}}$	$\overline{\text{C}}$	$\overline{\text{D}}$	$\overline{\text{M}}$

EXERCISE 1A.3

1 What numbers are represented by the following symbols?

a VIII **b** XIV **c** XVI **d** XXXI **e** CX
f LXXXI **g** CXXV **h** CCXVI **i** LXII **j** MCLVI
k $\overline{\text{D}}\,\overline{\text{L}}$DCV **l** DCCXX **m** CDXIX **n** $\overline{\text{D}}\,\overline{\text{L}}\,\overline{\text{V}}$DI **o** $\overline{\text{M}}\,\overline{\text{M}}\,\overline{\text{C}}\,\overline{\text{C}}\,\overline{\text{C}}$

2 Write the following numbers in Roman numerals:

a 18 **b** 34 **c** 279 **d** 902 **e** 1046 **f** 2551

3 **a** Which Roman numeral less than one hundred is written using the greatest number of symbols?

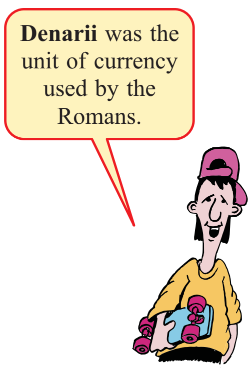
b What is the highest Roman numeral between M and MM which uses the least number of symbols?

c Write the year 1999 using Roman symbols.

4 Use Roman numerals to answer the following questions.

a Each week Octavius sharpens CCCLIV swords for his general. How many will he need to sharpen if the general doubles his order?

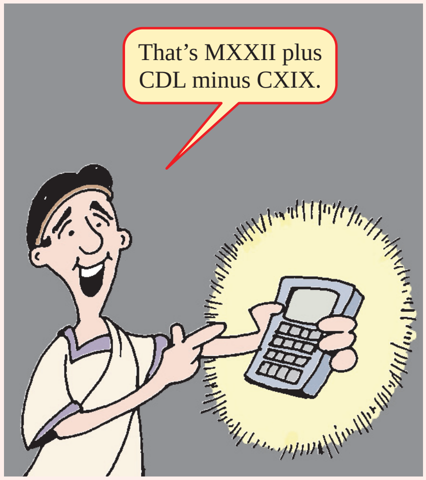
b What would it cost Claudius to finish his courtyard if he needs to pay for CL pavers at VIII denarii each and labour costs XCIV denarii?



ACTIVITY 1 — IF YOU LIVED IN ROMAN TIMES

What to do:

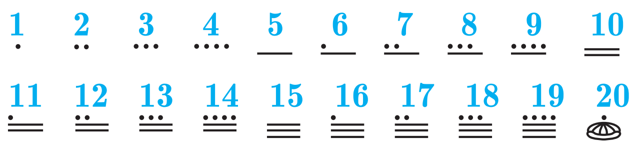
1 Use Roman numerals to write:
- **a** your house number and postcode
- **b** your height in centimetres
- **c** your phone number
- **d** the number of students in your class
- **e** the width of your desk in centimetres.



2 Use a calendar to help you write in Roman numerals:
- **a** your date of birth, for example XXI-XI-MCMXLVI
- **b** what the date will be when you are:
  i XV **ii** L **iii** XXI **iv** C

THE MAYAN SYSTEM

The Mayans originally used pebbles and sticks to represent numbers. They later recorded them as dots and strokes. A stroke represented the number 5.



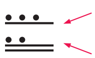
Unlike the Egyptians and Romans, the Mayans created a **place value** by placing one symbol *above* the other.

The Hindu-Arabic system we use in this course involves base 10.

The number 172 is 17 'lots of' 10 plus 2 'lots of' 1.

In contrast, the Mayan system used base 20.

Consider 

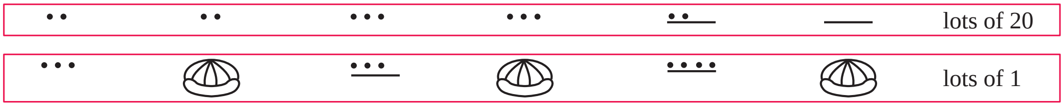


this upper part represents 8 'lots of' 20 or	160
the lower part represents 12 'lots of' 1 or	12
So, the number represented is	172

The Mayans also recognised the need for a number zero to show the difference between 'lots of 1' and 'lots of 20'. The symbol  which represented a mussel shell, works like our zero.

Compare these symbols:

43 **40** **68** **60** **149** **100**



EXERCISE 1A.4

1 Write these numbers using Mayan symbols:

a 23 **b** 50 **c** 99 **d** 105 **e** 217 **f** 303

2 Convert these Mayan symbols into Hindu-Arabic numbers:

a **b** **c** **d** **e** **f**

RESEARCH OTHER WAYS OF COUNTING

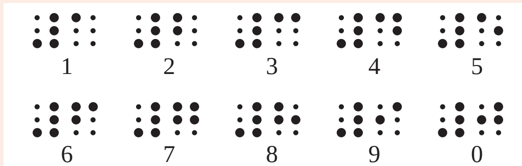
Find out:

a how the Ancient Egyptians and Mayans represented numbers larger than 1000

b whether the Egyptians used a symbol for zero

c what **Braille** numbers are and what they feel like

d how deaf people 'sign' numbers.



THE CHINESE - JAPANESE SYSTEM

The Chinese and Japanese use a similar place value system.

Their symbols are:

1	2	3	4	5	6
一	二	三	四	五	六
7	**8**	**9**	**10**	**100**	**1000**
七	八	九	十	百	千

This is how 4983 would be written:

四千 } 4 'lots' of 1000
\+
九百 } 9 'lots' of 100
\+
八十 } 8 'lots' of 10
\+
三 3

EXERCISE 1A.5

1 What numbers are represented by these symbols?

a 七百六十五

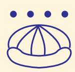
b 三千二百四十八

c 九千九百九十九

2 Write these numbers using Chinese-Japanese symbols:

a 497 **b** 8400 **c** 1111

3 Copy and complete:

	Words	*Hindu-Arabic*	*Roman*	*Egyptian*	*Mayan*	*Chinese-Japanese*
a	thirty seven	37				
b						
c			CLIX			
d						

ACTIVITY 2 MATCHSTICK PUZZLES

Use matchsticks to solve these puzzles. Unless stated otherwise, you are not allowed to remove a matchstick completely.

1 Move just one matchstick to make this correct: IV – II = V

2 Move one matchstick to make this correct: III – II = IV

3 Arrange 4 matchsticks to make a total of 15.

4 Make this correct moving just one matchstick: XI + I = X

5 Remove 3 matchsticks from this sum to make the equation correct. VII + I = I

B THE HINDU-ARABIC SYSTEM

The number system we will use throughout this course was developed in India 2000 years ago. It was introduced to European nations by Arab traders about 1000 years ago. The system was thus called the **Hindu-Arabic** system.

The marks we use to represent numbers are called **numerals**. They are made up using the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0, which are known as **digits**.

ordinal number	one	two	three	four	five	six	seven	eight	nine
Hindu-Arabic numeral									
modern numeral	1	2	3	4	5	6	7	8	9

The digits 3 and 8 are used to form the numeral 38 for the number 'thirty eight' and the numeral 83 for the number 'eighty three'.

The numbers we use for counting are called **natural numbers** or sometimes just **counting numbers**. The possible combination of natural numbers is endless. There is no largest natural number, so we say the set of all natural numbers is **infinite**.

If we include the number **zero** or 0, then our set now has a new name, which is the set of **whole numbers**.

The Hindu-Arabic system is more useful and more efficient than the systems used by the Egyptians, Romans, and Mayans.

- It uses only 10 digits to construct all the natural numbers.
- It uses the digit 0 or zero to show an empty place value.
- It has a **place value** system where digits represent different numbers when placed in different place value columns.

Each digit in a number has a place value.

For example: in 567 942

hundred thousands	ten thousands	thousands	hundreds	tens	units
5	6	7	9	4	2

Example 2 — Self Tutor

What number is represented by the digit 7 in:

a 374 **b** 5709 **c** 127 624?

a In 374, the 7 represents '7 lots of 10' or 70.
b In 5709, the 7 represents '7 lots of 100' or 700.
c In 127 624, the 7 represents '7 lots of 1000' or 7000.

EXERCISE 1B

1 What number is represented by the digit 8 in the following?

a 38 **b** 81 **c** 458 **d** 847
e 1981 **f** 8247 **g** 2861 **h** 28 902
i 60 008 **j** 84 019 **k** 78 794 **l** 189 964

2 Write down the place value of the 3, the 5 and the 8 in each of the following:

a 53 486 **b** 3580 **c** 50 083 **d** 805 340

3 **a** Use the digits 6, 4 and 8 once only to make the largest number you can.
b Write the largest number you can using the digits 4, 1, 0, 7, 2 and 9 once only.
c What is the largest 6 digit numeral you can write using each of the digits 2, 7 and 9 twice?
d How many different numbers can you write using the digits 3, 4 and 5 once only?

4 Put the following numbers in *ascending* order:

a 57, 8, 75, 16, 54, 19
b 660, 60, 600, 6, 606
c 1080, 1808, 1800, 1008, 1880
d 45 061, 46 510, 40 561, 46 051, 46 501
e 236 705, 227 635, 207 653, 265 703
f 554 922, 594 522, 545 922, 595 242

Ascending means from smallest to largest.
Descending means from largest to smallest.

5 Write the following numbers in *descending* order:

a 361, 136, 163, 613, 316, 631
b 7789, 7987, 9787, 8779, 8977, 7897, 9877
c 498 231, 428 931, 492 813, 428 391, 498 321
d 563 074, 576 304, 675 034, 607 543, 673 540

Example 3 **Self Tutor**

a Express $3 \times 10\,000 + 4 \times 1000 + 8 \times 10 + 5 \times 1$ in simplest form.
b Write 9602 in expanded form.

a $3 \times 10\,000 + 4 \times 1000 + 8 \times 10 + 5 \times 1 = 34\,085$
b $9602 = 9 \times 1000 + 6 \times 100 + 2 \times 1$

6 Express the following in simplest form:

a $8 \times 10 + 6 \times 1$
b $6 \times 100 + 7 \times 10 + 4 \times 1$
c $9 \times 1000 + 6 \times 100 + 3 \times 10 + 8 \times 1$
d $5 \times 10\,000 + 2 \times 100 + 4 \times 10$
e $2 \times 10\,000 + 7 \times 1000 + 3 \times 1$
f $2 \times 100 + 7 \times 10\,000 + 3 \times 1000 + 9 \times 10 + 8 \times 1$
g $3 \times 100 + 5 \times 100\,000 + 7 \times 10 + 5 \times 1$
h $8 \times 100\,000 + 9 \times 1000 + 3 \times 100 + 2 \times 1$

7 Write in expanded form:

a 975 **b** 680 **c** 3874 **d** 9083
e 56 742 **f** 75 007 **g** 600 829 **h** 354 718

8 Write the following in numeral form:

a twenty seven
b eighty
c six hundred and eight
d one thousand and sixteen
e eight thousand two hundred
f nineteen thousand five hundred and thirty eight
g seventy five thousand four hundred and three
h six hundred and two thousand eight hundred and eighteen.

9 What number is:

a one less than eight
b two more than eleven
c four more than seventeen
d one less than three hundred
e seven greater than four thousand
f 3 less than 10 000
g four more than four hundred thousand
h 26 more than two hundred and nine thousand?

10 The number 372 474 contains two 7s and two 4s.

first 7 second 7

a How many times larger is the first 7 compared with the second 7?
b How many times smaller is the second 7 compared with the first 7?
c Which of the 4s represents a larger number? By how much is it larger than the other one?

C BIG NUMBERS

Commas or **,** are sometimes used to make it easier to read numbers greater than 3 digits.

For example: 2, 954 two thousand, nine hundred and fifty four
4, 234, 685 four million, two hundred and thirty four thousand, six hundred and eighty five

When typed, we usually use a space instead of the comma. Can you suggest some reasons for this?

Millions			*Thousands*			*Units*		
hundreds	tens	units	hundreds	tens	units	hundreds	tens	units
	5	3	4	7	9	6	8	2

The number displayed in the place value chart is 53 million, 479 thousand, 682. To make the number easier to read the digits are arranged into the units, the thousands, and the millions. With spaces now used to separate the groups, the number on the place value chart is written 53 479 682.

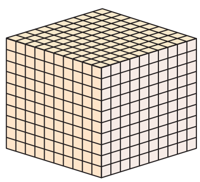
A MILLION

One million is written 1 000 000. Just how large is one million?

DEMO

Consider the following:

is a diagram of a cube with sides 1 mm.



is a diagram of a cube with sides 1 cm. Each 1 cm = 10 mm.
This cube contains 1000 cubes with sides 1 mm.

A cube which has sides 10 cm is made up of $10 \times 10 \times 10 = 1000$ cubes with sides 1 cm, and each cube with sides 1 cm is made up of 1000 cubes with sides 1 mm. So, it is made up of 1000×1000 or 1 000 000 cubes with sides 1 mm.

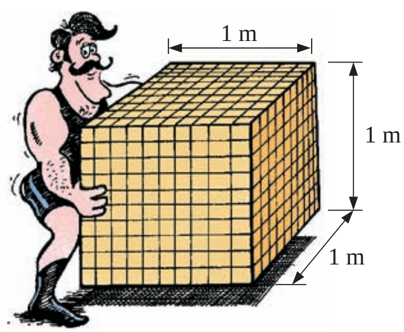
A BILLION AND A TRILLION

A **billion** is 1000 million or 1 000 000 000.

We saw previously that a 10 cm $\times$ 10 cm $\times$ 10 cm cube contains 1 000 000 cubic millimetres.

A billion cubic millimetres are contained in a cube which is 1 m $\times$ 1 m $\times$ 1 m.

A **trillion** is 1000 billion or 1 000 000 000 000.



Trillions			*Billions*			*Millions*			*Thousands*			*Units*		
H	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*
	6	3	5	8	4	2	0	1	5	7	1	9	2	6

The number displayed in the place value chart is
63 trillion, 584 billion, 201 million, 571 thousand 9 hundred and 26.

EXERCISE 1C

1 In the number 53 479 682, the digit 9 has the value 9000 and the digit 3 has the value 3 000 000. Give the value of the:

a 8 **b** 5 **c** 6 **d** 4 **e** 7 **f** 2

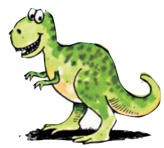
2 Write the value of each digit in the following numbers:

a 3 648 597 **b** 34 865 271

3 Read the following stories about large numbers. Write each large number using numerals.

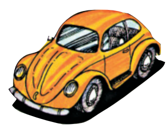
a A heart beating at a rate of 70 beats per minute would beat about thirty seven million times in a year.

b Austria's largest hamburger chain bought two hundred million bread buns and used seventeen million kilograms of beef in one year.



c The Jurassic era was about one hundred and fifty million years ago.

d One hundred and eleven million, two hundred and forty thousand, four hundred and sixty three dollars and ten cents was won by two people in a Powerball Lottery in Wisconsin USA in 1993.



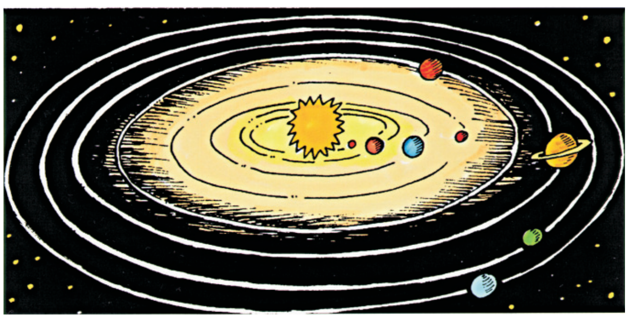
e A total of twenty one million, two hundred and forty thousand, six hundred and fifty seven Volkswagen 'Beetles' had been built to the end of 1995.

f In a lifetime the average person will blink four hundred and fifteen million times.

g One Megabyte of data on a computer is one million, forty eight thousand, five hundred and seventy six bytes.

4 Arrange these planets in order of their distance from the Sun starting with the closest.

Venus	108 200 000 kms
Saturn	1 427 000 000 kms
Earth	149 600 000 kms
Uranus	2 870 000 000 kms
Mercury	57 900 000 kms
Jupiter	778 300 000 kms
Pluto	5 900 000 000 kms
Neptune	4 497 000 000 kms
Mars	227 900 000 kms



5 **a** Use the table to answer the following:

  i Which continent has the greatest area?

  ii Name the continents with an area greater than 20 million square kilometres.

b Which continents are completely in the Southern Hemisphere?

Continent	*Area in square km*
Africa	30 271 000
Antarctica	13 209 000
Asia	44 026 000
Australia	7 682 000
Europe	10 404 000
North America	24 258 000
South America	17 823 000

ACTIVITY 3 NUMBER SEARCH PROBLEMS

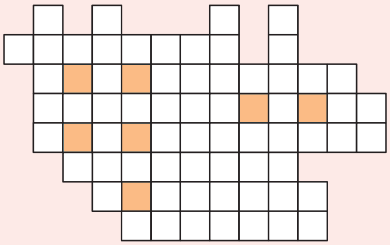
PRINTABLE WORKSHEET

Number searches are like crossword puzzles with numbers going across and down.

The aim is to fit all of the numbers into the grid using each number once. There is only one way in which all of the numbers will fit.

Draw or click on the icon to print these grids then insert the given numbers.

Search 1:



2 digits
89, 92, 56

3 digits
183

4 digits
6680

5 digits
69 235

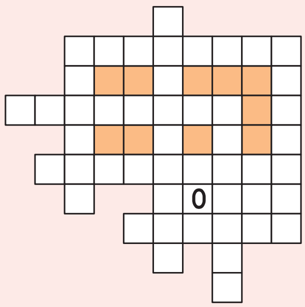
6 digits
949 875

7 digits
8 097 116
3 291 748
6 709 493
7 264 331
4 387 096
3 872 095

8 digits
62 658 397
79 408 632
10 343 879
91 863 432
81 947 368

Search 2:

- seven hundred and nine
- five hundred and eighty six
- sixty thousand, two hundred and eighty four
- seven hundred and ninety three thousand and forty two
- four hundred and forty nine thousand, seven hundred and sixty eight
- three million eight hundred and two thousand, seven hundred and forty eight
- two million six hundred and eighty three thousand, one hundred and forty eight
- seventy million, two hundred and eighty three thousand, six hundred and forty two
- nineteen million, three hundred and eighty four thousand, and three
- five hundred and eighty three million, seventy nine thousand, six hundred and forty six
- three hundred and forty five million, six hundred and ninety seven thousand and fifty one



Did you know?

The milk from 1 000 000 litre cartons would fill a 50 metre long by 20 metre wide pool to a depth of 1 metre.

KEY WORDS USED IN THIS CHAPTER

- Ancient Greek system
- counting number
- Hindu-Arabic system
- million
- numeral
- tally
- billion
- digit
- infinite
- natural number
- place value
- trillion
- Chinese-Japanese system
- Egyptian system
- Mayan system
- number system
- Roman numeral
- whole number

REVIEW SET 1A

1 Give the numbers represented by the Ancient Greek symbols:

a Η𐅄ΔΓ **b** ΧΧ𐅅ΗΔΔΔΙΙΙΙ

2 Write the following numbers using Egyptian symbols:

a 27 **b** 569

3 Give the numbers represented by the Roman numerals:

a XVIII **b** LXXIX

4 Write the year 2012 using Roman numerals.

5 Write the following numbers using the Mayan system:

a 46 **b** 273

6 Give the numbers represented by the Chinese-Japanese symbols:

a
四
百
七
十
六

b
三
百
五
十
九

7 Give the number represented by the digit 4 in: **a** 3409 **b** 41 076

8 What is the place value of the 8 in the following numbers?

a 3894 **b** 856 042

9 Use the digits 8, 0, 4, 1, 7 to make the largest number you can.

10 Write these numbers in ascending order (smallest first):

569 207, 96 572, 652 097, 795 602, 79 562

11 Express $2 \times 1000 + 4 \times 100 + 9 \times 10 + 7 \times 1$ in simplest form.

12 Write seventeen thousand three hundred and four in numeral form.

13 Write the value of each digit in the number 4 532 681.

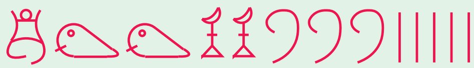
14 The total area of Canada is approximately nine million, nine hundred and eighty four thousand, seven hundred square kilometres. Write this number using numerals.

REVIEW SET 1B

1 Write the following numbers using Ancient Greek symbols:

a 78 **b** 245

2 Give the numbers represented by the Egyptian symbols:

a  **b** 

3 Which Roman numeral between 100 and 200 uses the greatest number of symbols?

4 Write these numbers using the Chinese-Japanese system:

a 386 **b** 2113

5 What number is represented by the digit 7 in the following?

a 3174 **b** 207 409

6 What is the largest 6 digit number you can write using each of the digits 0, 5 and 8 twice?

7 Write in descending order (largest first):

680 969, 608 699, 6 080 699, 698 096, 968 099

8 Write the following numbers in expanded form:

a 2159 **b** 306 428

9 What number is:

a five more than eighteen **b** nine less than one thousand?

10 Write the value of each digit in the number 37 405 922.

11 The average person will travel five million, eight hundred and ninety thousand kilometres in a lifetime. Write this number using numerals.

12 Consider the number 2 000 000 000.

a Write this number in words.

b Copy and complete: 2 000 000 000 is ...... lots of one million.

13 The number 2 552 667 contains two 2s, two 5s and two 6s.

(Labels above the number: 1st, 2nd pointing to the two 5s; labels below: 1st, 2nd pointing to the two 2s.)

a How many times larger is the first 2 compared with the second 2?

b How many times smaller is the second 5 compared with the first 5?

c Which of the 6s represents a larger number? By how much is it larger than the other 6?

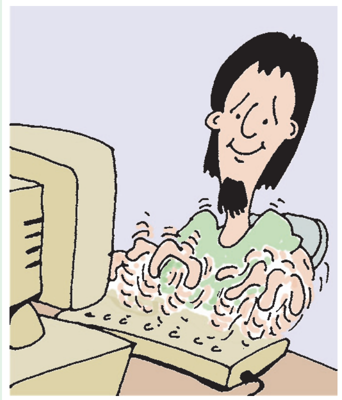
Chapter 2

Operations with whole numbers

Contents:

OPENING PROBLEM

Andreas is typing an essay on his computer. He is very quick at typing.



Things to think about:

- If Andreas types 70 words each minute, how many words will he type in 6 minutes?
- If Andreas types 378 words in 6 minutes, how many words per minute has he typed?
- If Andreas types 72 words per minute for 2 minutes and then 80 words per minute for 3 minutes, how many words has he typed in the 5 minute period, and what was his overall rate of typing?

A ADDING AND SUBTRACTING WHOLE NUMBERS

When we add or subtract whole numbers it is often easier to write the numbers in columns so that the place values are lined up.

Example 1 Self Tutor

Find: $32 + 427 + 3274$

$$\begin{array}{r} 32 \\ 427 \\ +\ 3274 \\ {\scriptstyle 1\ 1\ \ } \\ \hline 3733 \end{array}$$

Example 2 Self Tutor

Find: **a** $207 - 128$ **b** $4200 - 326$

a
$$\begin{array}{r} {\scriptstyle 1\ 9\ 17} \\ \not{2}\not{0}7 \\ -\ 128 \\ \hline 79 \end{array}$$

b
$$\begin{array}{r} {\scriptstyle 3\ 11\ 9\ 10} \\ \not{4}\not{2}\not{0}\not{0} \\ -\ 326 \\ \hline 3874 \end{array}$$

EXERCISE 2A.1

1 Do these additions:

a $\begin{array}{r} 392 \\ +\ 415 \\ \hline \end{array}$ **b** $\begin{array}{r} 601 \\ +\ 729 \\ \hline \end{array}$ **c** $\begin{array}{r} 1917 \\ +\ 2078 \\ \hline \end{array}$

d $\begin{array}{r} 913 \\ 24 \\ +\ 707 \\ \hline \end{array}$ **e** $\begin{array}{r} 217 \\ 106 \\ +\ 1274 \\ \hline \end{array}$ **f** $\begin{array}{r} 9004 \\ 216 \\ 23 \\ +\ 3816 \\ \hline \end{array}$

2 Find:

a $42 + 37$ **b** $72 + 35$ **c** $421 + 327$

d $624 + 72$ **e** $921 + 1234$ **f** $6214 + 324 + 27$

g $90 + 724$ **h** $32 + 627 + 4296$ **i** $912 + 6 + 427 + 3274$

3 Do these subtractions:

a $\begin{array}{r} 97 \\ -\ 15 \\ \hline \end{array}$ **b** $\begin{array}{r} 63 \\ -\ 19 \\ \hline \end{array}$ **c** $\begin{array}{r} 247 \\ -\ 138 \\ \hline \end{array}$

d $\begin{array}{r} 602 \\ -\ 149 \\ \hline \end{array}$ **e** $\begin{array}{r} 713 \\ -\ 48 \\ \hline \end{array}$ **f** $\begin{array}{r} 6005 \\ -\ 2349 \\ \hline \end{array}$

4 Find:

a $47 - 13$ **b** $62 - 14$ **c** $33 - 27$

d $40 - 18$ **e** $214 - 32$ **f** $623 - 147$

g $503 - 127$ **h** $5003 - 1236$

WORD PROBLEMS

We will now look at solving some **word problems** where the solution depends on **addition** or **subtraction**. When we answer the problem, we write a **mathematical sentence** which involves numbers.

Example 3 **Self Tutor**

Clive filled a wheelbarrow with 5 kg of potatoes, 3 kg of carrots, 7 kg of onions and 25 kg of pumpkin. What was the total weight of Clive's vegetables?

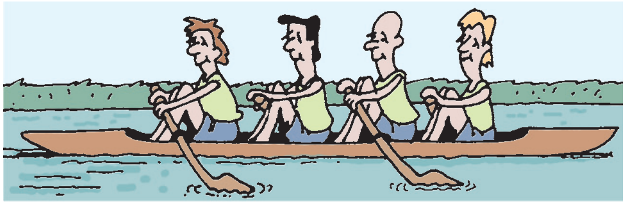
$$\begin{aligned} \text{Total weight} &= 5 + 3 + 7 + 25 \\ &= 40 \text{ kg} \end{aligned}$$

EXERCISE 2A.2

1 Jack bought 4 separate lengths of timber. Their lengths were 5 m, 1 m, 7 m, and 9 m. What was the total length of timber that Jack bought?

2 Xuen bought a Wii for \$255. She also purchased another controller for \$50, a game for \$95, and a bag to store these in for \$32. How much did she pay altogether?

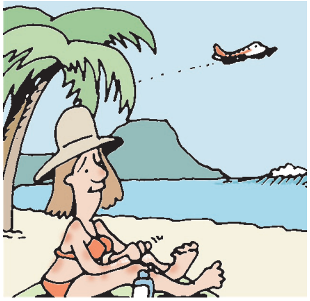
3 Kerry needs to lose some weight to be chosen in a light weight rowing team. He currently weighs 60 kg but needs to weigh 54 kg. How much weight does he need to lose?



4 Stefanie made €72 worth of phone calls in one month. Her parents said they would only pay €31 of this. How much did Stefanie have to pay?

5 Erika had 65 minutes of time left on her prepaid cellphone. She made a 10 minute call to Hiroshi, a 7 minute call to her mother, and a 26 minute call to her boyfriend Marino. How many minutes did she have left after making these calls?

6 Rima went on an overseas trip that required three plane flights. The first flight was 2142 km long, the next one was 732 km long, and the third one was 1049 km long. What was the total distance that Rima flew?



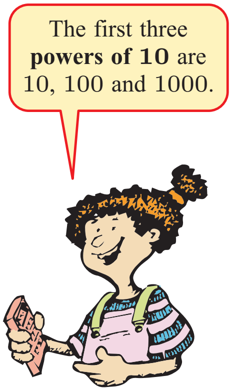
7 Bill measured out a straight line 6010 cm long on the school grounds. He actually went too far. The line should have been 4832 cm long. How much of the line will he need to rub out?

B MULTIPLYING AND DIVIDING WHOLE NUMBERS

MULTIPLYING BY POWERS OF 10

When we multiply by:	10	we make a number 10 times larger
	100	we make a number 100 times larger
	1000	we make a number 1000 times larger.

When we multiply by 100 we add two zeros onto the end of the whole number.



Example 4 Self Tutor

Find: **a** 23×10 **b** 89×100 **c** 381×1000

a 23×10
$= 230$

b 89×100
$= 8900$

c 381×1000
$= 381\,000$

MULTIPLYING LARGER WHOLE NUMBERS

Example 5 — Self Tutor

Find: **a** 67×4 **b** 53×16 **c** 428×54

a
$$\begin{array}{r} 6\,7 \\ \times\ {}_{2}4 \\ \hline 2\,6\,8 \end{array}$$

b
$$\begin{array}{r} 5\,3 \\ \times\ 1\,6 \\ {}_{1}\ \ \\ \hline 3\,1\,8 \\ 5\,3\,0 \\ \hline 8\,4\,8 \end{array}$$

c
$$\begin{array}{r} 4\,2\,8 \\ \times\ {}_{1}5\,4 \\ {}_{1\ 4\ 3} \\ \hline 1\,7\,1\,2 \\ 2\,1\,4\,0\,0 \\ {}_{1}\ \ \ \ \ \\ \hline 2\,3\,1\,1\,2 \end{array}$$

EXERCISE 2B.1

1 Find:

a 50×10	**b** 50×100	**c** 50×1000	**d** 69×100
e 69×1000	**f** $69 \times 10\,000$	**g** 123×100	**h** 246×1000
i 960×100	**j** $49 \times 10\,000$	**k** 490×100	**l** 4900×100

2 Find:

a 24×5	**b** 37×4	**c** 62×8	**d** 53×24
e 27×15	**f** 56×49	**g** 324×45	**h** 642×36
i 274×21	**j** 958×47	**k** 117×89	**l** 368×73

DIVIDING BY POWERS OF 10

When we divide by:	10	we make a number 10 times smaller
	100	we make a number 100 times smaller
	1000	we make a number 1000 times smaller.

When we divide by 100 we remove two zeros from the end of the whole number.

Example 6 — Self Tutor

Find: **a** $34\,000 \div 10$ **b** $34\,000 \div 100$ **c** $34\,000 \div 1000$

a $34\,000 \div 10$
$= 3400$

b $34\,000 \div 100$
$= 340$

c $34\,000 \div 1000$
$= 34$

DIVISION BY A SINGLE DIGIT NUMBER

Example 7 — Self Tutor

Find: **a** $256 \div 4$
b $2502 \div 6$

a
$$\begin{array}{r} 6\ \ 4 \\ 4\,\overline{)\,2\ \ 5\ {}^{1}6} \end{array}$$

b
$$\begin{array}{r} 4\ \ 1\ \ 7 \\ 6\,\overline{)\,2\ \ 5\ {}^{1}0\ {}^{4}2} \end{array}$$

EXERCISE 2B.2

1 Find:

a $2000 \div 10$ **b** $2000 \div 100$ **c** $2000 \div 1000$
d $57\,000 \div 10$ **e** $57\,000 \div 100$ **f** $57\,000 \div 1000$
g $243\,000 \div 10$ **h** $243\,000 \div 100$ **i** $243\,000 \div 1000$
j $45\,000 \div 10$ **k** $45\,000 \div 100$ **l** $45\,000 \div 1000$
m $720\,000 \div 10$ **n** $720\,000 \div 100$ **o** $720\,000 \div 1000$
p $6\,000\,000 \div 10$ **q** $6\,000\,000 \div 100$ **r** $6\,000\,000 \div 1000$

2 Do these divisions:

a $3\overline{)\,4\,2}$ **b** $4\overline{)\,2\,1\,6}$ **c** $8\overline{)\,1\,6\,8}$
d $5\overline{)\,3\,7\,5}$ **e** $7\overline{)\,6\,3\,0\,7}$ **f** $11\overline{)\,6\,8\,0\,9}$

3 Find:

a $24 \div 4$ **b** $125 \div 5$ **c** $312 \div 6$
d $240 \div 5$ **e** $624 \div 3$ **f** $7353 \div 9$

WORD PROBLEMS

Example 8 Self Tutor

How long would a satellite orbiting the earth at 8000 km per hour take to fly 1 million km?

The time taken $= 1$ million $\div 8000$ hours
$= 1\,000\,000 \div 1000 \div 8$ hours $\{8000 = 1000 \times 8\}$
$= 1000 \div 8$ hours $\{$dividing by 1000 first$\}$
$= 125$ hours

Example 9 Self Tutor

Jason works for a supermarket chain. He arranges to buy 217 baskets of fresh cherries at a price of £38 for each basket. How much does the supermarket have to pay?

Total cost $= 217 \times$ £38
$=$ £8246

$$\begin{array}{r} 2\,1\,7 \\ \times \;\; 3\,8 \\ \hline 1\,7\,3\,6 \\ 6\,5\,1\,0 \\ \hline 8\,2\,4\,6 \end{array}$$

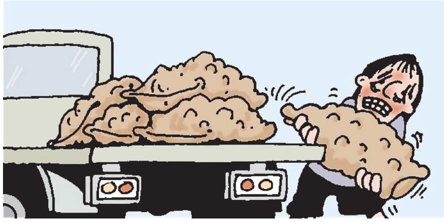
(carries: 1, 2 5 in the multiplication; 1 in the addition)

EXERCISE 2B.3

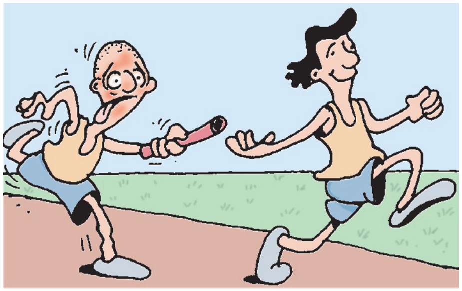
1. A fighter jet travels at 1000 km per hour. How long will it take the jet to fly non-stop for 1 000 000 km?

2. How long would a car, travelling non-stop at 100 kilometres per hour, take to travel a million kilometres?

3. Carlos lifted five 18 kg bags of potatoes onto a truck. How many kg of potatoes did he lift altogether?



4. My three brothers and I received a gift of $320. If we share the money equally amongst ourselves, how much will each person receive?

5. 

A relay team of nine people took 738 minutes to complete a charity relay race. If each team member ran for exactly the same time, for how long did each team member run?

6. This maths textbook is 245 mm long. If I put 10 books end to end, how far would they stretch?

7. 24 people each travelled 28 km to play sport. How far did they travel in total?

8. If I write 8 words per minute, how long will it take me to write 648 words?

9. How long would a motor cyclist, riding non-stop at 50 kilometres per hour, take to travel one million kilometres?

10. A sporting ground has a capacity of 50 000 people. How many capacity crowds would be needed so that 1 000 000 people will have visited the ground?

11. A supermarket chain places an order for 12 000 kg of onions. The onions arrive in bags weighing 50 kg each. How many bags arrive?

12. In the following questions, how many times does the given container need to be filled to make a total of 1 000 000 units?

    a. a fuel tank holds 50 litres
    b. a packet contains 250 sugar cubes
    c. a school hall seats 400 students
    d. a rainwater tank holds 2000 litres
    e. a case is packed with 100 oranges
    f. a carriage holds 80 passengers
    g. a restaurant feeds 125 diners
    h. a DVD rack stores 40 disks

NOTATION

The following symbols are commonly inserted between numbers to show how they compare:

$=$	reads 'is equal to'
$\approx$	reads 'is approximately equal to'
$>$	reads 'is greater than'
$<$	reads 'is less than'

EXERCISE 2B.4

1 In each of the following, replace □ by $=$ or $\approx$:

- **a** $375 + 836$ □ 1200
- **b** 79×8 □ 640
- **c** $978 - 463$ □ 515
- **d** $7980 \div 20$ □ 400
- **e** $455 + 544$ □ 999
- **f** 50×400 □ $20\,000$
- **g** $2000 - 1010$ □ 990
- **h** $3000 \div 300$ □ 10

2 In each of the following, replace Δ by $>$ or $<$:

- **a** $5268 - 3179 \ \Delta \ 4169$
- **b** $29 \times 30 \ \Delta \ 900$
- **c** $672 + 762 \ \Delta \ 1444$
- **d** $720 \div 80 \ \Delta \ 8$
- **e** $20 \times 80 \ \Delta \ 160$
- **f** $700 \times 80 \ \Delta \ 54\,000$
- **g** $5649 + 7205 \ \Delta \ 12\,844$
- **h** $6060 - 606 \ \Delta \ 5444$

C TWO STEP PROBLEM SOLVING

Sometimes we need to perform more than one operation to solve a problem. In these situations it may be easier to solve the problem in two steps.

For example, how much change would you receive from €50 if you bought three bags of potatoes at €14 a bag?

Step 1: Total cost of potatoes is €14 × 3 = €42

Step 2: So, the change is €50 − €42 = €8

Example 10 **Self Tutor**

Each week Clancy is paid \$350 as a retainer and \$65 for each vacuum cleaner he sells. How much does Clancy earn if he sells 13 vacuum cleaners in a week?

Money from sales = \$65 × 13
= \$845

So, the total earned = \$845 + \$350
= \$1195

$$\begin{array}{r} 65 \\ \times\ 13 \\ \hline 195 \\ 650 \\ \hline 845 \end{array}$$

EXERCISE 2C

1 Deloris bought a shirt costing $29 and a pair of jeans costing $45. How much change did she get from $100?

2 Rahman bought three T-shirts costing RM42 each and a pair of shoes costing RM75. Find the total cost of his purchases.

3 Maria bought five 3 kilogram bags of oranges. The numbers of oranges in the bags were 10, 11, 12, 12, and 10. Find the average number of oranges in a bag.

4 Mafinar had a herd of 183 goats. He put 75 in his largest paddock and divided the rest equally between two smaller paddocks. How many goats were put in each of the smaller paddocks?

5 George had £436 in his bank account. He was given £30 cash for his birthday. How much money did he have left if he bought a bicycle costing £455?

6 The cost of placing an advertisement in the local paper is €10, plus €4 for each line of type. If my advertisement takes 5 lines, how much will I pay?

7 How much would June pay for 8 iced buns if 3 buns cost her 54 cents?

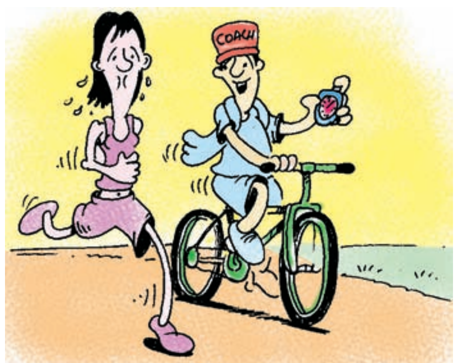
8 Marcia saved $620 during the year and her sister saved twice that amount. How much money did they save in total?

9 Anastasia had €463 in her savings account and decided to bank €20 a week for 14 weeks. How much was in the account at the end of that time?

10 Juen worked 45 hours at one job for €24 an hour, and 35 hours at another for €26 an hour. He hoped to earn €2000 over this period. Did he succeed?

11 Tony's wages for the week were $496. He was also paid for 3 hours overtime at $18 per hour. How much did he earn in total?

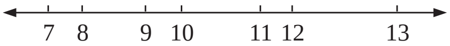
12 Alicia ran 8 km each day from Monday to Saturday, and 12 km on Sunday. How far did she run during the week?



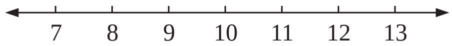
13 A plastic crate contains 100 boxes of ball point pens. The boxes of pens each weigh 86 grams. If the total mass of the crate and pens is 9200 g, find the mass of the crate.

D NUMBER LINES

A line on which **equally** spaced points are marked is called a number line.

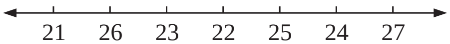


not a number line

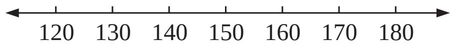


correct number line

A number line allows the **order** and **relative positions** of numbers to be shown.



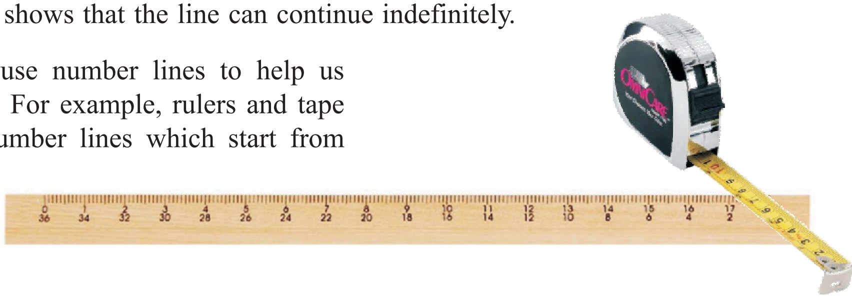
order and positions not relative



order and relative positions

The arrow head shows that the line can continue indefinitely.

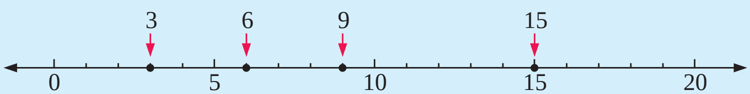
We sometimes use number lines to help us measure things. For example, rulers and tape measures are number lines which start from zero.

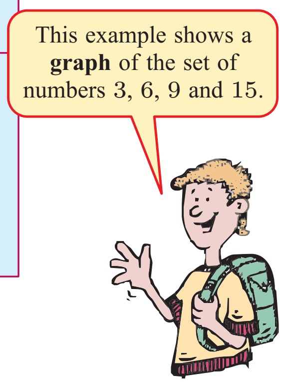


Example 11 — Self Tutor

Show the numbers 9, 15, 3 and 6 with dots on a number line.

We arrange 9, 15, 3 and 6 in ascending order: 3, 6, 9, 15.





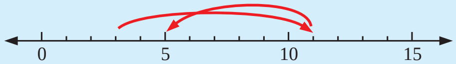
Number lines can also be used to show the four basic **operations** of adding, subtracting, multiplying, and dividing with whole numbers.

Example 12 — Self Tutor

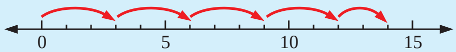
Perform the following operations on a number line:

a $3 + 8 - 6$ **b** $4 \times 3 + 2$ **c** $23 \div 5$

a $3 + 8 - 6 = 5$



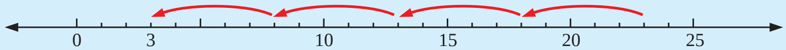
b $4 \times 3 + 2 = 14$



DEMO

c $23 \div 5$

Choose a suitable scale. $\div$ is the opposite of $\times$, so we start from the right side.



$23 \div 5 = 4$ with a remainder of 3.

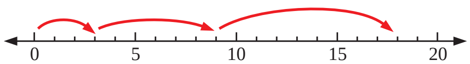
EXERCISE 2D

1 Use dots to show the following numbers on a number line:

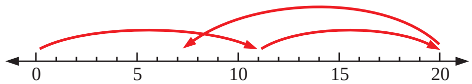
a 9, 4, 8, 2, 7
b 14, 19, 16, 18, 13
c 70, 30, 60, 90, 40
d 250, 75, 200, 25, 125
e 4000, 3000, 500, 2500, 1500

2 What operations do the following number lines show? Give a final answer.

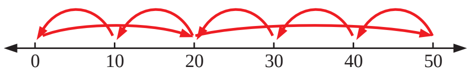
a



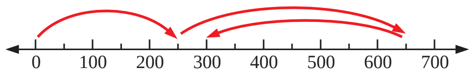
b



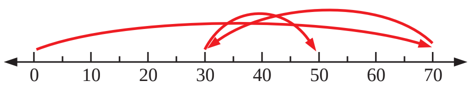
c



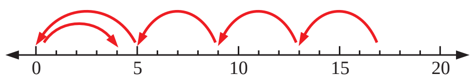
d



e



f



3 Draw a number line and show the following operations. Give a final answer.

a $9 + 8 - 6$
b $2 + 4 + 8 - 2$
c $40 + 70 + 90 - 50$
d $55 + 60 + 75 - 40$
e $3 \times 9 - 8$
f $4 \times 6 \div 5$

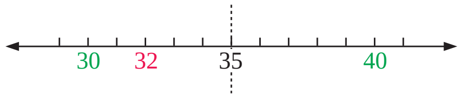
E ROUNDING NUMBERS

Often we are not really interested in the *exact* value of a number, but we want a reasonable **estimate** or **approximation** for it.

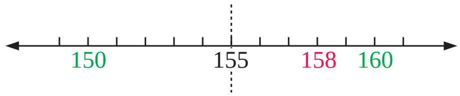
For example, suppose there were 306 competitors at an athletics carnival. We might say "there were about 300 competitors" since 300 is a good approximation for 306. In this case 306 has been "rounded" to the nearest hundred.

In order to approximate a number to the nearest ten, we start by finding the multiples of the ten on each side of the number.

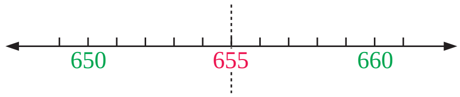
- 32 lies between 30 and 40.
  It is nearer to 30, so
  32 is approximately 30.
  We say 32 is **rounded down** to 30.



- 158 lies between 150 and 160.
  It is nearer to 160 than to 150, so
  158 is approximately 160.
  We say 158 is **rounded up** to 160.



- 655 lies between 650 and 660.
  It is half way between 650 and 660.
  In this case we agree to round up.
  So, 655 is approximately 660, to the nearest 10.
  We say 655 is **rounded up** to 660.



The rules for rounding off are:

- If the digit after the one being rounded off is **less than 5**, i.e., 0, 1, 2, 3 or 4, then we round **down**.
- If the digit after the one being rounded off is **5 or more**, i.e., 5, 6, 7, 8, or 9, then we round **up**.

Example 13 Self Tutor

Round off to the nearest 10:

a 63 **b** 475 **c** 3029

a 63 lies between 60 and 70.
It is nearer to 60, so we round down.
63 is approximately 60.

b 475 lies halfway between 470 and 480, so we round up.
475 is approximately 480.

c 3029 lies between 3020 and 3030.
It is nearer to 3030, so we round up.
3029 is approximately 3030.

EXERCISE 2E.1

1 Write the nearest multiple of 10 on each side of the number:

a 21 **b** 46 **c** 65 **d** 82 **e** 93 **f** 199
g 461 **h** 785 **i** 1733 **j** 2801 **k** 3947 **l** 6982

2 Which of the two outer numbers is nearer to the number in bold type?

a 30, **38**, 40 **b** 70, **71**, 80 **c** 90, **95**, 100
d 130, **132**, 140 **e** 450, **457**, 460 **f** 730, **735**, 740
g 810, **818**, 820 **h** 1220, **1225**, 1230 **i** 6740, **6743**, 6750

3 Round off to the nearest 10:

a 17	**b** 35	**c** 53	**d** 71	**e** 97	**f** 206
g 311	**h** 502	**i** 888	**j** 3659	**k** 7444	**l** 8705
m 9606	**n** 14 075	**o** 30 122	**p** 47 777	**q** 69 569	**r** 70 099

Example 14 **Self Tutor**

Round off to the nearest 100:

a 63 **b** 249 **c** 1655

a 63 lies between 0 and 100.
It is nearer to 100, so we round up.
63 is approximately 100.

b 249 lies between 200 and 300.
It is nearer to 200, so we round down.
249 is approximately 200.

c 1655 lies between 1600 and 1700.
It is nearer to 1700, so we round up.
1655 is approximately 1700.

To approximate a number to the nearest hundred, look at the multiples of one hundred on each side of the number.

4 Write the nearest multiple of 100 on each side of the number:

a 89 **b** 342 **c** 755 **d** 1694 **e** 3050 **f** 6219

5 Which of the outer numbers is nearer to the number in bold type?

a 500, **547**, 600 **b** 7600, **7631**, 7700 **c** 2900, **2985**, 3000

6 Round off to the nearest 100:

a 75	**b** 211	**c** 572	**d** 793	**e** 1050	**f** 2684
g 6998	**h** 13 208	**i** 27 660	**j** 38 457	**k** 55 443	**l** 85 074

Example 15 **Self Tutor**

Round off to the nearest 1000:

a 932 **b** 4500 **c** 44 482

a 932 lies between 0 and 1000.
It is nearer to 1000, so we round up.
932 is approximately 1000.

b 4500 lies midway between 4000 and 5000,
so we round up.
4500 is approximately 5000.

c 44 482 lies between 44 000 and 45 000.
It is nearer to 44 000, so we round down.
44 482 is approximately 44 000.

To approximate a number to the nearest thousand, look at the multiples of one thousand on either side of the number.

7 Round off to the nearest 1000:

a 834	**b** 495	**c** 1089	**d** 5485
e 7800	**f** 6500	**g** 9990	**h** 9399
i 13 095	**j** 7543	**k** 246 088	**l** 499 859

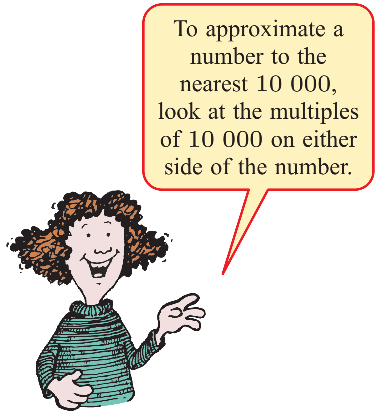
Example 16 Self Tutor

Round off to the nearest 10 000:

a 42 635 **b** 99 981

a 42 635 lies between 40 000 and 50 000.
It is nearer to 40 000, so we round down.
42 635 is approximately 40 000.

b 99 981 lies between 90 000 and 100 000.
It is nearer to 100 000, so we round up.
99 981 is approximately 100 000.



8 Round off to the nearest 10 000:

a 18 124	**b** 47 600	**c** 54 500	**d** 75 850
e 89 888	**f** 52 749	**g** 90 555	**h** 99 776

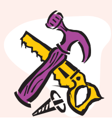
9 Round off to the nearest 100 000:

a 181 000	**b** 342 000	**c** 654 000	**d** 709 850
e 139 888	**f** 450 749	**g** 290 555	**h** 89 512

10 Round off to the accuracy given:

- **a** 37 musicians in an orchestra (to the nearest 10)
- **b** 55 singers in a youth choir (to the nearest 10)
- **c** a payment of €582 (to the nearest €10)
- **d** a tax bill of $4095 (to the nearest $10)
- **e** a load of bricks weighs 687 kg (to the nearest 100 kg)
- **f** a car costs $24 995 (to the nearest $100)
- **g** the journey was 35 621 km (to the nearest 100 km)
- **h** the circumference of the earth is 40 008 km (to the nearest 10 000 km)
- **i** the cost of a house is £463 590 (to the nearest £10 000)
- **j** the population of Manhattan is 1 537 195 (to the nearest 100 000)

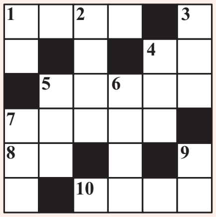
PUZZLE **ROUNDING WHOLE NUMBERS**



Click on the icon to obtain a printable version of this puzzle.

Round the numbers to the given amount.

PUZZLE



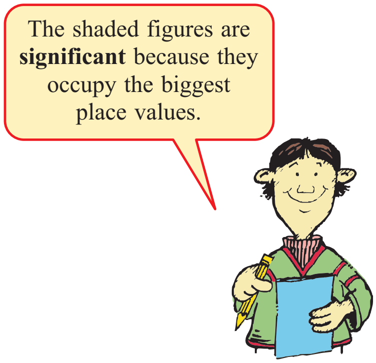
Across

1	4866	to the nearest 10
4	64	to the nearest 10
5	10 938	to the nearest 100
7	27 194	to the nearest 1000
8	85	to the nearest 10
10	2629	to the nearest 1000

Down

1	44	to the nearest 10
2	7247	to the nearest 100
3	751	to the nearest 100
4	550	to the nearest 100
5	165	to the nearest 10
6	8500	to the nearest 1000
7	293	to the nearest 10
9	45	to the nearest 10

ROUNDING TO A NUMBER OF FIGURES



When we round to a number of **significant figures**, this is the number of digits from the left hand side that we believe are important.

For example, if we round

37 621 to **two** significant figures, we notice that 37 621 is closer to 38 000 than it is to 37 000.

So, $37\,621 \approx 38\,000$ (to 2 significant figures)

Example 17 Self Tutor

Round:

a 3442 to one significant figure **b** 25 678 to two significant figures.

a 3442 is closer to 3000 than to 4000, so $3442 \approx 3000$.

b 25 678 is closer to 26 000 than to 25 000, so $25\,678 \approx 26\,000$.

EXERCISE 2E.2

1 Round off to one significant figure:

a 79 **b** 298 **c** 392 **d** 351
e 978 **f** 2666 **g** 6833 **h** 59 500

2 Round off to two significant figures:

a 781 **b** 267 **c** 750 **d** 339
e 1566 **f** 6649 **g** 8750 **h** 34 621

3 Round off to the accuracy given:

a €46345 (to one significant figure)
b a distance of 8152 km (to two significant figures)
c a weekly salary of £475 (to one significant figure)
d last year a company's profit was $307 882 (to three significant figures)
e the population of a town is 6728 (to two significant figures)
f the number of people at a football match is 32 688 (to two significant figures)

RESEARCH — ROUNDING

Research the following and round off to the accuracy requested. Record the name and date of publication of the reference (book or magazine title, or internet URL), the value given in the reference and your rounded value.

1. The population of your nearest capital city (nearest 10 000).
2. The speed of light (nearest 1000 km per hour).
3. The railway distance between Cairo and Luxor (nearest 100 km).
4. The population of Zimbabwe (nearest 100 000).
5. The population of the world (nearest billion).
6. The distance to the sun (nearest million km).

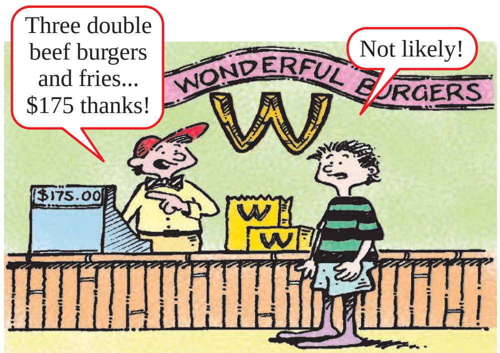
F ESTIMATION AND APPROXIMATION

Calculators and computers are part of everyday life. They save lots of time, energy and money by the speed and accuracy with which they complete different operations.

However the people operating the computers and calculators often make mistakes when keying in the information.

It is therefore very important that we can make an **estimate** of what the answer should be. An estimate is not a guess. It is a quick and easy **approximation** of the correct answer.

By making an estimate we can tell if the computed answer is **reasonable**.



ONE FIGURE APPROXIMATIONS

A quick way to estimate an answer is to use a **one figure approximation**. To do this we use the following rules:

- Leave single digit numbers as they are.
- Round all other numbers to single figure approximations.

For example, $3789 \times 6 \approx 4000 \times 6$
$\approx 24\,000$

Example 18 Self Tutor

Estimate the product: **a** 29×8 **b** 313×4

a 29×8
$\approx 30 \times 8$ {29 is rounded to 30, 8 is kept as 8}
≈ 240

b 313×4 {313 is rounded to 300, 4 is kept as 4}
$\approx 300 \times 4$
≈ 1200

EXERCISE 2F.1

1 Estimate these products:

a 89×3 **b** 57×9 **c** 62×6 **d** 28×8
e 113×7 **f** 6895×6 **g** 8132×8 **h** $29\,898 \times 9$

2 Estimate the cost of:

a 59 pens at $4 each **b** 77 books at $8 each
c 208 staplers at $7 each **d** 4079 rolls of tape at $9 each

Example 19 Self Tutor

Estimate the product: 511×38

511×38
$\approx 500 \times 40$ {511 is rounded to 500 and 38 is rounded to 40}
$\approx 20\,000$ {$5 \times 4 = 20$ and then add on the 3 zeros}

3 Using one figure approximations to estimate:

a 39×51 **b** 58×43 **c** 69×69 **d** 82×81
e 213×18 **f** 391×22 **g** 189×41 **h** 189×197

4 Estimate the cost of:

a 48 books at $19 each **b** 82 calculators at €28 each
c 69 concert tickets at £48 each **d** 89 CD players at $59 each.

Example 20 Self Tutor

Estimate the product: 414×692

414×692
$\approx 400 \times 700$ {414 is rounded to 400 and 692 is rounded to 700}
$\approx 280\,000$ {$4 \times 7 = 28$ and then add on the 4 zeros}

5 Use one figure approximations to estimate:

a 192×304 **b** 4966×41 **c** 607×491 **d** 885×990

e 3207×8 **f** 1966×89 **g** $39\,782 \times 5$ **h** 3814×7838

6 Estimate the cost of:

a 79 computers at \$1069 each **b** 683 game consoles at €198 each

c 388 monitors at \$578 each **d** 4138 tennis tickets at £59 each.

Example 21 Self Tutor

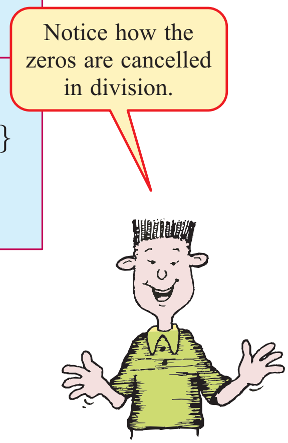
Find the approximate value of $4078 \div 81$.

$4078 \div 81$

$\approx 4000 \div 80$ {rounding each number to one significant figure}

$\approx 400 \div 8$ {dividing both numbers by 10}

≈ 50



7 Use one figure approximations to estimate:

a $498 \div 5$ **b** $897 \div 3$ **c** $78\,916 \div 4$

d $5908 \div 31$ **e** $697 \div 72$ **f** $8132 \div 41$

g $3164 \div 98$ **h** $6075 \div 29$ **i** $6177 \div 104$

j $9091 \div 58$ **k** $4862 \div 195$ **l** $3956 \div 1970$

8 **a** Estimate how much each person would get if a €489 555 lottery prize is equally divided amongst 18 friends

b Estimate how long it would take Andreas to type a 7328 word article for a magazine if he types at 68 words per minute.

9 One of the numbers given in a rectangle is correct. Use one figure estimation to find the correct answer for:

a 489×29	**A** 9681	**B** 14 181	**C** 134 081
b 317×482	**A** 1 258 094	**B** 83 694	**C** 152 794
c 2814×614	**A** 1 727 796	**B** 942 096	**C** 17 207 796
d $3172 \div 52$	**A** 71	**B** 61	**C** 81

10 In each of the following, use one figure approximation to estimate the answer.

a Helga can type at a rate of 58 words per minute. Estimate the number of words she can type in 38 minutes.

b An apple orchard contains 72 rows of apple trees with 38 trees per row.

i Estimate the number of apple trees in the orchard.

ii Suppose each tree has yields on average of 278 apples. Estimate the total number of apples picked from the orchard.

c A wine vat holds 29 675 litres of wine. The wine is siphoned into barrels for ageing. If each barrel holds 1068 litres of wine, how many barrels are needed?

d Michael wants to tow his boat on a trailer between two cities which are 468 km apart. It is a big boat so his car can only average 52 km per hour while towing. How long will the journey take Michael?

e A concert hall has 87 rows of seats, each with 58 seats. The cost per seat is $32.

  i Estimate the total number of seats

  ii Estimate the total income if all seats are sold.

ESTIMATION OF NUMBERS OF OBJECTS

When we conduct a **counting** process, for example, count the number of sheep in a field, or the number of people in a crowd, we do not always need to know the *exact* answer. We can estimate the answer using the following method:

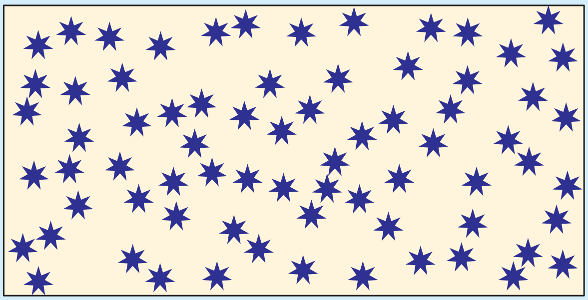
Step 1: Divide the area into equal parts.

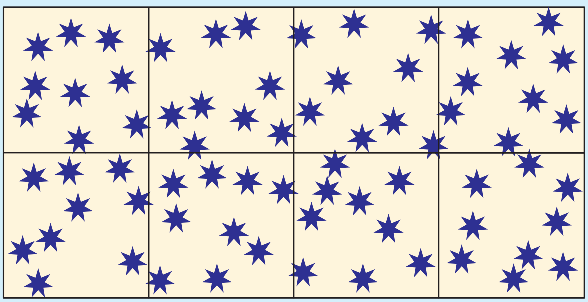
Step 2: Count the number of objects in one part.

Step 3: Multiply the number in one part by the total number of parts.

Example 22 Self Tutor

Estimate the number of stars on the poster:





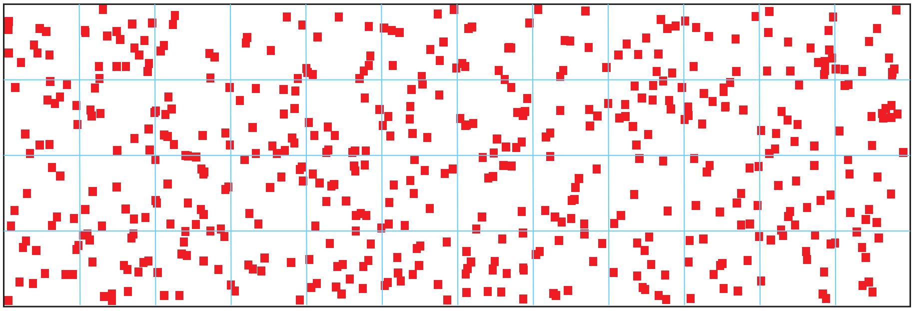
In the top left part there are 9 stars. The number of stars is:
stars in 1 part $\times$ number of parts $= 9 \times 8 = 72$ stars.

We estimate that there are 72 stars on the poster.

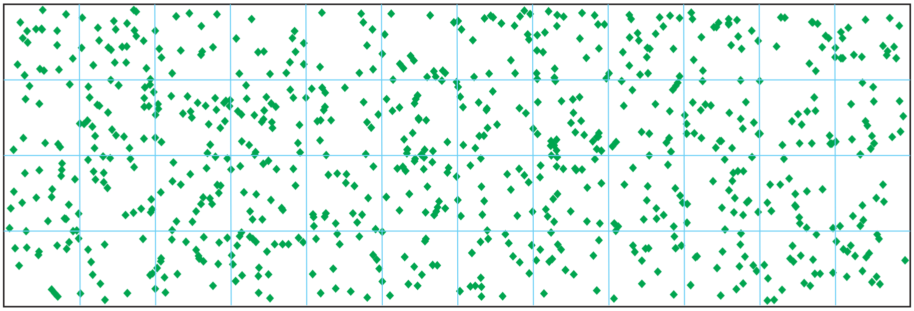
EXERCISE 2F.2

1 Estimate the number of objects in:

a



b



2 Click on the icon to load more objects to be estimated. Play with the software to improve your estimation skills. Note that the coloured square which appears is chosen at random by the computer.

KEY WORDS USED IN THIS CHAPTER

- addition
- estimate
- round down
- approximation
- multiplication
- round up
- division
- quotient
- subtraction

TENNIS RANKINGS

Areas of interaction:
Human ingenuity, Approaches to learning

REVIEW SET 2A

1 Find:

a $46 + 178$ **b** $311 - 39$

2 Damien bought some shorts for \$39 and a polo shirt for \$32. How much change did he get from \$100?

3 Would €200 be enough to pay for a €69 budget flight to Munich, a €114 return ticket, and an €18 ticket to the football? Show your working.

4 Find:

a 34×100 **b** $59\,000 \div 1000$

5 Find:

a 29×18 **b** $768 \div 6$

6 Are the following statements true or false?

a $1\,000\,000 = 100 \times 100 \times 100$ **b** $10\,000\,000 = 10 \times 100 \times 1000$

c $468\,751 > 468\,577$ **d** $1000 \times 4612 \neq 2306 \times 2000$

7 Replace $\square$ by $>$ or $<$:

a $60 \times 1000 \ \square\ 59\,000$ **b** $499\,994 \ \square\ 499\,949$

8 Find the cost of 24 opera tickets at £112 each.

9 Nine office workers form a syndicate and buy lottery tickets. If they win €4275, how much does each person receive?

10 Kathryn was paid wages of $608 for the week. She also earned $24 an hour for 5 hours overtime. How much did Kathryn earn in total?

11 The population of Australia is approximately 20 000 000 and the population of New Zealand is approximately 4 000 000.

a What is the difference in population size?

b How many times larger is the population of Australia than the population of New Zealand?

12 How long would an aeroplane flying non-stop at 250 km per hour take to fly 1 million kilometres? Give your answer in days and hours.

13 Round off:

a 35 to the nearest 10 **b** 4384 to the nearest 1000

c 463 994 to one significant figure.

14 Estimate the cost of 28 books at £19 each.

15 Find the approximate value of 197×234.

16 Estimate the mass of one carton of cat food if a crate containing 196 cartons weighs 402 kilograms.

REVIEW SET 2B

1 Find:

a $206 + 47 + 195$ **b** $3040 - 197$

2 Sally bought a jacket for €95, a pair of shoes for €78, and a scarf for €19. How much did Sally pay in total?

3 During 3 days of practice, a golfer hit 24 more balls each day than the previous day. How many golf balls did he hit in the 3 days if he hit 376 on the first day?

4 Find:

a 532×100 **b** $46\,000 \div 1000$

5 Are the following statements true or false?

a $4\,863\,663 < 4\,863\,363$ **b** $8703 - 6679 = 2124$

c $504 \times 1998 \approx 1\,000\,000$ **d** $3036 \div 6 = 506$

6 Replace $\square$ by $=$ or $\approx$:

a $237 + 384 \square 620$ **b** $8020 \div 20 \square 400$

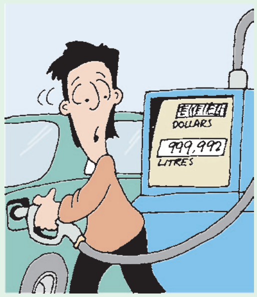
7 Find:

a 23×39 **b** $408 \div 8$

8 If Derek takes 6 minutes to construct one section of fence, how many sections can he construct in one hour?

9 A recycle depot pays 5 cents for each empty bottle. How much would a school's fundraising committee get if it collects and fills 154 crates with two dozen bottles in each?

10 A fuel tank holds 50 litres of fuel. How many times would the tank need to be filled from empty in order to use 1 000 000 litres of fuel?



11 **a** Round £39 758 to the nearest £100.

b Round 56 082 to the nearest 10 000.

12 Round off to the accuracy given:

a a phone bill for $82 (to one significant figure)

b the number of people in a sporting ground is 16 310 (to two significant figures).

13 Estimate the total cost of 3 dozen pizzas costing $9 each.

14 Find an approximate value for 687×231.

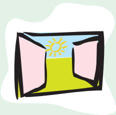
15 The area of Africa is approximately 30 271 000 km^2 and the area of Europe is approximately 10 404 000 km^2. Approximately how many times larger than Europe is Africa?

Chapter 3

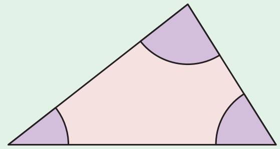
Points, lines and angles

Contents:

OPENING PROBLEM



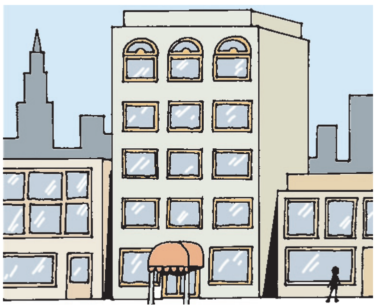
A **triangle** is a closed shape made from three straight sides.



- Is there any special property which the angles of a triangle have?
- If the triangle has two sides which are equal in length, can we say anything special about the angles of this triangle?

GEOMETRICAL SHAPES

Buildings and structures such as bridges and towers contain different geometrical shapes for visual appeal or strength. When we look at buildings we see many square corners, and windows which are rectangles. Triangles are often seen in bridges.



A POINTS AND LINES

DISCUSSION WHAT IS A POINT?

In groups of 4 or 5, for at most 10 minutes, discuss the following questions:

1. What is meant by a *point*?
2. Give examples of things which could be used to represent a point.
3. How small can a point be?

Each group could make a brief report to the class.

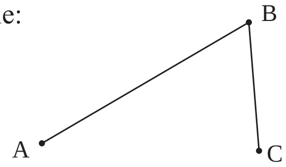
POINTS IN GEOMETRY

Good examples of points in the classroom are:

- a corner of the room where two walls and the floor all meet
- a speck of dust in the room at a particular instant in time.

In **geometry**, a point is represented by a small dot. To help identify it we name it with a capital letter.

For example:



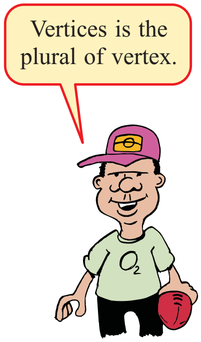
The letters A, B and C are useful because we can identify then refer to a particular point.

We can make statements like:
"the distance from A to B is ....." or "the angle at B measures .....".

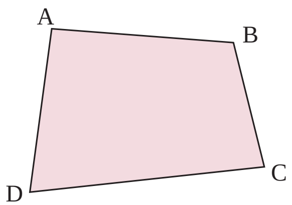
In mathematics: a **point** marks a position and does not have any size.

In order to see where a point is, we use a **dot** which has both size and colour.

FIGURES



A **figure** is a drawing which shows things we are interested in.



The figure alongside contains four points which have been labelled A, B, C and D.

These corner points are known as **vertices**.

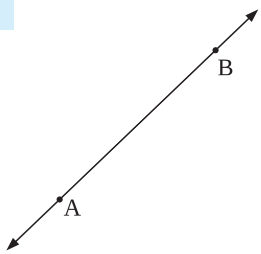
Vertices is the plural of vertex, so point B is a **vertex** of the figure.

LINES

A **straight line**, usually just called a **line**, is a continuous infinite collection of points with no beginning and no end.

This line passes through points A and B. It continues indefinitely in both directions, but because we cannot draw a line of infinite length, we use arrow heads to show it continues endlessly.

A line actually has no width, but in order to see it we give it thickness.



DISCUSSION — LINES

What to do:

Discuss the following questions:

1. How many different straight lines could be drawn through the single point A?
2. Suppose A and B are two separate points. How many straight lines could be drawn which pass through both A and B?
3. Suppose P, Q and R are three different points. How many straight lines can be drawn which pass through all three points P, Q and R? Explain your answer.

Each group could report their findings to the class.

NOTATION

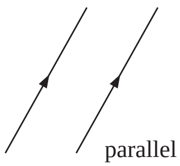
(AB) is the **line** which passes through A and B and continues indefinitely in both directions.

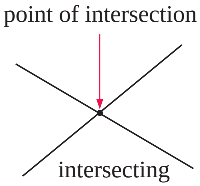
[AB] is the **line segment** which joins the two points A and B. It is only a part of the line (AB).

[AB) is the **ray** which starts at A, passes through point B, and continues on indefinitely.

PARALLEL AND INTERSECTING LINES

In Mathematics, a **plane** is a flat surface like a table top or a sheet of paper. Two straight lines down a plane are either **parallel** or **intersecting**.



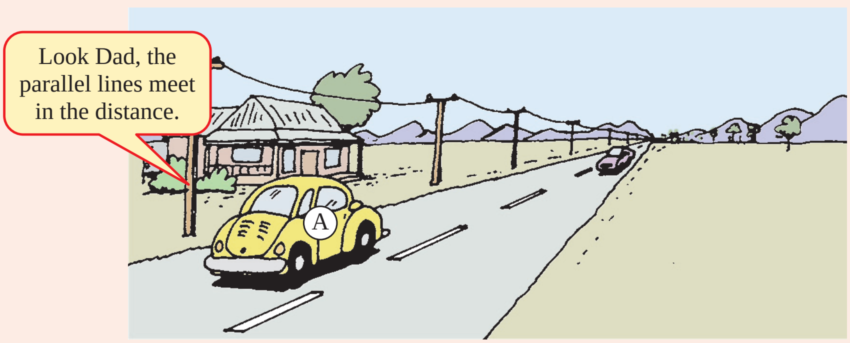


Parallel lines are lines which are always a fixed distance apart and never meet.

DISCUSSION

The edges of a long straight road are parallel lines. To the people in car A, the parallel lines appear to meet in the distance.

Discuss the picture. Does it represent the real world?



EXERCISE 3A

1 Give two examples in the classroom which indicate:

a a point **b** a line

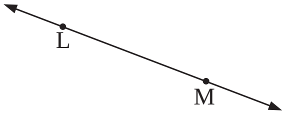
2 In geometry, what is meant by:

a a vertex **b** a point of intersection **c** parallel lines?

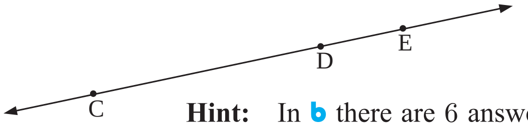
Draw diagrams to illustrate each.

3 Give all ways of naming the straight lines shown:

a

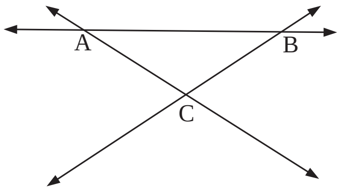


b



Hint: In **b** there are 6 answers.

4



Find the intersection of:

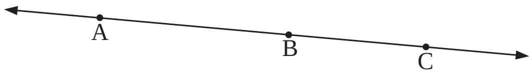
a lines (AB) and (BC)

b lines (CB) and (CA).

5 Find the intersection of:

a [AB] and [BC]

b [AB] and [AC].



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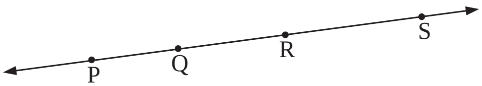


Find the intersection of:

a [PQ] and [QR]

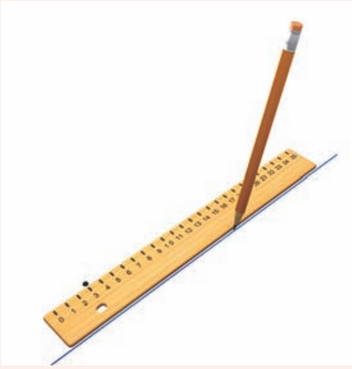
b [QR] and [PS]

c [PR] and [QS].

ACTIVITY 1 STRAIGHT LINE SURPRISES

Part 1:

You will need: a sheet of blank paper, a ruler and a sharp pencil.



What to do:

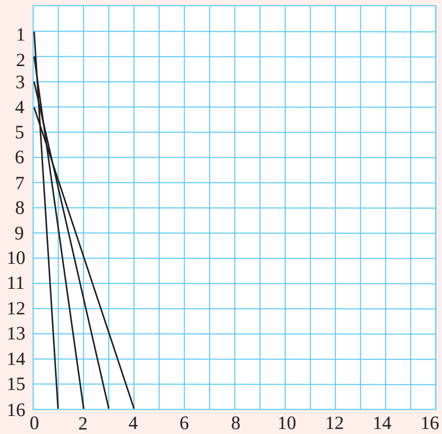
1. Mark a point somewhere near the centre of the paper.
2. Line up one edge of your ruler so it passes through the point. Draw a line along the *other* edge of the ruler across the paper.
3. Change the position of the ruler, keeping one edge passing through the point. Draw a second line so that it intersects with your first line.
4. Change the position of the ruler again, keeping one edge passing through the point. The other edge will allow you to form a triangle. Draw the third line.
5. Rule lots more lines like this.
6. Describe what happens to the shape formed by the intersecting lines as more lines are drawn.
7. Why is this shape forming?

DEMO

Part 2:

You will need: a sheet of 5 mm graph paper, a ruler, and a sharp pencil.

What to do:



1 On the graph paper draw a horizontal base line. Mark the numbers from 0 to 16 on it as shown in the diagram on the next page.

2 Draw a vertical line at 0. Mark on it the numbers from 1 to 16 at the intersections with the horizontal lines, as shown.

3 Rule a straight line from 1 to 1 as shown.

4 Rule a straight line from 2 to 2 as shown. Repeat this process until all the points have been joined.

DEMO

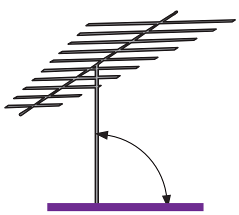
5 Now draw a vertical line at 16 on the base line and repeat the pattern.

6 A real challenge is to turn the page upside down and repeat the pattern so that you have drawn 4 sets of straight lines.

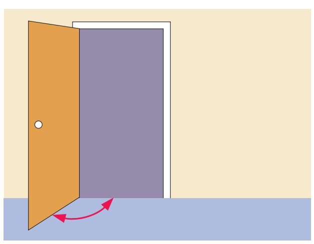
B ANGLES

Whenever two lines or edges meet, an **angle** is formed between them.

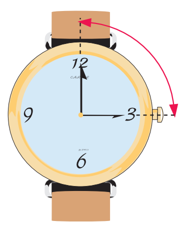
The angle between the pole and the ground.



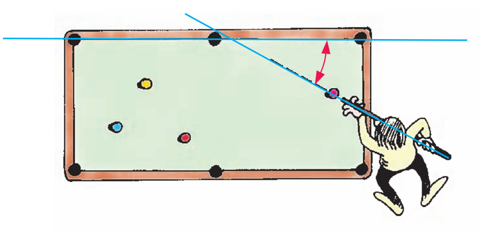
The angle between the wall and the door.



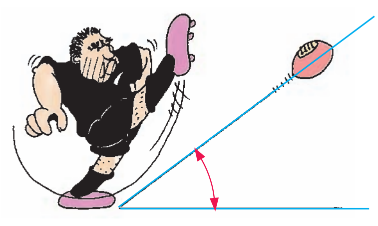
The angle between the hands of a clock.



The angle between the line of the ball's motion and the edge of the cushion.

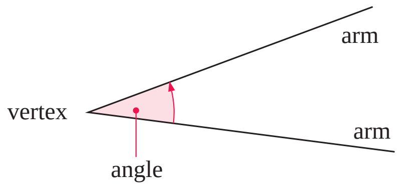


The angle between the ground and the direction of the ball.



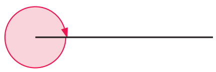
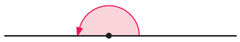
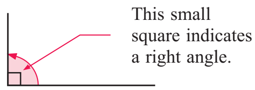
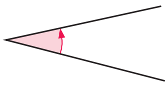
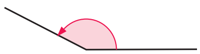
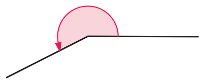
An **angle** is made up of two arms which meet at a point called the **vertex**.

The **size** of the angle is measured by the amount of turning or rotation from one arm to the other.



CLASSIFYING ANGLES

We use the following names to classify angles according to their sizes:

Revolution	**Straight Angle**	**Right Angle**
		
One complete turn.	A $\frac{1}{2}$ turn.	A $\frac{1}{4}$ turn.
Acute Angle	**Obtuse Angle**	**Reflex Angle**
		
Less than a $\frac{1}{4}$ turn.	Between a $\frac{1}{4}$ and $\frac{1}{2}$ turn.	Between a $\frac{1}{2}$ and 1 turn.

MEASURING ANGLES

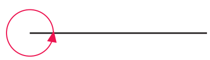
In order to accurately find the size or measure of an angle, we need a unit of measurement. The unit we will use is the **degree**. It was decided that there would be 360 degrees in a full turn. 360 was probably chosen because it can be divided by 2, 3, 4, 5, 6, 8, 9, 10, 12, and 15, to give whole number answers.

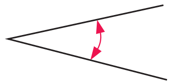
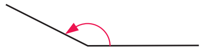
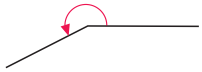
So, a **straight angle** or **half turn** will measure $\frac{1}{2}$ of 360 degrees, or 180 degrees.

We write this as 180^o. ← This small circle is used to indicate degrees and saves us writing the full word.

A **right angle** or **quarter turn** will measure $\frac{1}{4}$ of 360^o, or 90^o.

We can now classify angles in **degree measure**:

Name	*Figure*	*Degrees*
Revolution		360^o
Straight angle		180^o
Right angle		90^o

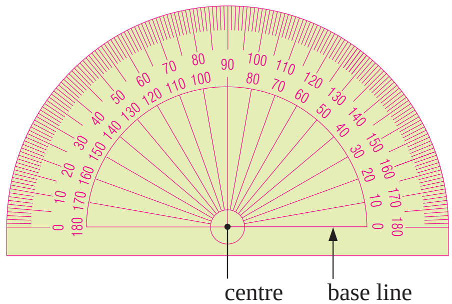
Name	*Figure*	*Degrees*
Acute angle		between 0^o and 90^o
Obtuse angle		between 90^o and 180^o
Reflex angle		between 180^o and 360^o

MEASURING DEVICES

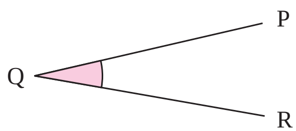
In order to measure angles we use a **protractor** with tiny 1^o markings on it.

To use a protractor to measure angles we:

- place it so its centre is at the angle's vertex and 0^o lies exactly on one arm
- start at 0^o and follow the direction the angle turns through to reach the other arm.



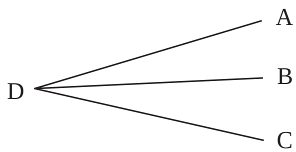
NAMING ANGLES



The angle at Q is written as $P\widehat{Q}R$ or $R\widehat{Q}P$.

Notice that Q is in the middle.

This method of writing angles is called **three point notation**.



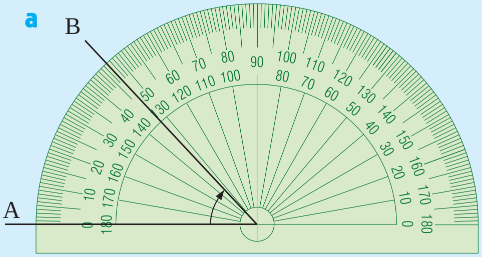
The figure alongside shows why three point notation is essential.

If we say 'the angle at D' we could be referring to $A\widehat{D}B$, $A\widehat{D}C$ or $B\widehat{D}C$.

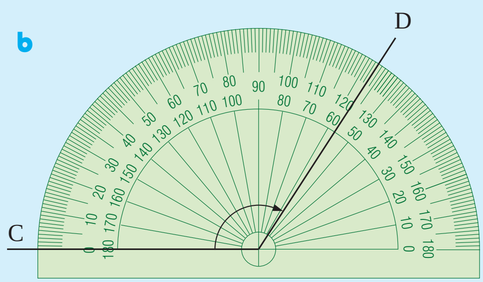
Example 1 — **Self Tutor**

Measure these angles:

a



b



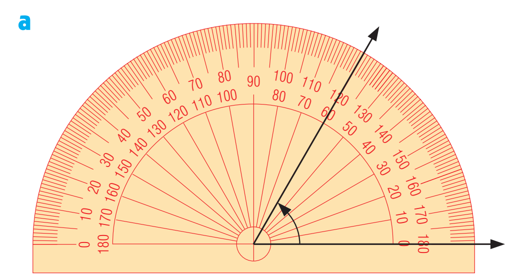
a $A\widehat{O}B$ has size 47^o.

b $C\widehat{O}D$ has size 123^o.

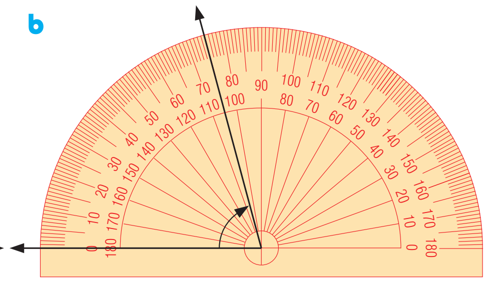
EXERCISE 3B

1 What is the size of the angle being measured?

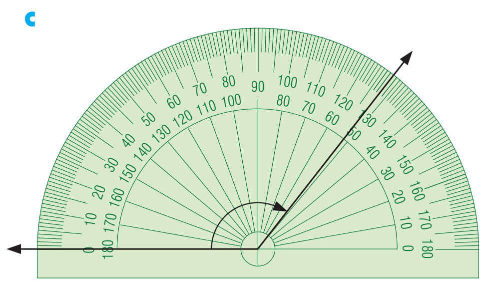
a



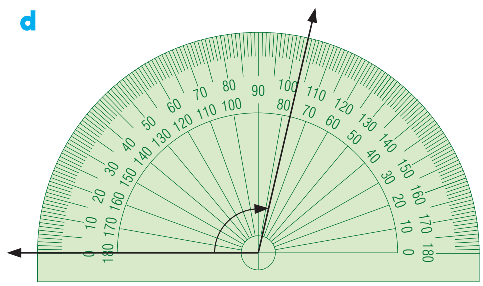
b



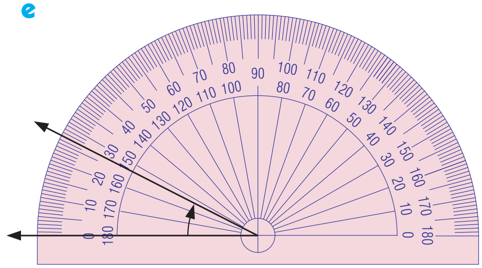
c



d



e



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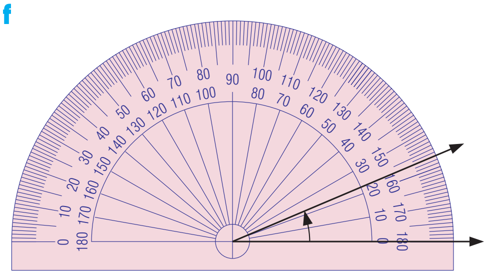

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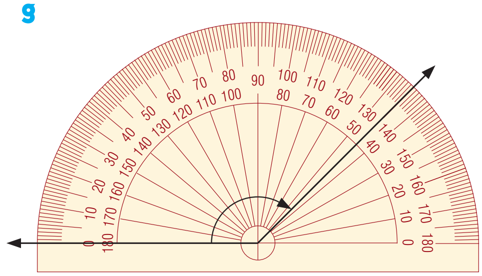

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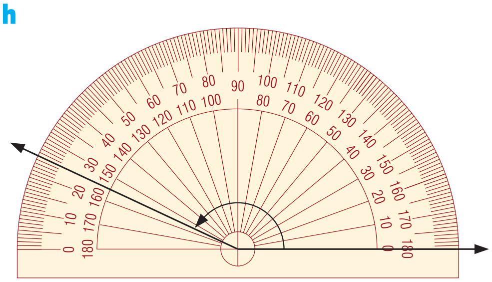

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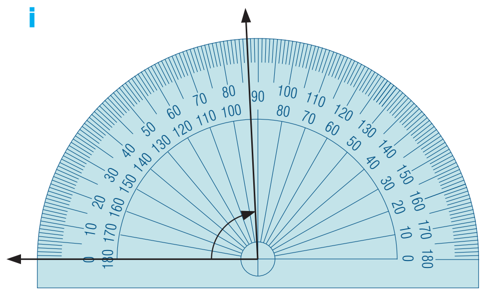

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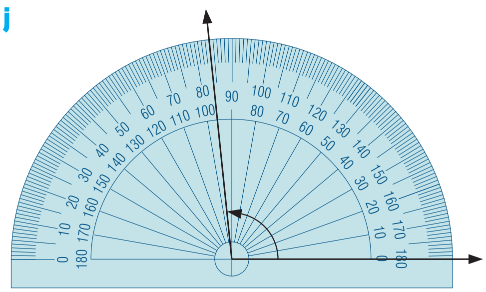


2 Draw a diagram to illustrate:

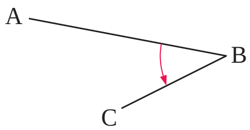
a a $\frac{1}{2}$ turn **b** a $\frac{1}{4}$ turn **c** a full turn.

3 Draw a diagram to illustrate:

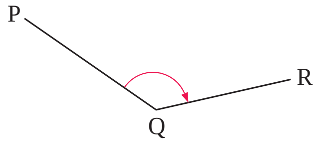
a a straight angle **b** a right angle **c** an obtuse angle.

4 Name the following angles in three point notation. State the type of angle in each case.

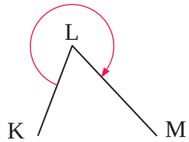
a



b



c

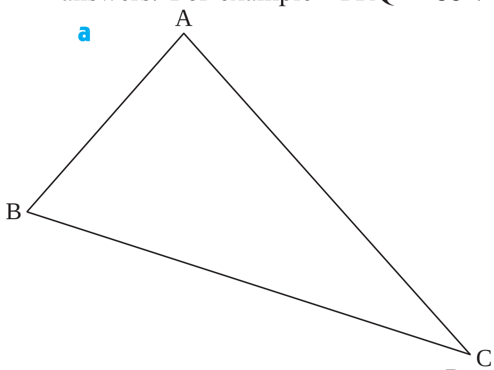


5 Draw an angle appropriate to the name:

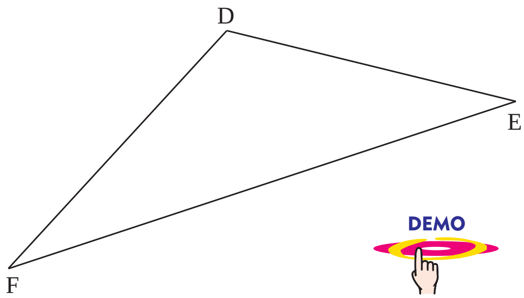
a $C\widehat{D}E$ **b** $Q\widehat{P}T$ **c** $M\widehat{T}D$ **d** reflex $S\widehat{N}P$.

6 Measure all angles of the following figures. Use three point notation to write down your answers. For example $P\widehat{R}Q = 38^o$.

a



b



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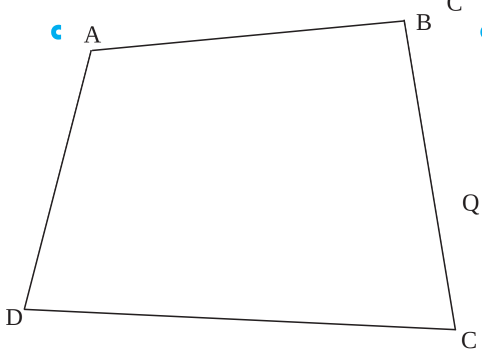
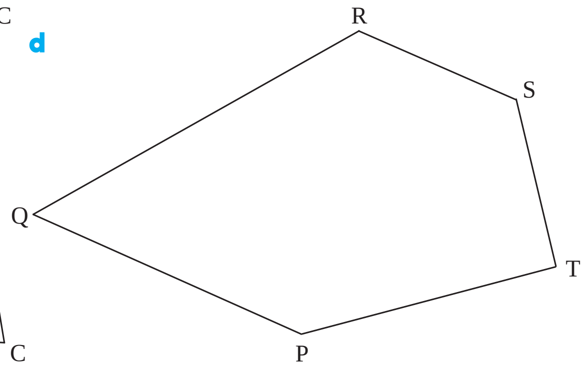
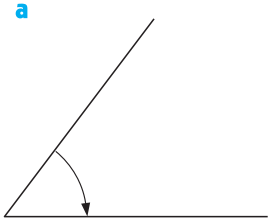


d

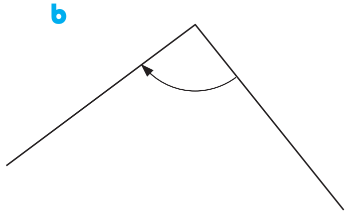


7 Estimate the size of the following angles. Check how good or bad your estimations are by using your protractor.

a



b



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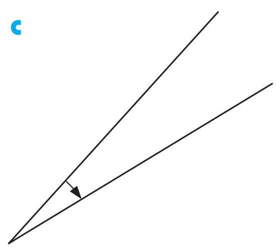
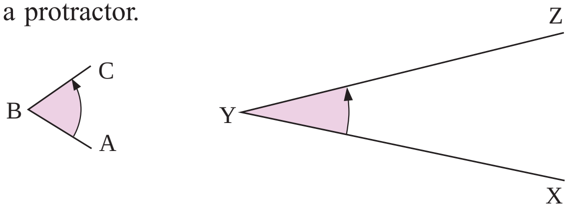


8 Using only a *ruler and pencil*, draw angles you *estimate* to be:

a 90^o **b** 45^o **c** 30^o **d** 60^o **e** 135^o

Check your estimations using a protractor.

9 Which is the larger angle, $\widehat{ABC}$ or $\widehat{XYZ}$?

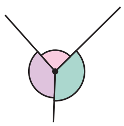


ACTIVITY 2 ANGLE GUESSING COMPETITION

Organise a competition between groups where 3 different angles are drawn. Each group guesses the size of each angle. The angles are then carefully measured with a protractor. A scoring system could be:

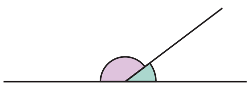
correct answer	$\pm 1^o$	5 points
correct answer	$\pm 2^o$	4 points
correct answer	$\pm 3^o$	3 points
correct answer	$\pm 4^o$	2 points
correct answer	$\pm 5^o$	1 point

C ANGLES AT A POINT OR ON A LINE



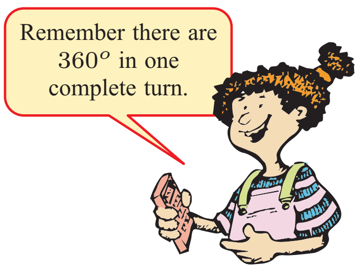
These angles are **angles at a point**.

Angles at a point add to 360^o.



These angles are **angles on a line**.

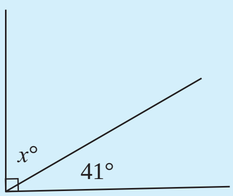
Angles on a line add to 180^o.



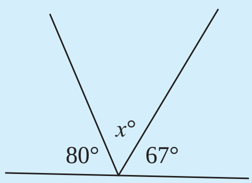
Example 2 Self Tutor

Find the value of x in the following. **Do not use a protractor**.

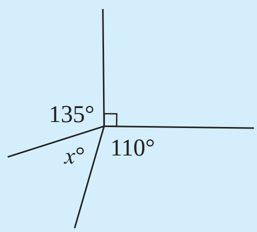
a



b



c



a The angles add to 90^o

so $x + 41 = 90$

But $49 + 41 = 90$,

so $x = 49$

b The angles add to 180^o

so $x + 80 + 67 = 180$

$\therefore$ $x + 147 = 180$,

But $33 + 147 = 180$,

so $x = 33$

c The angles add to 360^o

so

$x + 135 + 90 + 110 = 360$

$\therefore$ $x + 335 = 360$

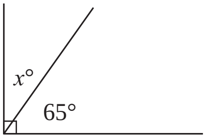
But $25 + 335 = 360$

so $x = 25$

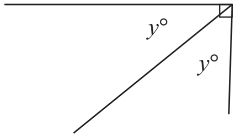
EXERCISE 3C

1 Find the size of the unknown angle in:

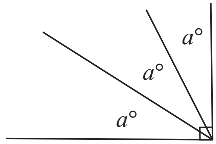
a



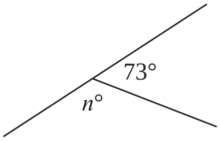
b



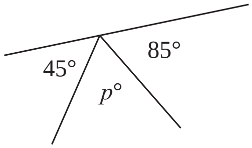
c



d



e



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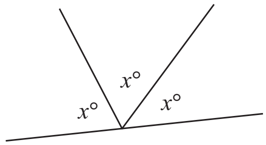

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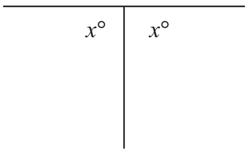

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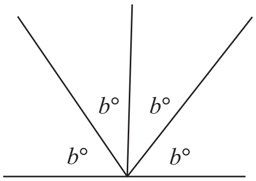

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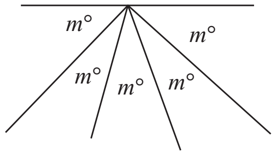
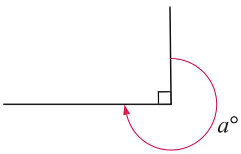
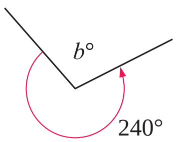


2 Find the size of the unknown angle in:

a



b



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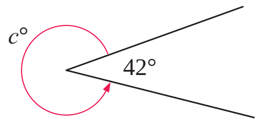
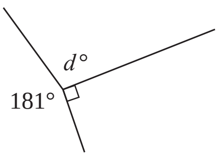
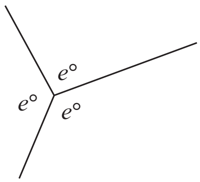


d



e



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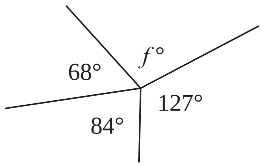

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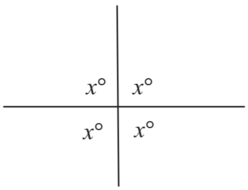

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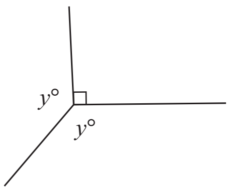

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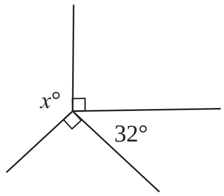

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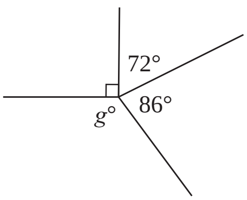

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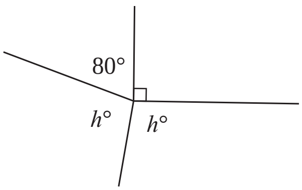

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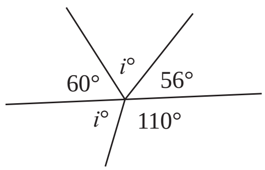


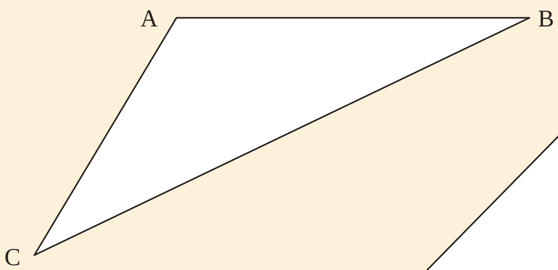
D ANGLES OF A TRIANGLE

INVESTIGATION 1 ANGLES OF A TRIANGLE

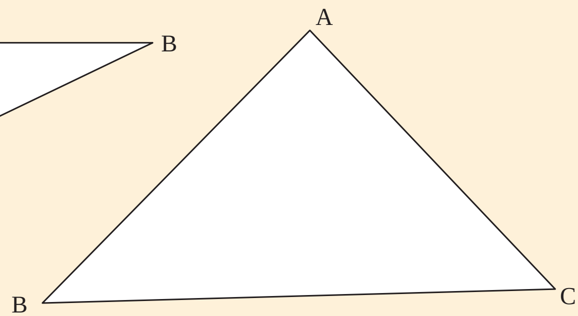
What to do:

1 Use a protractor to measure, to the nearest degree, the sizes of the angles of triangle ABC:

a



b



2 Copy and complete the following table. Use the results of **a** and **b** above, and draw *two* other triangles of your own choice (**c** and **d**).

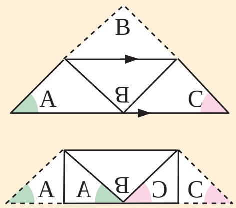
	$A\widehat{B}C$	$B\widehat{C}A$	$C\widehat{A}B$	*sum of the 3 angles*
a				
b				
c				
d				

3 From your results in **2**, copy and complete:

"*The sum of the angles of a triangle is ......*".

DEMO

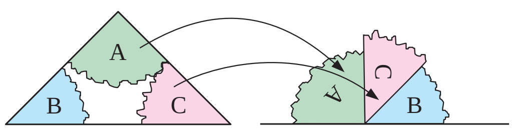
4 Now draw any triangle ABC and carefully cut it out.
Fold down the angle at B to meet the side [AC].
Fold corner A along a vertical line to meet B.
Fold corner C along a vertical line to meet B also.
What do you notice?
Repeat with another triangle of your choosing.



From the **Investigation** you should have noticed that:

The sum of the angles of a triangle is always 180^o.

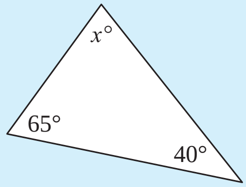
We can also see this result by tearing the angles of the triangle and rearranging them to sit on a line.



DEMO

Example 3 **Self Tutor**

Find the third angle x^o of the given triangle.



The angles of a triangle add to 180^o,

so $x + 40 + 65 = 180^o$

$\therefore$ $x + 105 = 180$

Now $75 + 105 = 180$

so $x = 75$

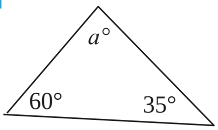
So, the third angle measures 75^o.

Note that when diagrams are not drawn to scale, we cannot use a protractor to measure the angles.

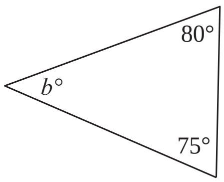
EXERCISE 3D

1 Find the unknowns in the following which *have not been drawn to scale*:

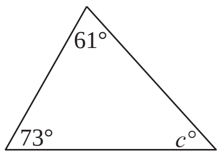
a



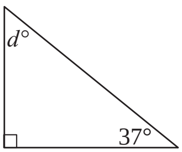
b



c



d



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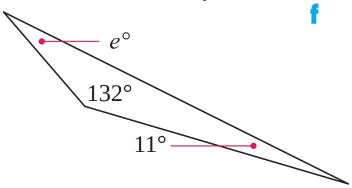

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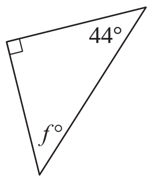

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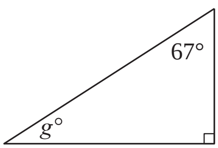

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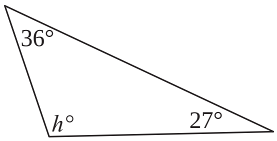

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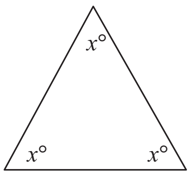

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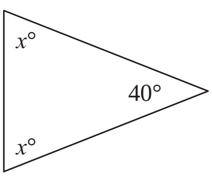

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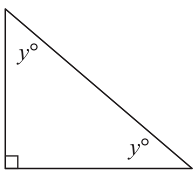

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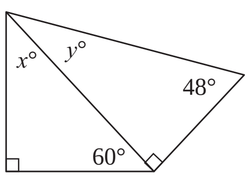
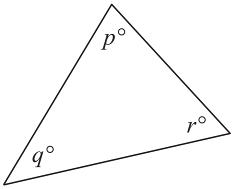
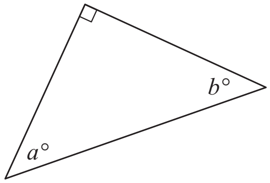


2 Write down a rule connecting the unknown angles in:

a



b



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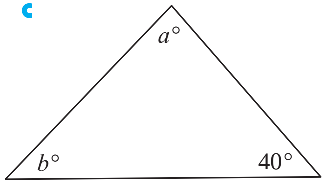
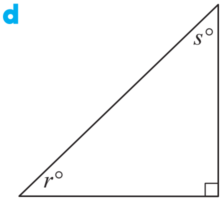
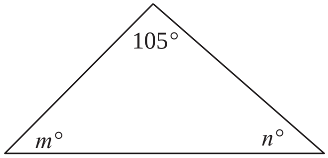


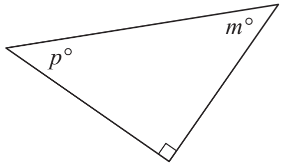
d



e



f



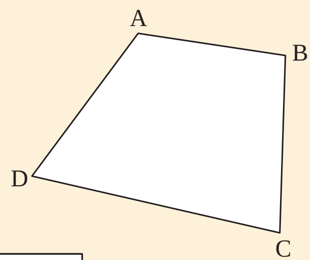
E ANGLES OF A QUADRILATERAL

INVESTIGATION 2 ANGLES OF A QUADRILATERAL

What to do:

1 Draw 4 half page size quadrilaterals and label the vertices A, B, C and D.

2 Accurately measure the angles at each vertex with a protractor.



3 Copy and complete the following table:

Diagram	$D\hat{A}B$	$A\hat{B}C$	$B\hat{C}D$	$C\hat{D}A$	*sum of the angles*
a					
b					
c					
d					

4 From your results in **3**, copy and complete:

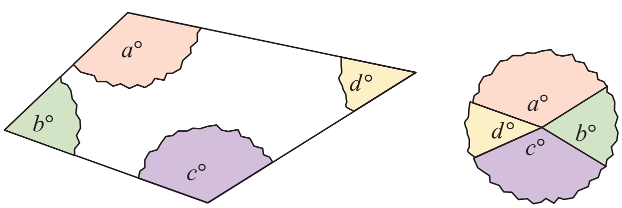
"The sum of the angles in a quadrilateral is ......"

From the **Investigation** you should have discovered that:

The sum of the angles of a quadrilateral is always 360°.

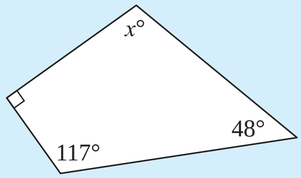
DEMO

We can also see this result by tearing the angles of a quadrilateral and rearranging them at a point.



Example 4 | **Self Tutor**

Find the value of x in:



The angles of a quadrilateral add to 360^o,

so $x + 48 + 117 + 90 = 360$

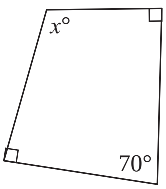
$\therefore\ x + 255 = 360$

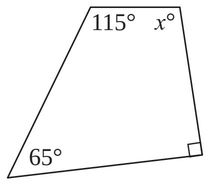
Now $105 + 255 = 360$

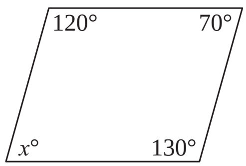
so $x = 105$

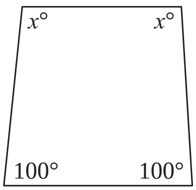
EXERCISE 3E

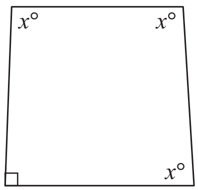
1 Find the unknowns in the following which *have not been drawn to scale*:

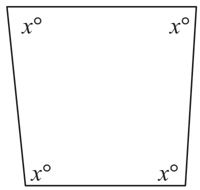
a


b


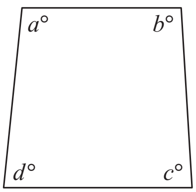
c


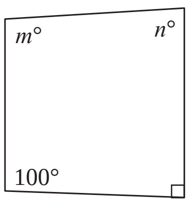
d


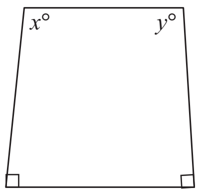
e


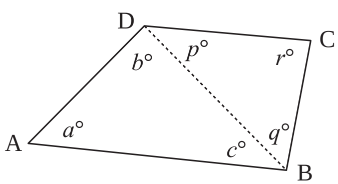
f


2 Write down a rule which connects the unknown angles in:

a


b


c


3


a Copy and complete:
$a + b + c =$ ...... and $p + q + r =$ ......
So, $a + b + p + r + q + c =$ ......

b What have you shown in **a** about the quadrilateral ABCD?

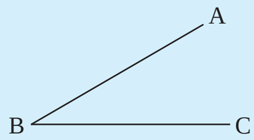
c Why is this argument 'stronger' than using paper tearing?

F BISECTING ANGLES

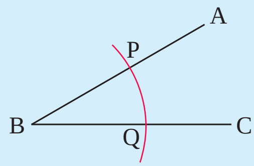
When we **bisect** an angle with a straight line, we divide it into two angles of equal size. In the following example we show how to bisect an angle using a *compass and ruler only*. A diagram drawn using a compass and ruler is called a **construction**.

Example 5 **Self Tutor**

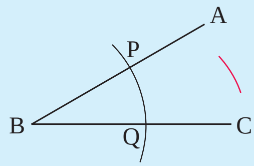
Bisect angle ABC.



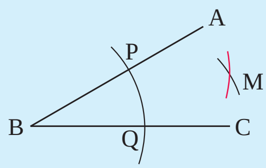
Step 1: With centre B, draw an arc which cuts [BA] and [BC] at P and Q respectively.



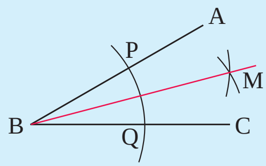
Step 2: With Q as centre, draw an arc within the angle ABC.



Step 3: Keeping the *same* radius and with centre P, draw another arc to intersect the previous one at M.



Step 4: Join B to M.
[BM] bisects angle ABC,
so $A\widehat{B}M = C\widehat{B}M$.



EXERCISE 3F

1 Use your protractor to draw an angle ABC of size 80^o.
- **a** Bisect angle ABC using a compass and ruler only.
- **b** Check with your protractor the size of each of the two angles you constructed.

2 Draw an acute angle XYZ of your own choice.
- **a** Bisect the angle without using a protractor.
- **b** Check your construction using your protractor.

3 Draw an obtuse angle ABC of your own choice.
- **a** Bisect the angle using a compass only.
- **b** Check your construction using your protractor.

4
- **a** Using a ruler, draw any triangle with sides greater than 5 cm.
- **b** Bisect each angle using a compass and ruler only.
- **c** What do you notice about the three angle bisectors?

KEY WORDS USED IN THIS CHAPTER

- acute angle
- intersecting lines
- obtuse angle
- quadrilateral
- revolution
- vertex
- angle
- line
- parallel lines
- ray
- right angle
- degree
- line segment
- point
- reflex angle
- straight angle

LINKS
click here

MAKING A PROTRACTOR

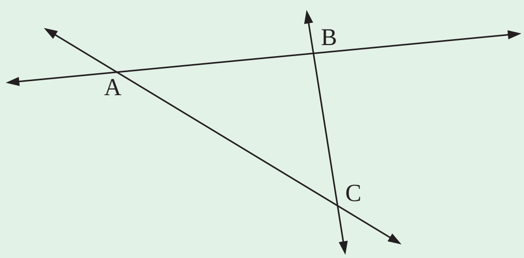
Areas of interaction:
Human ingenuity

REVIEW SET 3A

1 Name the intersection of:

a (AB) and (AC)

b (BC) and (CB).



2 Draw a diagram to illustrate:

a a $\frac{3}{4}$ turn

b an acute angle.

3 Using only a *ruler and pencil*, draw angles you estimate to be:

a 45^o

b 270^o

Check your estimations using a protractor.

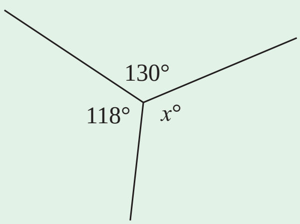
4 What angle is formed by the hands of a clock showing:

a 3 o'clock

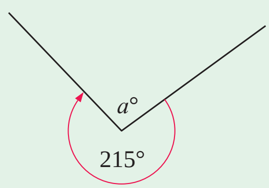
b 6 o'clock

5 Find, giving reasons, the value of the unknowns in the following which *are not drawn to scale*:

a

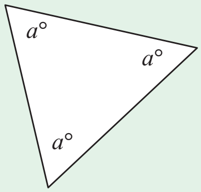


b

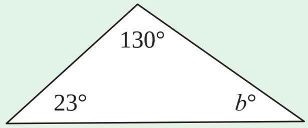


6 Find the sizes of the missing angles in the following which *are not drawn to scale:*

a

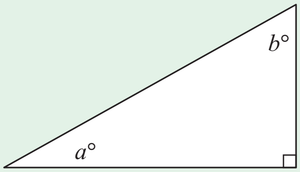


b

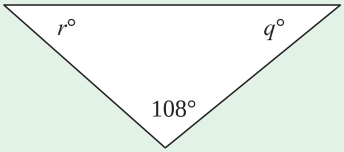


7 Write down a rule connecting the unknown angles:

a

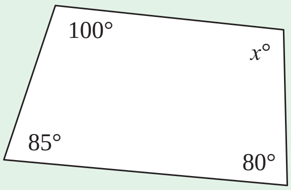


b

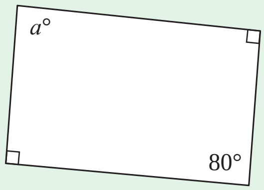


8 Find the unknowns in the following which *have not been drawn to scale:*

a



b



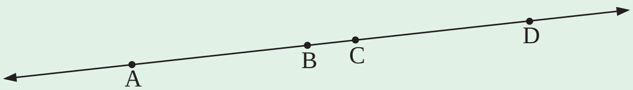
9 Use your protractor to draw an angle PQR of size 120^o. Bisect this angle using your compass and ruler only. Check that the two angles produced are each 60^o.

REVIEW SET 3B

1 Name the intersection of:

a [AC] and [BD]

b [BC] and [AD].



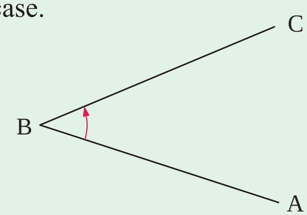
2 Draw a diagram to illustrate:

a a revolution

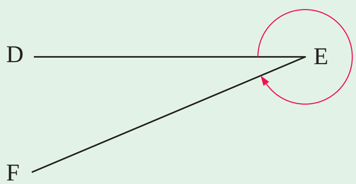
b a reflex angle.

3 Use three point notation to name the following angles. State the type of angle in each case.

a

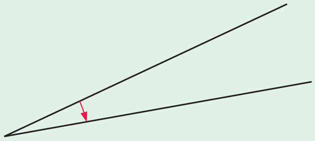


b

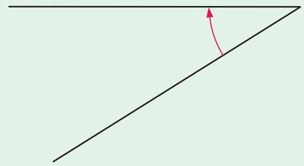


4 Estimate the size of the following angles, then check your estimations using a protractor:

a

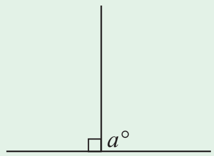


b

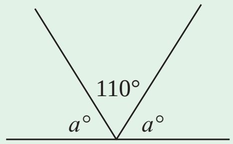


5 Find, giving reasons, the value of a in the following which *are not drawn to scale:*

a

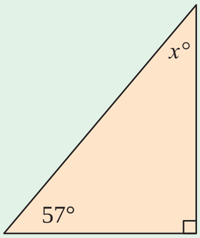


b

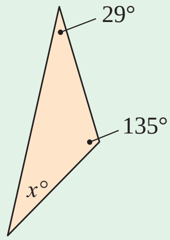


6 Find, giving reasons, the value of x in the following which *are not drawn to scale:*

a

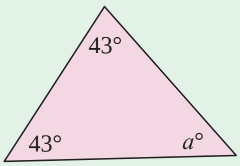


b

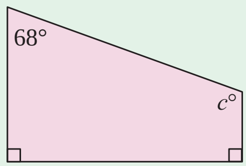


7 Find the sizes of the missing angles in the following which *are not drawn to scale*:

a

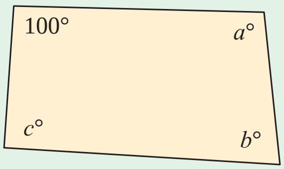


b

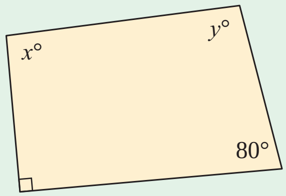


8 Write down a rule which connects the unknown angles in:

a



b



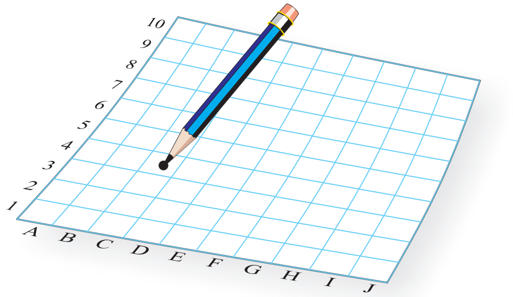
9 Draw an acute angle XYZ of your own choice. Bisect this angle using your compass and ruler only. Use your protractor to check the size of each of the two angles you have constructed.

Chapter 4

Location

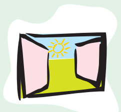
Contents:

Suppose your teacher gives each student in your class an identical blank sheet of paper and asks you to mark a point on that paper. Probably every student would mark a different point on the page. Now, what if your teacher wanted you to all mark exactly the same point on your sheets of paper? How can the exact position of the point be described?



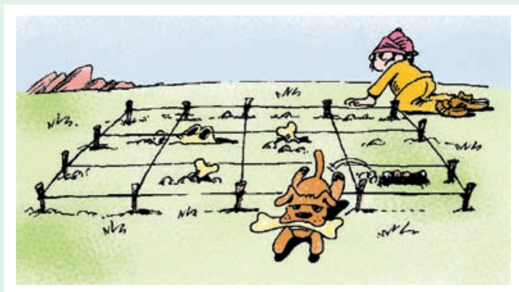
The answer to this question is to use a **map** or **grid reference**. We often see map references in street directories, and grid references in an atlas. They tell us where to look for a particular location.

OPENING PROBLEM



Archeologists use a grid to mark the positions of buildings and artefacts they discover while digging at a site.

Things to think about:



1 Mr Bone has used pegs and ropes to form a grid over his archeological 'dig'. What else does he need to do so that he can identify the positions of his discoveries?

2 How could Mr Bone improve his accuracy in identifying positions? Discuss your ideas.

3 Mr Bone wants to record the position of an object in his grid and the depth at which it is found. Suggest a way in which he could do this.

HISTORICAL NOTE

Frenchman **René Descartes** found a method for describing the position of a point in a plane. His work led to a new branch of mathematics called **coordinate geometry**.

One of his main rules was "never to accept anything as true which I do not clearly and distinctly see to be so", which is a good piece of advice for your own study of mathematics.

A MAP REFERENCES

In order to find the position of a street in a street directory, lines or **axes** of reference are added to the map. These lines are horizontal and vertical. Usually letters are used along one axis and numbers along the other. So, the position of a point or street on the map can be found using a pair such as D4 or E5.

An advantage of using letters on one axis and numbers on the other is that the order in which we list them is not important. For example, D3 and 3D are the same point.

Notice that the map reference does not tell us *precisely* where a feature is, but rather gives a square or region in which the feature is located.

Example 1 Self Tutor

This map is taken from a street directory of Perth, Western Australia.

a Name the street at A5.

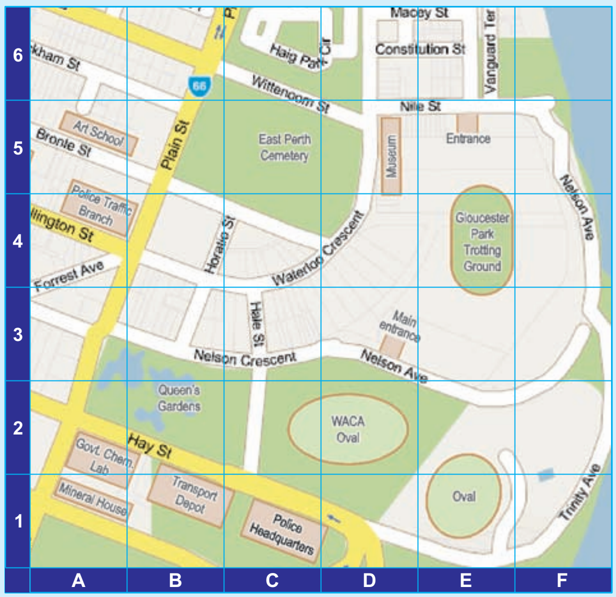
b Name the feature found at E1.

c Name the depot located at B1.

d State the location of Mineral House.

e Locate the Nile Street entrance to Gloucestor Park trotting ground.

f Locate the WACA oval.



a Bronte St **b** Oval **c** Transport depot

d A1 **e** E5 **f** D2

EXERCISE 4A

1 Use the street directory in **Example 1** to determine:

a the grid reference for:

i the Art School **ii** the Museum **iii** Government Chem Labs

b the feature located at:

i E4 **ii** C5 **iii** B2 **iv** A4 **v** C1

2 Use the map of Paris to determine:

a the grid reference for:

  i Opera

  ii the Arc de Triomphe

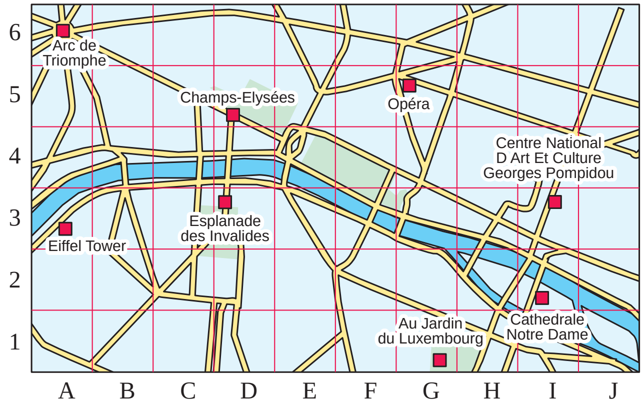
  iii Cathedrale Notre Dame

b the feature located at:

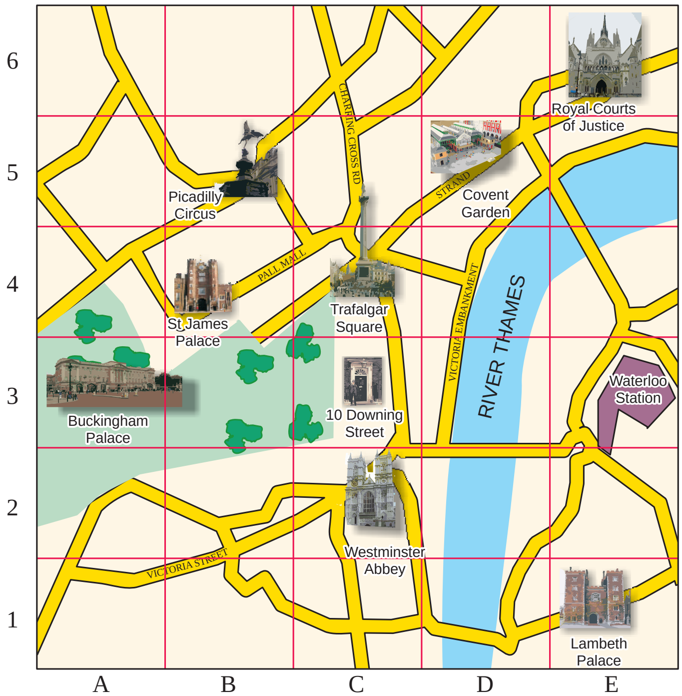
  i A3

  ii G1

  iii I3



3



Use the map of London alongside to determine:

a the grid reference for:

  i Westminster Abbey

  ii Trafalgar Square

  iii No. 10 Downing St.

  iv Lambeth Palace

  v St. James Palace

b the feature located at:

  i A3

  ii E6

  iii D5

  iv B5

  v E3

ACTIVITY 1 — STREET DIRECTORY

What to do:

1 In a street directory for your local area, find the page which shows the street where you live.

2 List 5 other streets on that page, perhaps the streets where your friends live.

3 Use the reference axes on the map to give references for each street.

4 Check your references with those in the index of your street directory.

5 Discuss any differences you found between the way you referenced the streets and the way they were referenced in the index.

DISCUSSION

What difficulties can arise when using a letter and a number to locate positions on a map?

How could you overcome them?

ACTIVITY 2 SCHOOL MAP

Next week some VIPs (very important people) will be visiting your school. It is your job to produce a map so they can find their way around.

What to do:

1 Draw a rough plan of your school grounds, including major landmarks and buildings.

2 On your plan, draw two labelled reference axes together with horizontal and vertical grid lines.

3 Provide a list of about 8 school landmarks or buildings together with references. For example: Library E4, Tennis Courts A7.

4 Compare your map with others produced in your class. Discuss the advantages and disadvantages of each map.

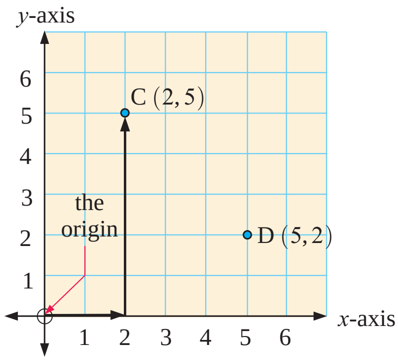
5 Display the maps around the classroom.

B NUMBER GRIDS

In **Exercise 4A** we saw how street directories and other maps use two reference lines or axes to direct us to a particular *region* on a map.

Another way of locating the exact position of a point in a plane is to use a **number grid**.

Here we have numbers on both axes.



The point of intersection is called the **origin**, **O**.
The horizontal axis is called the ***x*-axis**.
The vertical axis is called the ***y*-axis**.

To get from O to point C, we first move 2 units in the x-direction and then 5 units in the y-direction.

We say that C has **coordinates** (2, 5). The ***x*-coordinate** is 2 and the ***y*-coordinate** is 5.

To get from O to point D, we first move 5 units in the x-direction and then 2 units in the y-direction. So, D has coordinates (5, 2).

These number pairs are called **ordered pairs** because we move first in the x-direction and then in the y-direction.

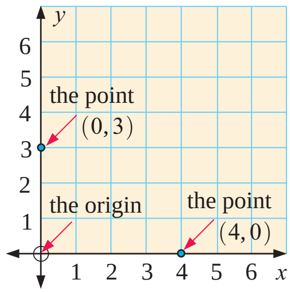
C(2, 5) and D(5, 2) are at different positions in the number plane.

POINTS ON THE AXES

If a point has an x-coordinate of 0 then it lies on the y-axis, because there is no movement to the right, only up.

If a point has a y-coordinate of 0 then it lies on the x-axis, because after we move to the right there is no movement up.

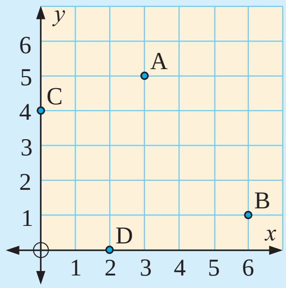
The **origin O** has coordinates (0, 0).



Example 2 **Self Tutor**

On the same set of axes plot and label the points with coordinates:

A(3, 5), B(6, 1),
C(0, 4), D(2, 0).



EXERCISE 4B

1 Use graph paper to draw a set of axes and plot and label the following points:

a A(2, 3)	**b** B(5, 7)	**c** C(4, 1)	**d** D(0, 5)
e E(3, 0)	**f** F(3, 2)	**g** G(8, 2)	**h** H(7, 0)
i I(1, 0)	**j** J(0, 8)	**k** K(1, 8)	**l** L(0, 0)

2 Copy and complete:

a All points on the x-axis have a y-coordinate equal to ....

b All points on the y-axis have an x-coordinate equal to ....

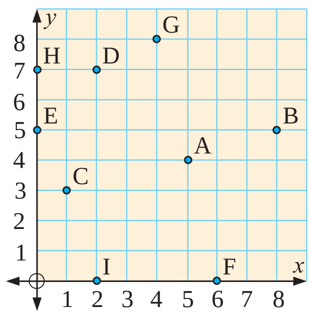
3 Write down:

a the x-coordinates of A, C, F and E

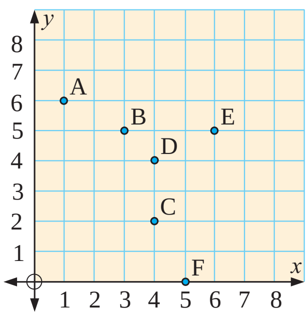
b the y-coordinates of C, G, H and I

c the coordinates of A, B, C, D, E, F, G, H and I

d the coordinates of the origin, O.



4

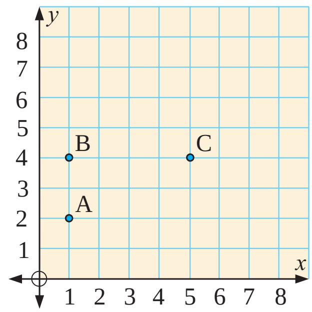


a Name two points which have the same x-coordinates as each other. What do you notice about these points?

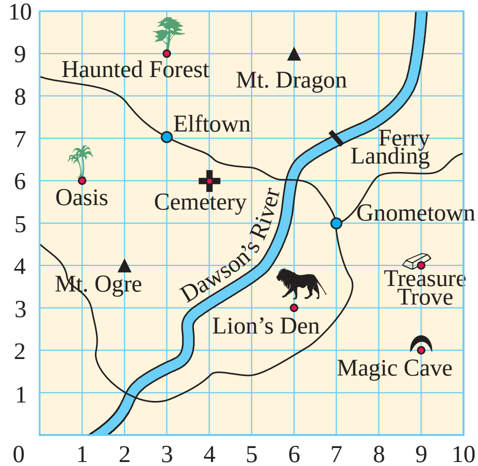
b Name two points which have the same y-coordinates as each other. What do you notice about these points?

c Name the point whose x-coordinate is equal to its y-coordinate.

5 ABCD is a rectangle. A, B and C are marked on the grid. Write down the coordinates of D.



6



Use the map to find:

- **a** the grid coordinates for:
  - **i** Gnometown
  - **ii** Magic Cave
  - **iii** Ferry Landing
  - **iv** where the roads cross Dawson's River
- **b** the places located at:
  - **i** (9, 4)
  - **ii** (6, 3)
  - **iii** (2, 4)
  - **iv** (1, 6)

7 On one set of axes, plot and label the points A(1, 2), B(2, 4), C(3, 6), D(4, 8).

- **a** Join these points. What do you notice?
- **b** If the pattern continues, what will the next point be?

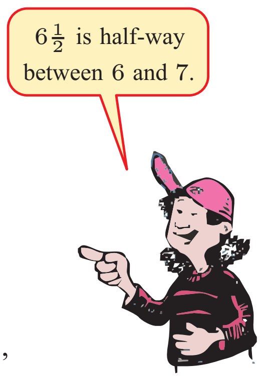
8 On one set of axes, plot and label the points A(1, 9), B(2, 8), C(3, 7), D(4, 6). If the pattern continues, what will the coordinates of the next three points be?

9 On 5 mm square graph paper, draw a set of axes. Number the horizontal x-axis from 0 to 20 and the vertical y-axis from 0 to 30. Plot and join these points with straight lines:

$(11, 20)$, $(10, 20)$, $(8, 20\frac{1}{2})$, $(6\frac{1}{2}, 22)$, $(6\frac{1}{2}, 23)$, $(8, 22\frac{1}{2})$, $(6\frac{1}{2}, 22\frac{1}{2})$.

Lift pencil. $(8, 22\frac{1}{2})$, $(9, 22)$, $(10, 22)$.

Lift pencil. $(6\frac{1}{2}, 23)$, $(6, 24)$, $(6\frac{1}{2}, 26)$, $(8, 27)$, $(11, 26)$, $(13, 24)$, $(14\frac{1}{2}, 21)$, $(14\frac{1}{2}, 20)$, $(13, 15)$, $(13, 13)$, $(14, 10\frac{1}{2})$, $(14\frac{1}{2}, 8)$, $(14, 4)$, $(14\frac{1}{2}, 2\frac{1}{2})$, $(13\frac{1}{2}, 3)$, $(13, 2)$, $(12\frac{1}{2}, 2\frac{1}{2})$, $(12, 1\frac{1}{2})$, $(11\frac{1}{2}, 2\frac{1}{2})$, $(10\frac{1}{2}, 1\frac{1}{2})$, $(10\frac{1}{2}, 3)$, $(9\frac{1}{2}, 3)$, $(10\frac{1}{2}, 4)$, $(10, 7\frac{1}{2})$, $(7, 7)$, $(6\frac{1}{2}, 4)$, $(7\frac{1}{2}, 2\frac{1}{2})$, $(6\frac{1}{2}, 2\frac{1}{2})$, $(6, 1\frac{1}{2})$, $(5, 2\frac{1}{2})$, $(4, 1\frac{1}{2})$, $(4, 3)$, $(3, 2)$, $(3, 3)$, $(2, 2\frac{1}{2})$, $(2, 3)$, $(3\frac{1}{2}, 4\frac{1}{2})$, $(3, 9)$, $(4, 13)$, $(9, 18)$, $(10, 20)$.



Lift pencil. (13, 24), ($14\frac{1}{2}$, $24\frac{1}{2}$), (14, 23), (15, $23\frac{1}{2}$), (15, 22), ($15\frac{1}{2}$, $22\frac{1}{2}$), (15, 21), (16, $21\frac{1}{2}$), (15, $19\frac{1}{2}$), (16, $19\frac{1}{2}$), ($14\frac{1}{2}$, 17), (13, 13).

Lift pencil. (14, $10\frac{1}{2}$), ($15\frac{1}{2}$, $10\frac{1}{2}$), ($16\frac{1}{2}$, 10), (17, 9), (16, 6), (17, 5), (17, $4\frac{1}{2}$), (16, 4), (16, $4\frac{1}{2}$).

Lift pencil. (16, 4), (15, 4), (15, $4\frac{1}{2}$). Lift pencil. (15, 4), (14, 4).

Lift pencil. (7, $3\frac{1}{2}$), (10, $3\frac{1}{2}$).

Lift pencil. (2, 3), (1, 3), (2, 4), ($1\frac{1}{2}$, 6), ($1\frac{1}{2}$, 9), (2, $9\frac{1}{2}$), ($2\frac{1}{2}$, $9\frac{1}{2}$), (3, 9).

Lift pencil. ($1\frac{1}{2}$, 9), (0, 11), (0, $12\frac{1}{2}$), (1, 14), (4, 15), (6, 16), (6, 17), ($3\frac{1}{2}$, 19), (3, $21\frac{1}{2}$), (2, 20), (2, 22), (3, 24), (4, 24), (5, 22), (5, 20), (4, $21\frac{1}{2}$), (4, $19\frac{1}{2}$), (7, 17), (7, 16).

Lift pencil. ($3\frac{1}{2}$, 11), (3, $11\frac{1}{2}$), (3, $12\frac{1}{2}$), (4, 13).

Lift pencil. (7, $25\frac{1}{2}$), ($7\frac{1}{2}$, 26), ($7\frac{1}{2}$, $25\frac{1}{2}$), (7, $25\frac{1}{2}$).

Lift pencil. (11, 24), (11, $24\frac{1}{2}$), ($11\frac{1}{2}$, $24\frac{1}{2}$), ($11\frac{1}{2}$, 24), (11, 24).

10 On 5 mm square graph paper draw a set of axes. Number the horizontal x-axis from 0 to 30 and the vertical y-axis from 0 to 20. Plot and join these points with straight lines:

(8, 10), (7, 8), (4, 12), (4, 14), (7, 17), (9, 17), (10, 18), (12, 18), (10, 17), (10, 16), (11, 16), (10, 15), (8, 15), (7, 13), (7, 12), (8, 10), (12, 12), (16, 12), (20, 10), (21, 12), (21, 13), (20, 15), (18, 15), (17, 16), (18, 16), (18, 17), (16, 18), (18, 18), (19, 17), (21, 17), (24, 14), (24, 12), (21, 8), (20, 10).

Lift pencil. (21, 8), (21, 6), (23, 6), (27, 10), (26, 11), (26, 10), (23, 7), (21, 7).

Lift pencil. (23, 6), (26, 6), (28, 4), (26, 5), (20, 5), (21, 6).

Lift pencil. (18, 4), (20, 5), (22, 3), (24, 2), (25, 1), (23, 2), (21, 3), (20, 4), (16, 4), (18, 2), (18, 1), (16, 1), (18, 0), (19, 0), (20, 1), (20, 3), (17, 4).

Lift pencil. (16, 4), (11, 4), (8, 3), (8, 1), (9, 0), (10, 0), (12, 1), (10, 1), (10, 2), (12, 4).

Lift pencil. (11, 4), (8, 4), (7, 3), (5, 2), (3, 1), (4, 2), (6, 3), (8, 5), (10, 4).

Lift pencil. (7, 8), (7, 6), (8, 5), (2, 5), (0, 4), (2, 6), (7, 6).

Lift pencil. (5, 6), (1, 10), (2, 11), (2, 10), (5, 7), (7, 7).

Lift pencil. (11, $11\frac{1}{2}$), (12, $11\frac{1}{2}$), (12, 12). Lift pencil (16, 12), (16, $11\frac{1}{2}$), (17, $11\frac{1}{2}$).

ACTIVITY 3 HOPPING AROUND A NUMBER PLANE

Click on the icon to obtain instructions and a printable grid to do this activity.

PRINTABLE GRID

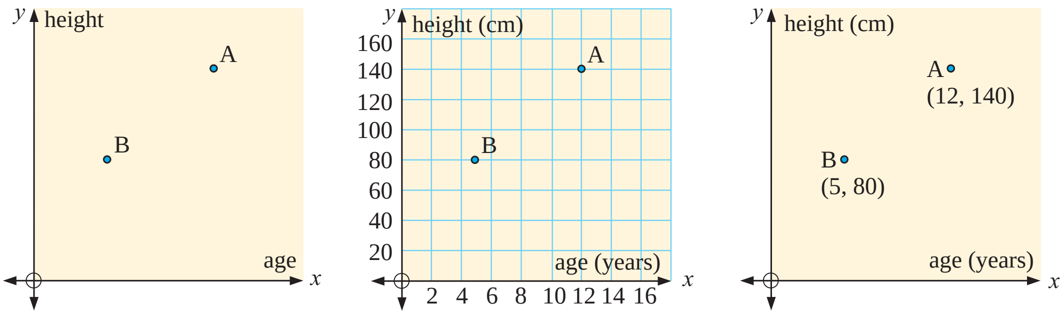
C INTERPRETING POINTS ON A GRID

A quantity whose value can change is called a **variable**.

A **graph** is a means of showing how two variables are related.

POINT GRAPHS

The following **point graphs** show the heights and ages of two girls, Anh and Belinda. The two points on the graph show this information. Point A gives us information about Anh's height and age, while point B gives the same information about Belinda.



The first graph just shows the two points and does not give any numerical information. All that we can deduce is that Anh is older than Belinda, since A lies to the right of B, and that Anh is taller than Belinda, since A lies above B.

The second graph is identical to the first, but it contains additional numerical information. It tells us that Anh is 12 years old and 140 cm tall, while Belinda is 5 years old and 80 cm tall.

The third graph shows the same numerical information as the second graph, but in a different format. Instead of giving scales on each of the axes, the values relating to each point are given as a pair of coordinates. Point A has coordinates (12, 140) and point B has coordinates (5, 80). Remember that the first number of the ordered pair relates to the horizontal axis. So, A(12, 140) means that Anh's age is 12 years and her height is 140 cm, while B(5, 80) means that Belinda's age is 5 years and height is 80 cm.

EXERCISE 4C

1 This graph shows the weights and ages of Christopher (C), Dragan (D) and Emilio (E).

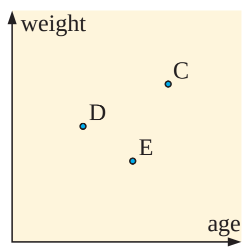
a **i** What are the two variables represented on this graph?

ii What variable is represented on the horizontal axis?

iii What variable is represented on the vertical axis?

b **i** Who is the heaviest? **ii** Who is the oldest?

iii Who is the youngest? **iv** Who is the lightest?



c Answer true or false to these statements, using the information on the graph:

i Emilio is older than Christopher. **ii** Dragan is younger than Emilio.

iii Dragan is heavier than Emilio.

iv Older boys are heavier than younger boys.

2 This graph relates the shoe sizes and ages of Sarah (S), Thao (T) and Voula (V). Using the graph, answer the following questions:

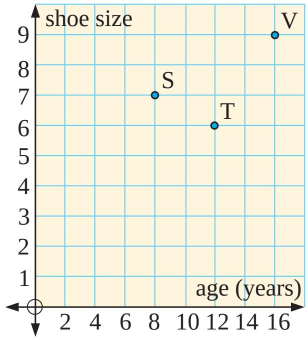
a Which girl is the youngest?

b Which girl has the largest feet?

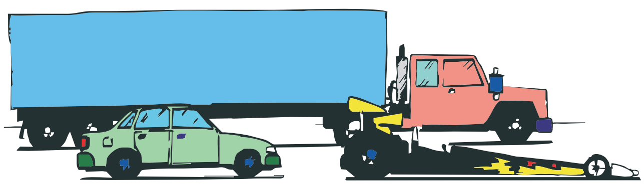
c What is Sarah's age and shoe size?

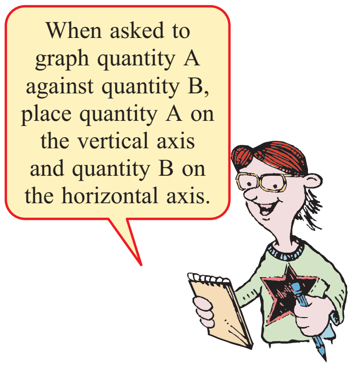
d What is Voula's age and shoe size?

e What is Thao's age and shoe size?



3 Consider the truck, the family sedan, and the drag racing car pictured below. The truck is the slowest vehicle. The drag racer is the lightest vehicle.





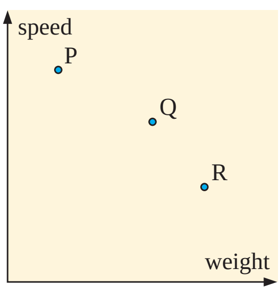
Draw point graphs to represent the following relationships. Remember to label your axes. Label the points with S for Sedan, D for Drag racer, and T for Truck.

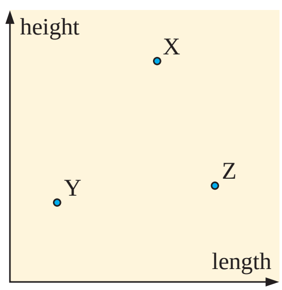
a Top speed against weight. **b** Top speed against height.

c Top speed against length. **d** Height against length.

e Height against weight. **f** Weight against length.

4 The cheetah is the fastest of all land animals. A fully-grown grizzly bear is heavier than a giraffe. The two graphs below relate the speed of each of these animals to its weight, and the height of each animal to its (horizontal) length. Use this information and the information shown in the pictures of these animals to answer the following questions:





a Identify each of the points P, Q, R, X, Y and Z with the animal that it represents.

b Which animal is faster, the giraffe or the grizzly bear?

c Draw a pair of axes with speed on the vertical axis and height on the horizontal axis. Mark in points to represent each of the animals.

d Draw a pair of axes with height on the vertical axis and weight on the horizontal axis. Mark in points to represent each of the animals.

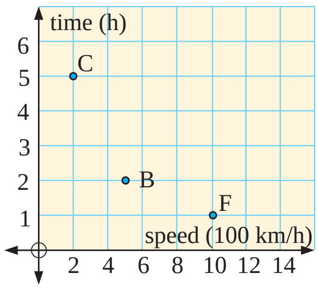
5 A Cessna aeroplane (C), a Boeing 747 (B), and an fighter plane (F) all fly from Dublin to Paris. The times taken to complete the journey and the speeds at which they fly are shown in this graph. Answer the following questions using the graph:

a What is the Cessna's speed and flight time?

b What is the Boeing 747's speed and flight time?

c What is the F111's speed and flight time?

d From the information in the graph, what can you say about the relationship between the flight time and the speed of an aeroplane?



6 This graph shows the height (in metres) and trunk diameter (in metres) of a Red Gum (G), a Californian Redwood (R), and a Baobab (B):

a Which tree is the tallest?

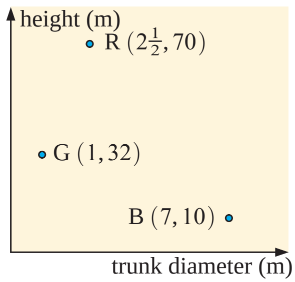
b Which tree has a trunk of smallest diameter?

c What is the trunk diameter and the height of the Red Gum?

d What is the height of the Californian Redwood?

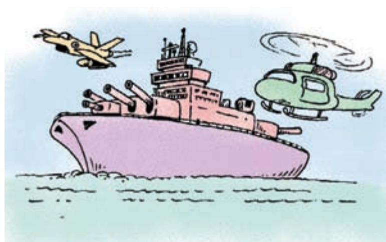
e What is the height of the Baobab?

f What is the trunk diameter of the Baobab?

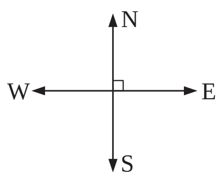


D BEARINGS AND DIRECTIONS

One of the most important applications of angles is in **navigation**. When flying an aeroplane, sailing a ship or hiking across land, you need to know the direction in which to travel.



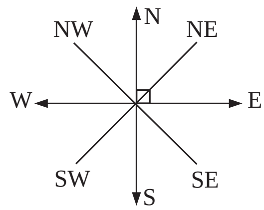
COMPASS POINTS



We are most familiar with the directions of the four main compass points: **N**orth, **S**outh, **E**ast and **W**est. They are often called the **cardinal** directions, and are 90^o apart.

We can divide each 90^o into 45^o angles to create the 'half-way' directions, NE, SE, SW and NW.
For example, Southwest (SW) is half-way between South and West.

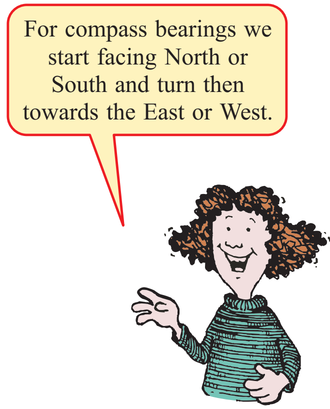
The directions NE, SE, SW and NW are sometimes called **ordinal** directions.



However, in order to navigate a ship more accurately, we need to be able to specify directions exactly. To do this we use either **compass bearings** or **true bearings**.

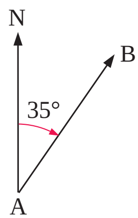
COMPASS BEARINGS

This method uses only **acute angles** between 0^o and 90^o. The angles are measured either clockwise or anticlockwise from either **North** or **South**.



For example, in the diagram shown, the bearing of B from A is N35^{o}E.

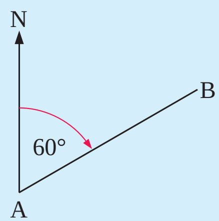
This means that an observer at A, facing North, needs to turn 35^o towards the East to face B.



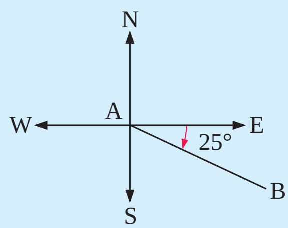
Example 3 **Self Tutor**

Find the compass bearing of B from A:

a



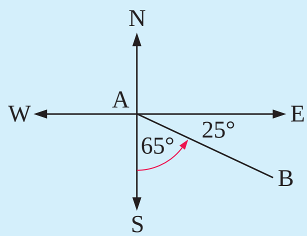
b



a The bearing of B from A is N60^{o}E.

b $90^o - 25^o = 65^o$

$\therefore$ the bearing of B from A is S65^{o}E.

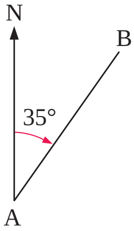


TRUE BEARINGS

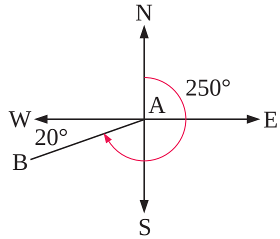
This method uses **clockwise** rotations from the **true north** direction and so angles between 0^o and 360^o are used.

When writing a true bearing we always use three digits. For example, we write 072^o instead of 72^o, and 009^o instead of 9^o.

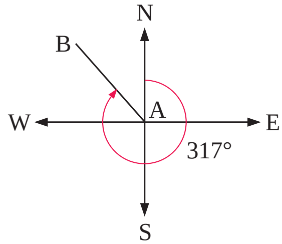
Here are some examples of true bearings:



The **true bearing of B from A** is 035^o



The **true bearing of B from A** is 250^o.

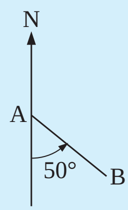


The **true bearing of B from A** is 317^o.

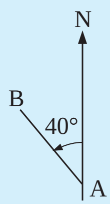
Example 4 | Self Tutor

Find the true bearing of B from A for:

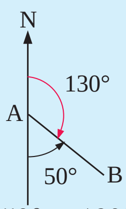
a



b



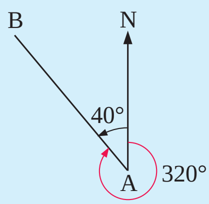
a



$180^o - 50^o = 130^o$

$\therefore$ the bearing is 130^o.

b



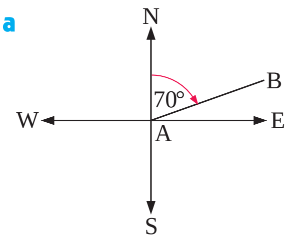
$360^o - 40^o = 320^o$

$\therefore$ the bearing is 320^o.

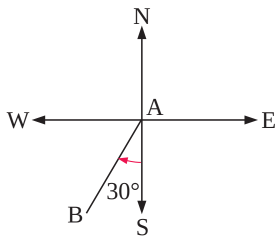
EXERCISE 4D

1 Give the compass bearing of B from A in each of the following:

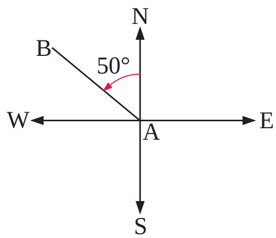
a



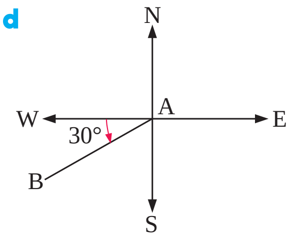
b



c



d



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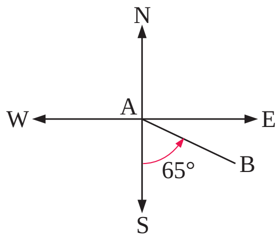

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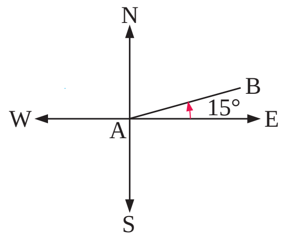


2 Use a protractor to draw fully labelled diagrams which show that the compass bearing of:

a B from A is N40°E **b** A from B is S50°W **c** C from D is S45°E
d P from Q is N65°W **e** X from Y is S80°E **f** M from N is N84°E.

3 Give true bearings for B from A for each of the diagrams in **1**.

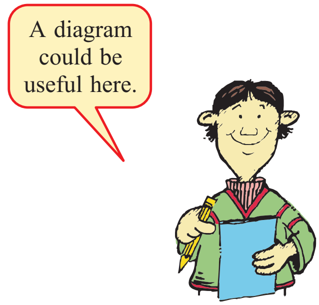
4 Use a protractor to draw fully labelled diagrams showing true bearings of:

a 070° **b** 160° **c** 213° **d** 312° **e** 096°

5 Describe the bearings: **a** 270° **b** 000°

6 Determine the true bearings of the eight main compass bearings:

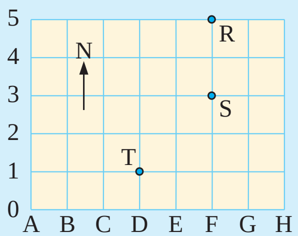
a North **b** Northeast
c East **d** Southeast
e South **f** Southwest
g West **h** Northwest



Example 5 Self Tutor

a Give the grid references of:
i point R **ii** point S **iii** point T

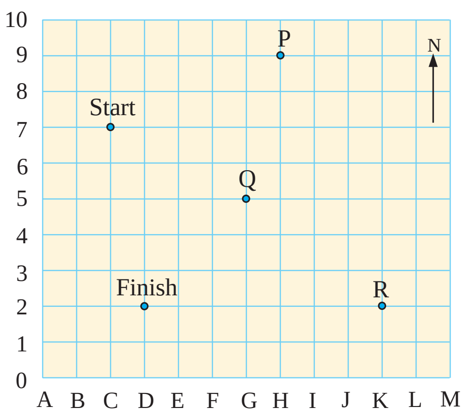
b Find the bearing of:
i R from S **ii** T from S



a **i** R has grid reference (F, 5).
ii S has grid reference (F, 3).
iii T has grid reference (D, 1).

b **i** R is North of S.
ii T is Southwest of S.

7



An orienteer must travel from the Start to P, then to Q, then to R, and finally to the Finish point.

a The Start is given by the grid reference C7. Find the grid references for:
i P **ii** Q **iii** R **iv** Finish.

b Use a protractor to find the true bearing of:
i P from the Start
ii R from Q
iii Q from R
iv the Start from the Finish.

PRINTABLE PAGE

8 P, Q and N are landmarks on the map and S is a ship at sea. The position of S is given by (7, 10).

a What is at the point given by (3, 1)?

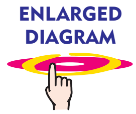
b What is the compass bearing of N from P?

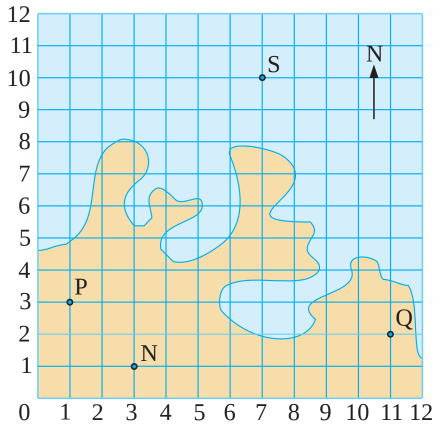
c What is the true bearing of:

i the ship from P

ii the ship from Q

iii Q from P?





RESEARCH — MAGNETIC COMPASS

Find out what a **magnetic compass** is and how it works.

ACTIVITY 4 — ORIENTEERING IN THE SCHOOL YARD

You will need: a magnetic compass, a trundle wheel or tape measure

What to do:

1 Find 4 or 5 places in the school grounds that are easily accessible and where you have a clear line of sight from one to the next. For example, you might choose the flagpole, goal posts, or the corner of a building.

2 Draw a rough sketch of the school grounds showing the objects you have selected.

3 From a starting point, measure the distances and directions from one point to the next. Record all distances and bearings on your rough sketch.

4 Accurately draw your pathway on clean paper, showing all distances and bearings.

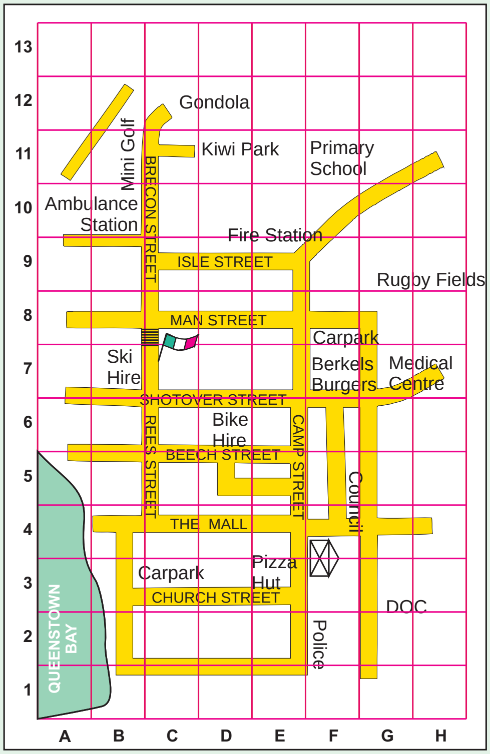
5 Give the detailed map to another student and see if he or she can follow your directions accurately.

KEY WORDS USED IN THIS CHAPTER

- cardinal direction
- coordinates
- ordinal direction
- true bearing
- x-axis
- y-coordinate
- clockwise
- number grid
- origin
- true north
- x-coordinate
- compass bearing
- ordered pair
- point graph
- variable
- y-axis

REVIEW SET 4A

1 Use the street map of Queenstown in New Zealand to answer the following questions.

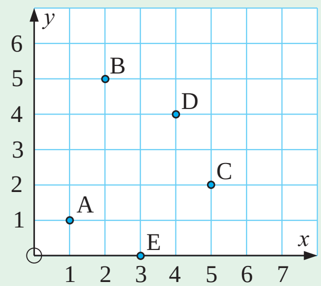


a Find what is at:
   i D11 **ii** G7.

b Give the location of
   i the Pizza Hut
   ii Queenstown Primary School.

c The Italian Restaurant has a flag as its symbol. What is its location?

2 Write the coordinates or ordered pairs for the following points:

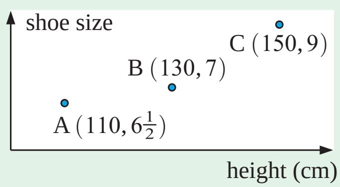


a A **b** B
c C **d** D **e** E

3 Plot the points A(4, 2), B(7, 2) and C(7, 5) on grid paper. Find the coordinates of D, the fourth vertex of rectangle ABCD.

4 Construct a 10 by 10 grid and label each axis from 0 to 10.
Follow the directions to locate the points and join them in the correct order.
Begin with (2, 3), (8, 3), (7, 2), (5, 2), (5, 1), (4, 1), (4, 2), (3, 2), (2, 3).
Lift pencil. (4, 3), (4, 10), (8, 4), (2, 4), (4, 10), (5, 10), (4, 9).

5 This graph shows the heights and shoe sizes of three girls: Anna, Briony and Claire.



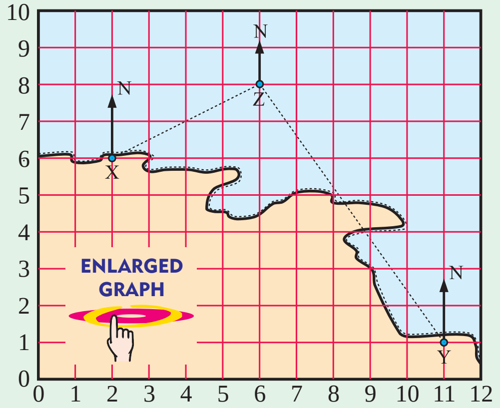
a Who has the largest shoe size?

b Who is the shortest?

c What is Briony's height and shoe size?

6 X and Y are radar stations on the coastline. Z represents a yacht.

- **a** What is at the point given by (2, 6)?
- **b** What is the grid reference for radar station Y?
- **c** What is the true bearing of the radar station X:
  - **i** from the yacht Z
  - **ii** from radar station Y?



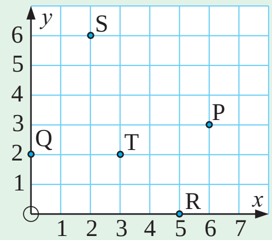
REVIEW SET 4B

1 Refer to the street directory map given in **Example 1** on page **69** to answer the following questions.

- **a** Find what is at: **i** D5 **ii** A5.
- **b** Give the location of: **i** Vanguard Terrace **ii** Queen's Gardens.

2 Write the coordinates or ordered pairs for the following points:

a P **b** Q **c** R **d** S **e** T



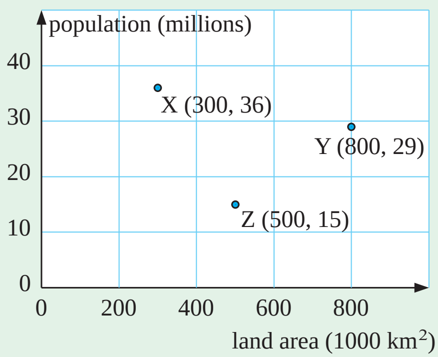
3 Rule up a grid with the vertical y-axis from 0 to 10 and the horizontal x-axis from 0 to 20. Follow the directions to locate the points and join them in the correct order.

Begin with (4, 8), (6, 9), (12, 9), (14, 8), (12, 7), (6, 7), (4, 8), (4, 3), (6, 2), (12, 2), (14, 3), (14, 8).

Lift pencil. (14, 7), (16, 7), (16, 4), (14, 4), (18, 3), (18, 1), (14, 0), (4, 0), (0, 1), (0, 3), (4, 4).

Lift pencil. (14, 6), (15, 6), (15, 5), (14, 5).

4



The graph alongside shows the population (in millions of people) and land area (in thousands of square km) of three countries: X, Y and Z.

- **a** Which country has the smallest population?
- **b** Which country is the largest?
- **c** What is the population of country Z?
- **d** What is the land area of country X?
- **e** Which country is the most crowded?

5 Use a protractor to draw fully labelled diagrams showing true bearings of:

a 085^o **b** 236^o

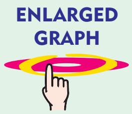
6 R and Q are two landmarks and S is a ship at sea.

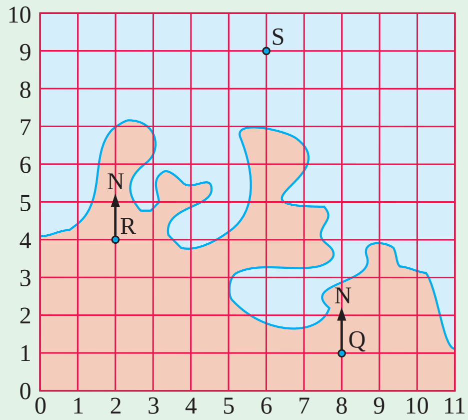
a Find the grid references for:

i Q **ii** R **iii** S.

b Using a protractor, find the true bearing of:

i the ship from R
ii the ship from Q
iii R from Q.



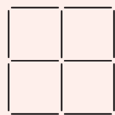


PUZZLE MORE MATCHSTICK PUZZLES

In playing with matches a number of interesting puzzles have been developed. It is impossible to state any general rules for solving puzzles with matches but the fun and the challenge remain.

Investigate the following puzzles:

1 In the configuration of 12 matches given:



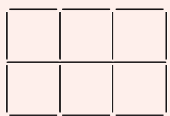
a remove 4 matches to leave 1 square

b remove 2 matches to leave

i 3 squares **ii** 2 squares

c shift 4 matches to obtain 3 squares.

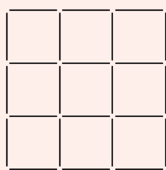
2 Using 17 matches we can obtain the rectangle shown:



a Remove 5 matches to leave 3 congruent squares.

b Remove 2 matches to leave 6 squares.

3 Using the 24 match rectangle shown:



a remove 4 matches to make 5 squares

b make 2 squares by removing 8 matches

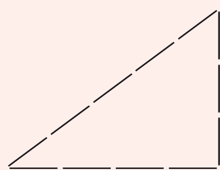
c remove 8 matches to leave

i 2 squares **ii** 3 squares **iii** 4 squares

d remove 12 matches to leave 3 squares

e shift 8 matches to make 3 squares.

4 The triangle alongside has area 6 units^2.



a Move two matches to make the area 5 units^2.

b See what other areas you can make by moving matches 2 at a time.

Chapter 5

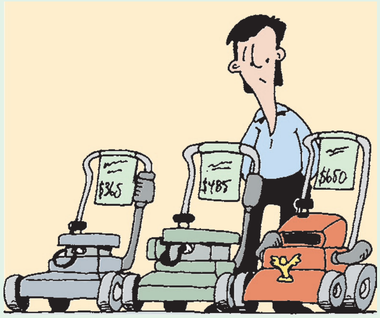
Number properties

Contents:

OPENING PROBLEM

Stanley's Mowers only sell lawn mowers, which come in three models.

The Standard model sells for \$365, the Advanced for \$485, and the Deluxe for \$650. In one month they sell 85 Standards, 46 Advanced, and 28 Deluxe mowers.



Things to think about:

a How many mowers did they sell for the month?

b What was the income for each mower type?

c What was the total income for the business?

d If the business costs including rent, salaries, and the mowers amount to \$36 580 for the month, how much profit was made?

A ADDITION AND SUBTRACTION

To find the **sum** of two or more numbers, we **add** them.

For example, the sum of 8 and 6 is $8+6$ which is 14.

To find the **difference** between two numbers we **subtract** the smaller one from the larger one.

For example, the difference between 5 and 12 is $12-5$ which is 7.

Example 1 **Self Tutor**

Find: **a** the sum of 7, 8 and 11 **b** the difference between 13 and 31.

a The sum of 7, 8 and 11
$= 7+8+11$
$= 26$

b The difference between 13 and 31
$= 31-13$ {larger − smaller}
$= 18$

When we add 3 or more numbers together we can rewrite them **in any convenient order** before we find the sum.

For example, in $8+39+12$ we notice that $8+12$ is 20

$$\begin{aligned}\text{So,}\quad & 8+39+12\\ &= 8+12+39\\ &= 20+39\\ &= 59\end{aligned}$$

Example 2 — Self Tutor

Find: **a** $74 + 23 + 7$ **b** $16 + 67 + 14$

a
$$\begin{aligned} &74 + 23 + 7 \\ =\ &23 + 7 + 74 \\ =\ &30 + 74 \\ =\ &104 \end{aligned}$$

b
$$\begin{aligned} &16 + 67 + 14 \\ =\ &16 + 14 + 67 \\ =\ &30 + 67 \\ =\ &97 \end{aligned}$$

EXERCISE 5A

1 Find the sum of:

a 8 and 11 **b** 19 and 13 **c** 24 and 17 **d** 56, 14 and 28.

2 Find the difference between:

a 7 and 3 **b** 27 and 18 **c** 19 and 38 **d** 123 and 280.

3 Find:

a the sum of 4, 6 and 13 **b** the difference between 18 and 37

c by how much 83 is greater than 66

d the sum of the whole numbers from 2 to 6.

4 Find the following sums by adding them in the most convenient order:

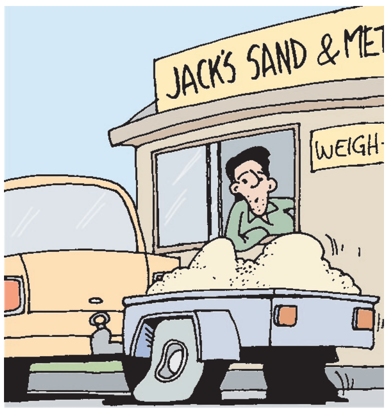
a $3 + 6 + 7$ **b** $19 + 8 + 2$ **c** $3 + 6 + 7 + 4$ **d** $21 + 98 + 19$

e $45 + 14 + 26$ **f** $98 + 57 + 102$ **g** $107 + 14 + 23$

h $28 + 13 + 12 + 37$

5 **a** What number must be increased by 13 to get 42?

b What number must be decreased by 13 to get 42?



6 At a tennis tournament the first prize was €175 000 and second prize was €108 500. How much more did the winner get than the runner-up?

7 The lifts by a weight-lifter in one event were 275 kg, 290 kg and 310 kg. How much less than 1000 kg is the total of the three lifts?

8 When measured on a weigh-bridge, a car and empty trailer weigh 1267 kg. Sand is poured into the trailer and the weighing process takes place again. The new weight is 2193 kg. How much does the sand weigh?

9 Herb's bank balance is £1793. He deposits £375 and then £418.

a How much does he have in his account now?

b If he then withdraws £895 to buy a kite-surfing kit, how much will be left in his account?

B MULTIPLICATION AND DIVISION

To find the **product** of two or more numbers we **multiply** them.

For example, the product of 6 and 7 is 6×7 which is 42.

To find the **quotient** of two numbers we divide the first one by the second one.

The number being divided is the **dividend** and the number we are dividing by is called the **divisor**.

For example, the quotient of 56 and 7 is $56 \div 7$ which is 8.

$$\underset{\text{dividend}}{56} \div \underset{\text{divisor}}{7} = \underset{\text{quotient}}{8}$$

Example 3 Self Tutor

Find: **a** the product of 7 and 12 **b** the quotient of 56 and 7.

a The product of 7 and 12
$= 7 \times 12$
$= 84$

b The quotient of 56 and 7
$= 56 \div 7$
$= 8$

When we multiply three or more numbers together, we can also rearrange their order to make the multiplication easier.

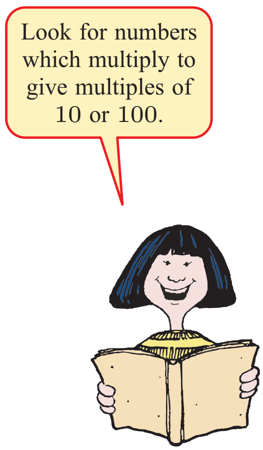
For example, $4 \times 47 \times 25$
$= 4 \times 25 \times 47$
$= 100 \times 47$
$= 4700$

Example 4 Self Tutor

Find: **a** $5 \times 19 \times 4$ **b** $16 \times 125 \times 8$

a $5 \times 19 \times 4$
$= 5 \times 4 \times 19$
$= 20 \times 19$
$= 380$

b $16 \times 125 \times 8$
$= 16 \times 1000$
$= 16\,000$



Example 5 — Self Tutor

Find the products: **a** 3×4 **b** 30×4 **c** 30×400

a 3×4
$= 12$

b 30×4
$= 3 \times 10 \times 4$
$= 12 \times 10$
$= 120$

c 30×400
$= 3 \times 10 \times 4 \times 100$
$= 12 \times 1000$
$= 12\,000$

EXERCISE 5B

1 Find the product of:

a 6 and 9 **b** 11 and 13 **c** 2, 5 and 7 **d** 3, 8 and 11
e the first five natural numbers.

2 Find the quotient of:

a 12 and 3 **b** 28 and 7 **c** 99 and 9 **d** 165 and 11.

3 Find the products by rearranging the numbers in a more convenient order:

a $5 \times 13 \times 2$ **b** $25 \times 19 \times 4$ **c** $50 \times 21 \times 2$ **d** $125 \times 19 \times 8$
e $4 \times 21 \times 5$ **f** $200 \times 97 \times 5$ **g** $40 \times 27 \times 5$ **h** $12 \times 125 \times 4$

4 Find:

a the product of 17 and 13 **b** the quotient of 120 and 6
c the sum of the products of 3 and 4, and 6 and 5.

5 Find the product:

a 3×2 **b** 30×2 **c** 30×20 **d** 300×20
e 5×7 **f** 5×70 **g** 50×70 **h** 50×700
i 3×11 **j** 30×11 **k** 300×11 **l** 300×1100

6 Find the quotient:

a $6 \div 2$ **b** $60 \div 2$ **c** $600 \div 2$ **d** $600 \div 20$
e $35 \div 7$ **f** $350 \div 7$ **g** $3500 \div 7$ **h** $350 \div 70$
i $12 \div 4$ **j** $120 \div 4$ **k** $120 \div 40$ **l** $12\,000 \div 40$

7 I buy 8 tennis racquets for \$175 each. What will it cost me?

8 What must I multiply £12 by to get £324?

9 150 rows of pine trees were planted, each row containing 80 trees. How many pine trees were planted altogether?

10 A hotel has 6 floors, each with 35 rooms. The hotel is fully occupied and the rooms cost €150 a night.

a How many rooms does the hotel have?
b How much income does the hotel have each night?
c What would be the total income over a 14 day period?

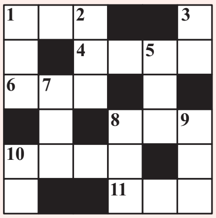
11 Paulo runs 42 000 m during a week while training for half marathons. How far does he run each day if he runs the same distance on each of:

a 7 days **b** 5 days **c** 3 days?

12 Revisit the **Opening problem** on page **86** and answer the questions.

PUZZLE OPERATIONS WITH WHOLE NUMBERS

Click on the icon to obtain a printable version of this puzzle.



Across		*Down*			
1	11×12	**1**	$445 - 249$	**9**	$1000 \div 5 - 1$
4	4×1234	**2**	$972 \div 4$	**10**	$204 \div 12$
6	$247 + 366$	**3**	7×8		
8	$1146 \div 6$	**5**	$845 - 536$		
10	427×4	**7**	$129 + 58$		
11	$347 - 128$	**8**	$85 \times 2 + 12$		

C ZERO AND ONE

Zero (0) and one (1) are two very special numbers.

ZERO

- When 0 is added to a number, the number remains the same.
- When 0 is subtracted from a number, the number remains the same.
- When a number is multiplied by 0, the result is 0.
- It is meaningless to divide by 0, so the result is **undefined**.

For example: $12 + 0 = 12$, $12 - 0 = 12$, $12 \times 0 = 0$, $12 \div 0$ is undefined.

ONE

If we multiply or divide a number by 1, it remains the same.

For example: $5 \times 1 = 1 \times 5 = 5$, $5 \div 1 = 5$.

EXERCISE 5C

1 Find, if possible:

a $7 + 0$ **b** $7 - 0$ **c** 7×0 **d** $7 \div 0$ **e** $18 - 0$

f $15 + 0 - 8$ **g** $18 \div 0$ **h** $0 \div 7$ **i** $8 + 7 - 0$ **j** $23 - 0 - 0$

2 Simplify, if possible:

a $0 + 73$ **b** $0 \div 12$ **c** $12 \div 0$ **d** $0 \div 30$ **e** $30 \div 0$

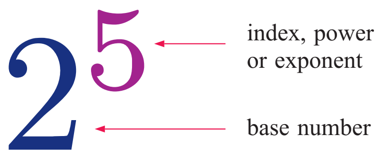
f 11×0 **g** 3×1 **h** 1×125 **i** $0 \div 8$ **j** $45 \div 1$

k 0×4 **l** 1×0 **m** 0×0 **n** $0 \div 1$ **o** $235 \div 1$

D INDEX OR EXPONENT NOTATION

Instead of writing $2 \times 2 \times 2 \times 2 \times 2$ we can write 2^5.

In 2^5, the 2 is called the **base number** and the 5 is the **index**, **power** or **exponent**. The index is the number of times the base number appears in the product.



This notation enables us to quickly write long lists of identical numbers being multiplied together.

For example:

- 3^4 is the short way of writing $3 \times 3 \times 3 \times 3$
- 10^6 is the short way of writing $1\,000\,000$ as $1\,000\,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

LANGUAGE

The following table demonstrates correct language when talking about index notation.

Natural number	*Factorised form*	*Index form*	*Spoken form*
2	2	2^1	two
4	2×2	2^2	two squared
8	$2 \times 2 \times 2$	2^3	two cubed
16	$2 \times 2 \times 2 \times 2$	2^4	two to the fourth
32	$2 \times 2 \times 2 \times 2 \times 2$	2^5	two to the fifth

Example 6 — **Self Tutor**

Write using index notation: **a** $5 \times 5 \times 5$ **b** $2 \times 2 \times 3 \times 3 \times 3 \times 3$

a $5 \times 5 \times 5$
$= 5^3$

b $2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= 2^2 \times 3^4$

EXERCISE 5D

1 Write using index notation:

a 7×7 **b** $8 \times 8 \times 8$ **c** $7 \times 7 \times 7 \times 7$

d $2 \times 2 \times 5 \times 5$ **e** $3 \times 3 \times 3 \times 11$ **f** $4 \times 4 \times 5 \times 5 \times 5$

g $2 + 3 \times 3 \times 3$ **h** $2 \times 2 + 3 \times 3$ **i** $7 \times 7 + 2 \times 2 \times 2$

j $2 \times 2 \times 2 - 2$ **k** $3 \times 3 - 2 \times 2 \times 2$ **l** $5 + 2 \times 2 \times 2 + 7 \times 7$

2 Write as a power of 10:

a 100 **b** 1000 **c** 10 000

d one million **e** one billion **f** one trillion

3 Write as an ordinary number:

a 2^4 **b** 5^4 **c** 7^4 **d** $2^3 \times 5^4$

e $2^3 + 5^4$ **f** $3^3 - 2^4$ **g** $5^3 - 3^4$ **h** $(5-3)^5$

4 Which is larger:

a 2^3 or 3^2 **b** 2^4 or 4^2 **c** 5^2 or 2^5 **d** 3^7 or 7^3?

E ORDER OF OPERATIONS

To find the value of $8 - 4 \div 2$,

Allan did the subtraction first and then the division:

$$\begin{aligned} & 8 - 4 \div 2 \\ &= 4 \div 2 \\ &= 2 \end{aligned}$$

Dale decided to do the division first and then the subtraction:

$$\begin{aligned} & 8 - 4 \div 2 \\ &= 8 - 2 \\ &= 6 \end{aligned}$$

We see that we can get different answers depending on the order in which we do the calculations. To avoid this problem, we agree to use a set of rules.

RULES FOR ORDER OF OPERATIONS

- Perform operations within **B**rackets first.
- Then, calculate any part involving **E**xponents.
- Then, starting from the left, perform all **D**ivisions and **M**ultiplications as you come to them.
- Finally, working from the left, perform all **A**dditions and **S**ubtractions.

The word **BEDMAS** may help you remember this order.

Note:
- If an expression contains only + and − operations we work from left to right.
- If an expression contains only × and ÷ operations we work from left to right.
- If an expression contains more than one set of brackets, work the innermost brackets first.

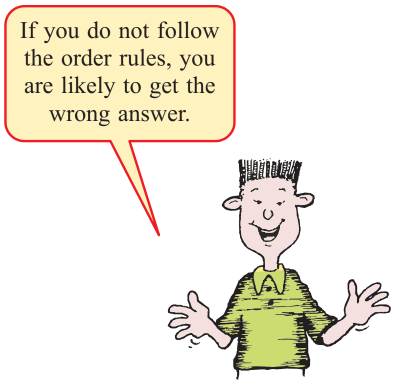
Example 7 Self Tutor

Find the value of:

a $11 + 6 - 8$ **b** $18 - 8 \div 2$

a
$$\begin{aligned} & 11 + 6 - 8 \\ &= 17 - 8 \\ &= 9 \end{aligned}$$

b
$$\begin{aligned} & 18 - 8 \div 2 \\ &= 18 - 4 \\ &= 14 \end{aligned}$$



EXERCISE 5E

1 Find:

a $12 - 6 + 8$ **b** $12 + 6 - 8$ **c** $12 \div 6 + 8$
d $2^3 \times 3 \div 3$ **e** $6 \times 2 \div 3$ **f** $9 + 8 \div 2^2$
g $12 \div 3 + 2$ **h** $12 \div 4 - 2$ **i** $6 \times 6 \div 2$
j $5 + 6 \div 3$ **k** $20 \div 2 \div 5$ **l** $17 - 7 \times 2$
m $3^3 + 3 \times 5$ **n** $5 \times (6 - 2)$ **o** $5 \times 6 - 2$
p $(8 - 4) \div 2$ **q** $8 - 4 \div 2$ **r** $11 - 2 + 3$
s $11 - (2 + 3)$ **t** $6 + (9 \div 3)$ **u** $(6 + 9) \div 3$

Example 8 Self Tutor

Find: **a** $7 + 3 \times 2 - 4$ **b** $9 \div 3 + 7 \times 2$

a
$$\begin{aligned} & 7 + 3 \times 2 - 4 \\ &= 7 + 6 - 4 \\ &= 13 - 4 \\ &= 9 \end{aligned}$$

b
$$\begin{aligned} & 9 \div 3 + 7 \times 2 \\ &= 3 + 14 \\ &= 17 \end{aligned}$$

2 Find the value of:

a $7 + 6 - 5 + 2$ **b** $18 \div 2 \times 3 - 1$ **c** $18 \div 3 + 10 \times 3$
d $7 + 3 \times 4 \times 2^2$ **e** $8 \times 3 - 4 \times 5$ **f** $2^3 \times 4 + 3 \times 2$
g $5 + 7 - 3 \times 4$ **h** $5 + (7 - 3) \times 4$ **i** $32 - (12 - 5) \times 3$
j $22 \div 2 + 5 \times 4$ **k** $21 \div (2 + 5) + 4$ **l** $(7 + 2) \times 5 - 4$
m $18 - (5 + 4) \div 3$ **n** $(14 + 6) \div (9 - 5)$ **o** $6 \times (5 - 2) + 1$
p $27 - 7 \times 2 + 2^3$

Example 9 Self Tutor

Find the value of $23 - [17 - (2 \times 5)]$.

$$\begin{aligned} & 23 - [17 - (2 \times 5)] \\ &= 23 - [17 - 10] \\ &= 23 - 7 \\ &= 16 \end{aligned}$$

3 Find the value of:

a $2 \times [(3 + 2) - 4]$ **b** $[2 \times (8 - 2)] \div 3$ **c** $[(4 \times 5) - 12] \div 8$
d $[4 \times 2 - 2] \times 5$ **e** $[4 \times (2 - 2)] \times 5$ **f** $2 + [(3 \times 7) - 11] \times 3$
g $[3 \times (8 - 2)] - 10$ **h** $5^2 - [(8 - 4) \times 2]$
i $[(3 \times 2) + (11 - 4)] \times 2$

Example 10 **Self Tutor**

Simplify:
$4 \times (7-4)^3$

$4 \times (7-4)^3$
$= 4 \times 3^3$
$= 4 \times 27$
$= 108$

4 Simplify:

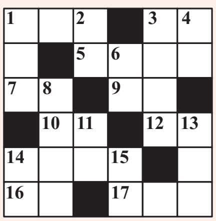
a $2^2 + 5^2$ **b** $(2+5)^2$ **c** 2×3^2 **d** $(2 \times 3)^2$
e $(4-2)^3 \div 8$ **f** $3 \times 2 + 2^2$ **g** $5 + (4+5)^2$ **h** $3^4 - (3 \times 2)^2$

5 Replace $*$ and ♦ by either $+$, $-$, $\times$ or $\div$ to make a correct statement

a $4 + 18 * 3 = 10$ **b** $6 * 7 - 12 = 30$ **c** $(17 * 3) \div 5 = 4$
d $(18 * 2)$ ♦ $8 = 2$ **e** $12 * 4 + 10$ ♦ $2 = 23$ **f** $12 * 4 - 10$ ♦ $2 = 43$

PUZZLE

Click on the icon to obtain a printable version of this puzzle.



Across
1 $40 \times 5 - 17$
3 $100 - (7-1)$
5 $(1 + 5 \times 50) \times 25$
7 $3 \times (3 + 20)$
9 $8 \times 11 - 7$
10 $100 - 9 \times 2$
12 $5 \times (6+7)$
14 $153 \div 3 + 3 \times 1000$
16 $90 - 4 \times 4$
17 $9 \times 100 + 8 \times 5$

Down
1 $100 + 24 \div 4$
2 $10 \times 4 - 20 \div 5$
3 $10\,000 - 3 \times 100 + 2 \times 8$
4 $7 \times 7 - 2 \times 2$
6 $(7-3) \times (6+1)$
8 $100 \times 100 - 14 \times 14$
11 $625 \div (20+5)$
13 $10 \times (9 \times 6)$
14 $70 - 3 \times 11$
15 $2 \times 5 + 3 \times 3$

ACTIVITY 1 NUMBER PUZZLES

In these number puzzles each letter stands for a different one of the digits 0, 1, 2, 3, ... to 9. There are several solutions to each puzzle. Can you find one of them? Can you find all of them?

a
$$\begin{array}{cccc} & D & O & G \\ + & C & A & T \\ \hline H & A & T & E \end{array}$$

b
$$\begin{array}{ccccc} & S & U & R & F \\ - & S & A & N & D \\ \hline & & S & E & A \end{array}$$

F POWERS WITH BASE 10

When we use 10 as a base, the index shows the place value or number of zeros following the one.

	Tens of millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
10^1							1	0
$10^2 = 10 \times 10 =$						1	0	0
$10^3 = 10 \times 10 \times 10 =$					1	0	0	0
$10^4 = 10 \times 10 \times 10 \times 10 =$				1	0	0	0	0
$10^5 = 10 \times 10 \times 10 \times 10 \times 10 =$			1	0	0	0	0	0
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$		1	0	0	0	0	0	0
$10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$	1	0	0	0	0	0	0	0

In **expanded notation** we write the number as the sum of its place values.

For example, $5042 = (5 \times 1000) + (4 \times 10) + (2 \times 1)$.

Power notation is expanded notation written with powers of 10.

For example, $5042 = (5 \times 10^3) + (4 \times 10^1) + (2 \times 1)$

Example 11 Self Tutor

Write the simplest numeral for these numbers:

$$(8 \times 10^4) + (7 \times 10^3) + (5 \times 10^2) + (3 \times 10^1) + (9 \times 1)$$

$$\begin{aligned} &(8 \times 10^4) + (7 \times 10^3) + (5 \times 10^2) + (3 \times 10^1) + (9 \times 1) \\ &= (8 \times 10\,000) + (7 \times 1000) + (5 \times 100) \times (3 \times 10) + (9 \times 1) \\ &= 87\,539 \end{aligned}$$

Example 12 Self Tutor

Expand 952 473 using power notation.

$$\begin{aligned} &952\,473 \\ &= (9 \times 100\,000) + (5 \times 10\,000) + (2 \times 1000) + (4 \times 100) + (7 \times 10) + (3 \times 1) \\ &= (9 \times 10^5) + (5 \times 10^4) + (2 \times 10^3) + (4 \times 10^2) + (7 \times 10^1) + (3 \times 1) \end{aligned}$$

EXERCISE 5F

1 Write the simplest numerals for each of the following:

a $(8 \times 100\,000) + (6 \times 10\,000) + (2 \times 1000) + (9 \times 100) + (5 \times 10) + (3 \times 1)$

b $(3 \times 1\,000\,000) + (5 \times 10\,000) + (7 \times 100) + (9 \times 1)$

c $(2 \times 10^7) + (3 \times 10^5) + (6 \times 10^4) + (9 \times 10^3) + (6 \times 10^1) + (8 \times 1)$

d $(10^6) + (10^4) + (10^3) + (10^2) + (9 \times 10^1)$

e 9 thousands and 8 hundreds and 3 tens and 6 units

f 8 hundred thousands + 9 ten thousands + 6 hundreds + 3 tens and seven units

g 5 ten millions + 8 hundred thousands + seven ten thousands + 5 thousands

2 Write these numbers using expanded notation:

a 9738 **b** 29 782 **c** 40 404 **d** 657 931

e 800 888 **f** 1 247 091 **g** 49 755 400 **h** 6 777 777

3 Expand these numbers using power notation:

a 658 **b** 3874 **c** 95 636 **d** 100 100

e 505 750 **f** 1 274 947 **g** 36 600 000 **h** 4 293 375

i four hundred thousand, six hundred and eighty seven

j twenty three million, six hundred and ninety seven thousand, five hundred

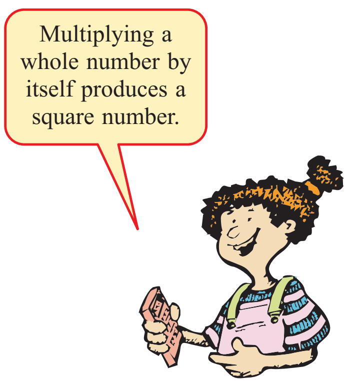
G SQUARES AND CUBES

SQUARE NUMBERS

The product of two identical whole numbers is a **square number**.

For example: $1 \times 1 = 1$ so $1^2 = 1$; $2 \times 2 = 4$ so $2^2 = 4$; $3 \times 3 = 9$ so $3^2 = 9$; $12 \times 12 = 144$ so $12^2 = 144$.

So, 1, 4, 9 and 144 are all square numbers.



SQUARE ROOTS

The square root of the square number 9 is written as $\sqrt{9}$.
It is the positive number which when squared gives 9.

Since $3^2 = 9$, $\sqrt{9} = 3$.

The **square root** of a is written as $\sqrt{a}$. $\sqrt{a} \times \sqrt{a} = a$

For example: $2^2 = 4$ so $\sqrt{4} = 2$. $5^2 = 25$ so $\sqrt{25} = 5$. $11^2 = 121$ so $\sqrt{121} = 11$. $15^2 = 225$ so $\sqrt{225} = 15$.

CUBE NUMBERS

The product of three identical whole numbers is a **cube number**.

For example, 8 is a cube number as $2^3 = 2 \times 2 \times 2 = 8$.

CUBE ROOTS

The cube root of 8 is written $\sqrt[3]{8}$.

It is the number when multiplied by itself twice gives 8

Since $2 \times 2 \times 2 = 2^3 = 8$, $\sqrt[3]{8} = 2$.

The **cube root** of a is written as $\sqrt[3]{a}$.

$$\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$$

EXERCISE 5G

1 Find the value of:

- **a** 4^2
- **b** 5^2
- **c** 7^2
- **d** 10^2
- **e** $2^2 + 4^2$
- **f** $(2+4)^2$
- **g** $5^2 - 2^2$
- **h** $(5-2)^2$

2 $4^2 = 16$ ends in a 6 and $5^2 = 25$ ends in a 5.

- **a** List all the possible numbers that a square number could end in.
- **b** Is 638 254 916 823 620 058 a square number?

3 Find the square root of:

- **a** 1
- **b** 16
- **c** 36
- **d** 81
- **e** 144

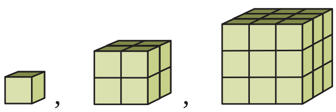
4 Find:

- **a** $\sqrt{49}$
- **b** $\sqrt{64}$
- **c** $\sqrt{100}$
- **d** $\sqrt{0}$
- **e** $\sqrt{400}$

5 Find the first 10 cube numbers, beginning with $1^3 = 1$.

6 Find:

- **a** $2^3 - 2^2$
- **b** $5^3 - 5$
- **c** $4^3 + 2^3$
- **d** $7^3 - 7^2$

7  represent the first 3 cube numbers.

- **a** Draw a sketch which represents 4^3.
- **b** Explain why these diagrams do represent 1^3, 2^3, 3^3 and 4^3.

8 Find:

- **a** $\sqrt[3]{27}$
- **b** $\sqrt[3]{64}$
- **c** $\sqrt[3]{125}$
- **d** $\sqrt[3]{1000}$

H FACTORS OF NATURAL NUMBERS

The **factors** of a natural number are the natural numbers which divide exactly into it.

For example, the factors of 6 are 1, 2, 3 and 6 since $6 \div 1 = 6$, $6 \div 2 = 3$, $6 \div 3 = 2$, and $6 \div 6 = 1$.

4 is not a factor of 6 as $6 \div 4 = 1$ with remainder 2.

All natural numbers can be split into **factor pairs**.

For example, $11 = 11 \times 1$ and $6 = 1 \times 6$ or 2×3.

12 has factors 1, 2, 3, 4, 6 and 12, so 12 can be split into 1×12, 2×6 or 3×4.

DIVISIBILITY

One number is **divisible** by another if the second number is a factor of the first.

12 is divisible by 1, 2, 3, 4, 6 and 12 since division by any of these numbers leaves no remainder.

EVEN AND ODD NUMBERS

A natural number is **even** if it has 2 as a factor and thus is divisible by 2.

A natural number is **odd** if it is not divisible by 2.

The units digit of an even number will be 0, 2, 4, 6 or 8.

The units digit of an odd number will be 1, 3, 5, 7 or 9.

EXERCISE 5H

1 **a** List the factors of the following natural numbers in ascending order (smallest to largest):

i 2 **ii** 3 **iii** 4 **iv** 5 **v** 7 **vi** 8
vii 9 **viii** 10 **ix** 11 **x** 13 **xi** 14 **xii** 15
xiii 16 **xiv** 17 **xv** 18 **xvi** 19 **xvii** 20 **xviii** 21

b Which of the natural numbers in **a** have exactly two different factors?

c List the natural numbers less than 22 which have:

i exactly 4 different factors **ii** more than 4 different factors.

2 List the factors of:

a 23 **b** 24 **c** 100 **d** 45 **e** 64 **f** 72

3 **a** Beginning with 8, write three consecutive even numbers.

b Beginning with 17, write five consecutive odd numbers.

4 **a** Write two even numbers which are not consecutive and which add to 10.

b Write all the sets of two non-consecutive odd numbers which add to 20.

c Write all the sets of three different even numbers which add to 20.

5 Use the words "even" and "odd" to complete these sentences correctly:
  a The sum of two even numbers is always ......
  b The sum of two odd numbers is always ......
  c The sum of an odd number and an even number is always ......
  d When an even number is subtracted from an odd number, the result is ......
  e When an odd number is subtracted from an odd number, the result is ......
  f The product of two odd numbers is always ....
  g The product of an even and an odd number is always ......

I DIVISIBILITY TESTS

How can we quickly decide whether one number is divisible by another? Obviously this can be done using a calculator provided the number is not too big. However, there are also some simple tests we can follow to determine whether one number is divisible by another, without actually doing the division!

STANDARD DIVISIBILITY TESTS

Number	Divisibility Test
2	If the last digit is 0 or even, then the original number is divisible by 2.
3	If the sum of the digits is divisible by 3, then the original number is divisible by 3.
4	If the number formed by the last *two* digits is divisible by 4, then the original number is divisible by 4.
5	If the last digit is 0 or 5 then the number is divisible by 5.
6	If a number is divisible by both 2 and 3 then it is divisible by 6.

Example 13 Self Tutor

Is 768 divisible by: **a** 3 **b** 6?

a The sum of the digits $= 7 + 6 + 8 = 21$
and 21 is divisible by 3 $\{$as $21 \div 3 = 7\}$
$\therefore$ 768 is divisible by 3.

b 768 ends in 8 which is even
$\therefore$ 768 is divisible by 2
We showed in **a** that 768 is divisible by 3.
$\therefore$ 768 is divisible by 6.

EXERCISE 5I

1 Which of these numbers are divisible by 2?

a 216 **b** 3184 **c** 827 **d** 4770 **e** 123 456

2 Which of these numbers are divisible by 3?

a 84 **b** 123 **c** 437 **d** 111 114 **e** 707 052

3 Which of these numbers are divisible by 5?

a 400 **b** 628 **c** 735 **d** 21 063 **e** 384 005

4 Which of these numbers are divisible by 4?

a 482 **b** 2556 **c** 8762 **d** 12 368 **e** 213 186

5 Which of these numbers are divisible by 6?

a 162 **b** 381 **c** 1602 **d** 2156 **e** 5364

6 Consider the numbers of the form 3□8. Which digits could be put in place of □ so that the number 3□8 is:

a even **b** divisible by 3 **c** divisible by 4 **d** divisible by 6?

7 Paul believes that the number forms alongside are always divisible by 6:

$2^3 - 1^3 - 1$
$3^3 - 2^3 - 1$
$4^3 - 3^3 - 1$
$5^3 - 4^3 - 1$
$\vdots$

a Check that the first four of them are divisible by 6.

b Check that $10^3 - 9^3 - 1$ is divisible by 6.

8 What digits could replace □ so that these numbers are divisible by 3?

a 3□2 **b** 8□5 **c** 3□14 **d** □229

J PRIME AND COMPOSITE NUMBERS

Some numbers have only two factors, one and the number itself.

For example, the only two factors of 5 are 5 and 1, and of 23 are 23 and 1.

Numbers of this type are called **prime numbers**.

A **prime** number is a natural number which has exactly two different factors, 1 and itself.

A **composite** number is a natural number which has more than two factors.

The first 14 prime numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, and 43

but the list extends on forever.

Since 1 has only one factor (itself), the number **1** is neither prime nor composite.

PRIME FACTORS

To find the prime factors of a composite number, we systematically divide the number by the prime numbers which are its factors, starting with the smallest.

All composite numbers can be written as a product of prime numbers in index form in exactly one way.

Example 14 Self Tutor

a Write 792 as a product of prime factors in index form.

b What are the prime factors of 792?

a $792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$

$= 2^3 \times 3^2 \times 11$

b 792 has prime factors 2, 3 and 11.

2	792
2	396
2	198
3	99
3	33
11	11
	1

EXERCISE 5J.1

1 **a** List all the prime numbers up to 29.

b Explain why 1 is not a prime number.

c How many even prime numbers are there?

d List the prime numbers between:

i 30 and 40 **ii** 60 and 70 **iii** 90 and 110.

2 What are the prime factors of:

a 7 **b** 12 **c** 50 **d** 42 **e** 108 **f** 210?

3 Give reasons why these numbers are not primes:

a 284 **b** 5615 **c** 2804 **d** 993 **e** 2709 **f** 111 111

4 Write in index form with the base number as small as possible

a 4 **b** 9 **c** 25 **d** 8 **e** 27 **f** 32

g 81 **h** 64 **i** 125 **j** 243 **k** 128 **l** 343

5 Write these numbers as a product of prime factors in index form:

a 72 **b** 160 **c** 180 **d** 968 **e** 3920 **f** 13 500

HIGHEST COMMON FACTOR (HCF)

A number which is a factor of two or more other numbers is called a **common factor** of these numbers.

For example, 5 is a common factor of 15 and 40 since 5 is a factor of both of these numbers.

We can use the method of finding prime factors to find the **highest common factor** (**HCF**) of two or more natural numbers.

Example 15 Self Tutor

Find the highest common factor of 18 and 30.

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ $18 = 2 \times 3 \times 3$
and $30 = 2 \times 3 \times 5$

So, the HCF of 18 and 30 is $2 \times 3 = 6$.

EXERCISE 5J.2

1 Find the HCF of:

a 2 and 3 **b** 4 and 10 **c** 6 and 30 **d** 12 and 20
e 18 and 36 **f** 18 and 27 **g** 42 and 14 **h** 32 and 24
i 24 and 60 **j** 24 and 72 **k** 33 and 77 **l** 26 and 52

2 Find the HCF of:

a 2, 3, 5 **b** 4, 8, 20 **c** 30, 12, 36 **d** 12, 18, 36

K MULTIPLES AND LCM

The **multiples** of any whole number have that number as a factor. They are obtained by multiplying it by 1, then 2, then 3, then 4, and so on.

The multiples of 4 are: 4×1, 4×2, 4×3, 4×4, 4×5,
4, 8, 12, 16, 20, ...

The multiples of 6 are: 6, 12, 18, 24, 30, ...

So, 12 and 24 are two **common multiples** of 4 and 6, and 12 is the **lowest common multiple (LCM)** of 4 and 6.

Example 16 — Self Tutor

a List the first ten multiples of 4 and 10.
b Find all common multiples of 4 and 10 which are less than or equal to 40.
c What is the lowest common multiple of 4 and 10?

a The first ten multiples of 4 are: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**
The first ten multiples of 10 are: 10, **20**, 30, **40**, 50, 60, 70, 80, 90, 100
b The common multiples of 4 and 10 are 20 and 40.
c The LCM is 20.

EXERCISE 5K

1 List the first 5 multiples of:
a 6 **b** 11 **c** 12 **d** 15 **e** 20 **f** 35

2 What is the 13th multiple of 7?

3 Find the LCM of:
a 4 and 6 **b** 4 and 8 **c** 6 and 8 **d** 5 and 7
e 5 and 9 **f** 2, 4 and 6 **g** 3, 4 and 8 **h** 2, 3, 4 and 6.

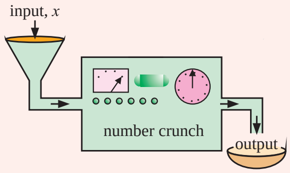
4 Find the largest multiple of 11 which is less than 200.

5 I am an odd multiple of 5 and the sum of my three digits is 18. What number am I?

ACTIVITY 2 NUMBER CRUNCHING MACHINE

Any positive integer can be fed into a number crunching machine which produces one of two results:

- If the integer fed in is **even**, the machine divides the number by 2.
- If the integer fed in is **odd**, the machine subtracts one from the number.



What to do:

1 Find the result when the following numbers are fed into the machine:
a 26 **b** 15 **c** 42 **d** 117

2 What was the input to the machine if the output is:
a 8 **b** 13?

It is possible to feed the output from the machine back into the input, and continue to do so until the output reaches zero.

For example, with an initial input of 11, the following would occur:

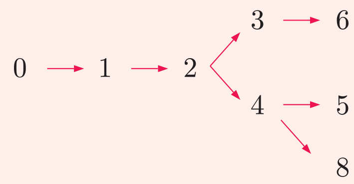
$$11 \longrightarrow 10 \longrightarrow 5 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0.$$

We see that 6 steps are required to reach zero.

3 Give the number of steps required to reach zero if you start with:

a 7 **b** 24 **c** 32

4 There are three 4-step numbers.
The method of finding them is to work in **reverse**.
The only 4-step numbers are: 5, 6 and 8.
Can you determine all the 5-step numbers?



5 By changing the rules for the number crunching machine, different outputs can be obtained. Try some different possibilities for yourself.
For example:
- If the number is divisible by 3, divide it by 3.
- If the number is not divisible by 3, subtract 1.

KEY WORDS USED IN THIS CHAPTER

- base
- cube root
- divisor
- factor
- lowest common multiple
- power
- quotient
- sum
- composite
- difference
- even number
- highest common factor
- number sequence
- prime
- square number
- term
- cube number
- dividend
- exponent
- index
- odd number
- product
- square root
- undefined

LINKS
click here

CICADAS

Areas of interaction:
Environments, Approaches to learning

REVIEW SET 5A

1 Simplify:

a $13 - 0 + 19$ **b** 23×0 **c** $31 + 238 + 69$ **d** $0 \div 18$

2 What number must be increased by 211 to get 508?

3 Find the sum of the first three square numbers.

4 Find **a** $\sqrt{49}$ **b** $\sqrt[3]{64}$

5 Simplify:

a $17 - 7 \times 2$ **b** $24 - (6 + 2) \times 2$ **c** $[5 \times (2 + 6)] \div 4$

6 Simplify:

a $396 \times 483 \times 0$ **b** $25 \times 17 \times 4$ **c** $23 \times 40 \times 5$

7 23 students each get 15 books at the start of the year. How many books were given out in total?

8 Replace $*$ by either $+$, $-$, $\times$, or $\div$ in $32 \div 8 * 4 + 4 = 12$ to make a correct statement.

9 List all the factors of 54.

10 What is the 13th odd number?

11 List the prime numbers between 20 and 30.

12 What is the 6th multiple of 7?

13 Find the HCF of 16 and 56.

14 Find the LCM of 8 and 6.

15 What numbers could $\square$ be if $2\square8$ is divisible by:

a 3 **b** 4?

16 Write $2^3 \times 3^2$ as a natural number.

17 Write as a product of prime numbers in index form:

a 45 **b** 144.

REVIEW SET 5B

1 Find:

a the difference between 17 and 35
b the sum of 13, 27 and 38
c the product of 13 and 8
d the quotient of 115 and 5.

2 What number must be increased by 124 to get 360?

3 Find:

a 3×0 **b** $0 \div 3$ **c** $3 \div 0$ **d** $298 + 117 + 2$

4 Sarah gives \$26 to each of her 5 children. If she had \$180 in her purse, how much money does she now have left?

5 Simplify:

a $6 \times 2 + 8 \times 3$ **b** $45 - 2 \times 3^2$ **c** $13 + [11 - (12 - 8)] \times 2$

6 Replace $*$ by either $+$, $-$, $\times$, or $\div$ in $2 \times 8 * 4 + 2 = 6$ to make a correct statement.

7 List all the: **a** factors of 48
b prime numbers between 50 and 60
c multiples of 7 between 13 and 28.

8 Write 92 as a product of prime numbers in index form.

9 Convert $2^5 \times 3^2$ to a natural number.

10 What results when two odd numbers are:

a added **b** multiplied?

11 The four year 6 classes at a school each contain 27 students. An extra class will be created if a further 17 students arrive. What will the class sizes be then?

12 For the numbers of the form □32, what values could □ have so that the number is divisible by:

a 3 **b** 4 **c** 6?

13 Find the value of:

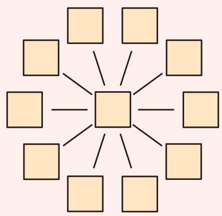
a 4^2 **b** 3^3 **c** $\sqrt{81}$ **d** $\sqrt[3]{125}$

14 Simplify: $(5 \times 10^5) + (3 \times 10^3) + (8 \times 10^2) + 6$

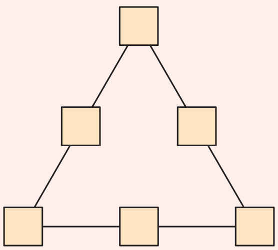
15 Find the prime factors of 392 and write 392 as a product of prime factors in index form.

PUZZLE NUMBER PUZZLES

1 In the eleven squares write each of the numbers from 1 to 11 so that every set of three numbers in a straight line adds up to 18.



PRINTABLE TEMPLATE

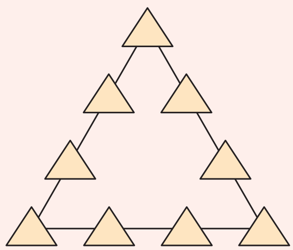
2 

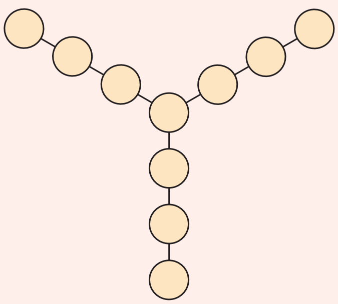
Draw **three** triangles like the one shown. Using each number once only, place the numbers 2 to 7 in the squares so that each side of the triangle adds up to:

a 12 **b** 13 **c** 14

3 Draw three triangles like the one shown. Using each number once only, place the numbers 11 to 19 in the triangles so that each side of the triangle adds up to:

a 57 **b** 59 **c** 63



4 

Draw three shapes as shown. Using each number once only, place the numbers 1 to 10 in the circles so that each line leading to the centre adds up to:

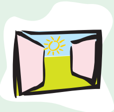
a 19 **b** 21 **c** 25

Chapter 6

Fractions

Contents:

OPENING PROBLEM



When Uncle Paulo died he left all his money to his sister's children, 3 of whom are girls and 4 are boys. They are each to get equal shares of the total amount of €350 000.

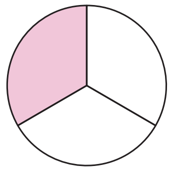
Things to think about:

a What part of the inheritance does each child receive?

b What part do the girls receive?

c How much do the boys receive in total?

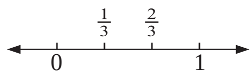
The circle alongside is *divided* into three equal portions. The one whole circle is divided into three, so the one portion that is shaded represents $1 \div 3$ of the whole circle.



We commonly write this as the **fraction** $\frac{1}{3}$.

Two of the three portions are unshaded, so this is $2 \div 3$ or $\frac{2}{3}$ of the circle.

On a number line, we have divided the segment from 0 to 1 into three equal parts. We can then place $\frac{1}{3}$ and $\frac{2}{3}$ on the number line.



In general, $a \div b$ can be written as the **fraction** $\dfrac{a}{b}$.

$\dfrac{a}{b}$ means we divide a whole into b equal portions, and then consider a of them.

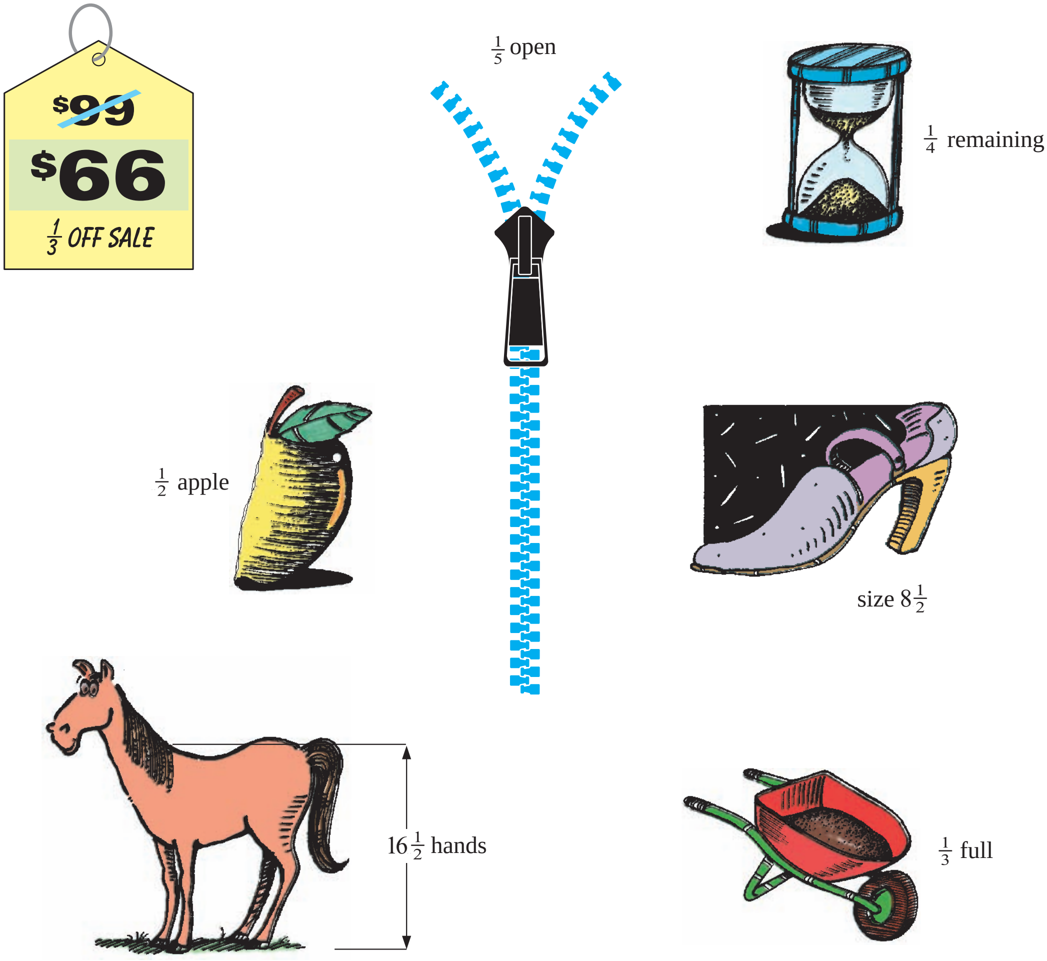
$\dfrac{a}{b}$
- the **numerator** is the number of portions considered
- the **bar** indicates division
- the **denominator** is the number of portions we divide a whole into.

The denominator cannot be zero, as we cannot divide a whole into zero pieces.

Other denominators we describe using different words:

Denominator	*Name of portions*
2	half
3	third
4	quarter
5	fifth
6	sixth

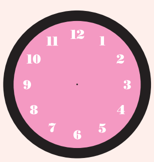
FRACTIONS ARE EVERYWHERE

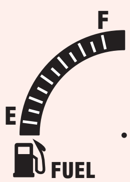


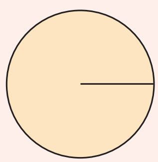
ACTIVITY 1 FRACTIONS WE ALL KNOW

What to do:

1 Copy and complete the following sketches to show:







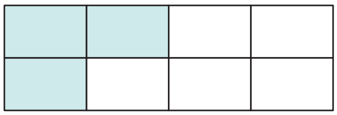
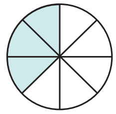
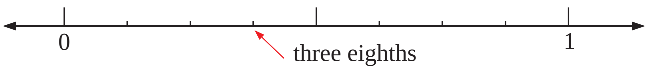
a half past twelve

b a petrol gauge showing the tank is almost three quarters full

c a pizza with one fifth of it missing.

A REPRESENTING FRACTIONS

The fraction three eighths can be represented in a number of different ways:

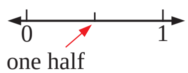
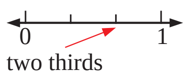
Words	three eighths
Diagram	as a shaded region *or* as pieces of a pie  
Number line	
Symbol	$\frac{3}{8}$ numerator, bar, denominator

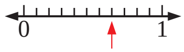
A fraction written in symbolic form with a bar is called a **common fraction.**

PRINTABLE WORKSHEET

EXERCISE 6A

1 Copy and complete the following table:

	Symbol	*Words*	*Numerator*	*Denominator*	*Meaning*	*Number Line*
a		one half		2	One whole divided into two equal parts and one is being considered.	
b	$\frac{3}{4}$	three quarters			One whole divided into four equal parts and three are being considered.	
c	$\frac{2}{3}$		2	3		
d		two sevenths		7		
e					One whole divided into nine equal parts and seven are being considered.	

	Symbol	Words	Numerator	Denominator	Meaning	Number Line
f			5	8		
g						

ACTIVITY 2 ESTIMATING FRACTIONS

What to do:

1 Make your own fraction wheel as follows:

a Use a drawing compass to draw two identical circles on two different coloured pieces of cardboard.

b Use your protractor to mark the fractions as shown on the second circle.

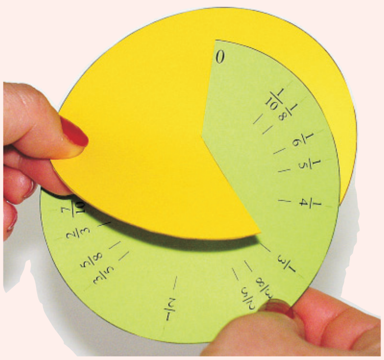
For example, $\frac{1}{10}$ is $(360^o \div 10) = 36^o$,

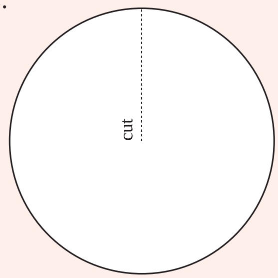
$\frac{1}{8}$ is $(360^o \div 8) = 45^o$,

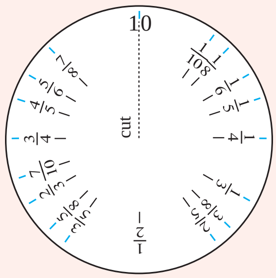
$\frac{3}{8}$ is $(360^o \div 8 \text{ then } \times 3) = 135^o$.

c Cut out both pieces.

d Mark and cut a radius on both circles as shown. Interlock the circles.







2 Challenge your partner to guess the fractions you make by looking at the reverse side which has no fractions written on it. Have your partner estimate the fraction which adds to yours to make one.

3 Click on the icon to load a game for estimating fractions. Play the game until you become good at recognising the size of different fractions.

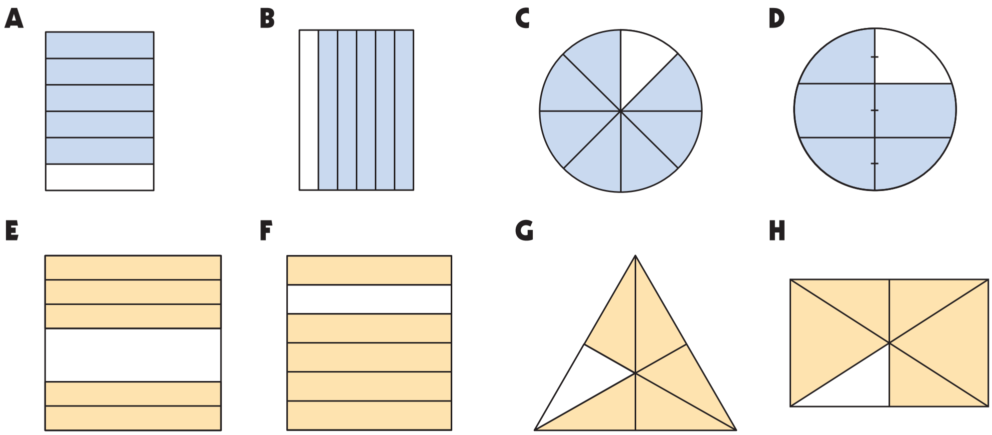
FRACTION ESTIMATOR

B FRACTIONS OF REGULAR SHAPES

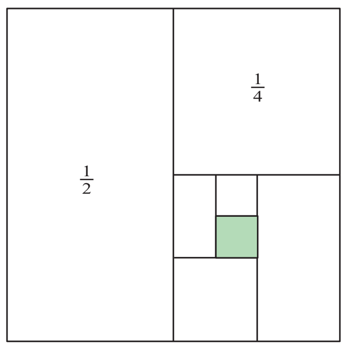
A good way to learn about fractions is to divide regular two dimensional shapes.

EXERCISE 6B

1 Which of the following shaded shapes does not show five sixths?



2 Copy the given shape exactly. Consider the large square to be a whole or 1.



a If each rectangle is half of the one before it, how much of the shape is unshaded?

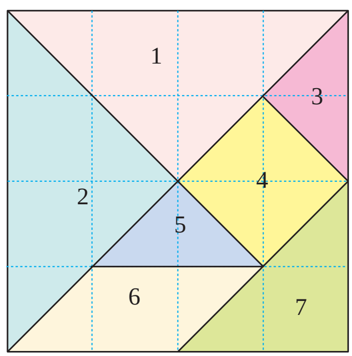
b Check your answer to **a** by drawing a grid within the large square. Use the boundaries of the shaded square as the dimensions of the smallest squares in your grid.

c How many of the smallest squares fit into your large square?

d What fraction of the whole is the shaded square?

e What fraction of the whole is the unshaded area?

3 Using identical square pieces of paper, make 2 copies of this tangram. Number the pieces on both sheets. Cut one of the sheets into its seven pieces. Use the pieces to help you work out the following:



a How many triangles like piece 1 would fit into the largest square?

b What fraction of the largest square is piece 1?

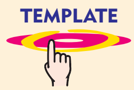
c What fraction of piece 1 is piece 3?

d What fraction of the largest square is each tangram piece?

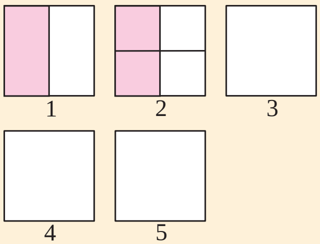
C EQUAL FRACTIONS

INVESTIGATION EQUAL FRACTIONS

What to do:



1 Use grid paper to construct 6 identical squares with sides 4 cm long, or click on the icon to obtain a template. Use the grid lines on the paper to guide you.



a Divide the first square into 2 equal parts. Each part is one half $\frac{1}{2}$. One half has been shaded. Divide the second square into quarters. Each half is now equivalent to two quarters or $\frac{2}{4}$. Shade in the same half as you did in the first square.

b Divide the third square into eighths. Shade in the one half of the big square.

c Divide the fourth square into sixteenths. Shade in the one half of the big square.

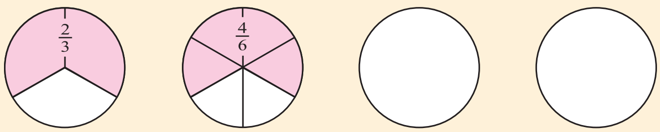
d In the fifth square show that one half equals $\frac{16}{32}$.

e Copy and complete: $\frac{1}{2} = \frac{2}{4} = \ldots\ldots = \ldots\ldots = \frac{16}{32}$.

2 **a** Use a protractor to outline 4 identical circles.

b From the centre of the first circle, measure and rule 3 lines, 120^o apart. Since $3 \times 120^o = 360^o$, you have divided the circle into thirds. Shade $\frac{2}{3}$.

c In the second circle draw 6 lines 60^o apart. Since $6 \times 60^o = 360^o$, you have divided the circle into sixths. Shade $\frac{4}{6}$.



d In the third circle draw 12 lines 30^o degrees apart. Shade the appropriate equal area.

e Continue the pattern in the fourth circle.

f Copy and complete: $\frac{2}{3} = \frac{4}{6} = \frac{\ldots}{12} = \ldots\ldots$.

In the investigation above, you should have found that:

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32} \quad \text{and} \quad \frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{16}{24}$$

Notice how these numbers are related:

$$\frac{1}{2} \overset{\times 2}{=} \frac{2}{4} \overset{\times 2}{=} \frac{4}{8} \overset{\times 2}{=} \frac{8}{16} \overset{\times 2}{=} \frac{16}{32} \qquad \frac{2}{3} \overset{\times 2}{=} \frac{4}{6} \overset{\times 2}{=} \frac{8}{12} \overset{\times 2}{=} \frac{16}{24}$$

(numerators and denominators each multiplied by 2 at every step)

This suggests that we can use **multiples** to find fractions that are equal.

For example: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = ...$ (numerator and denominator of $\frac{1}{2}$ multiplied by $\times 2$, $\times 3$, $\times 4$, $\times 5$)

> Multiplying or dividing both the numerator and the denominator by the same non-zero number produces an equal fraction.

For example: $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$ and $\frac{2}{5} = \frac{2 \times 12}{5 \times 12} = \frac{24}{60}$ and so $\frac{2}{5} = \frac{4}{10} = \frac{24}{60}$.

$\frac{12}{18} = \frac{12 \div 2}{18 \div 2} = \frac{6}{9}$ and $\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$ and so $\frac{12}{18} = \frac{6}{9} = \frac{2}{3}$.

Example 1 — Self Tutor

Express with denominator 18:

a $\frac{7}{9}$ **b** $\frac{5}{6}$

a $\frac{7}{9}$
$= \frac{7 \times 2}{9 \times 2}$ $\{$as $9 \times 2 = 18\}$
$= \frac{14}{18}$

b $\frac{5}{6}$
$= \frac{5 \times 3}{6 \times 3}$ $\{$as $6 \times 3 = 18\}$
$= \frac{15}{18}$

EXERCISE 6C

DEMO

1 Express with denominator 8:

a $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** 1

2 Express with denominator 30:

a $\frac{1}{2}$ **b** $\frac{4}{5}$ **c** $\frac{5}{6}$ **d** $\frac{3}{10}$
e $\frac{1}{5}$ **f** $\frac{2}{3}$ **g** 1 **h** $\frac{3}{5}$

3 Express in sixteenths:

a $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** 1 **d** 0
e $\frac{7}{8}$ **f** $\frac{3}{4}$ **g** $\frac{5}{8}$ **h** 2

4 Express in hundredths:

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{4}{5}$ **d** $\frac{9}{10}$
e $\frac{7}{25}$ **f** $\frac{13}{50}$ **g** 1 **h** $\frac{17}{20}$

5 Multiply to find equal fractions:

a $\frac{5}{6} = \frac{5 \times 2}{6 \times \square} = \frac{10}{12}$ **b** $\frac{8}{9} = \frac{8 \times 3}{9 \times \square} = \frac{24}{\square}$ **c** $\frac{5}{7} = \frac{5 \times \square}{7 \times 5} = \frac{25}{\square}$

d $\frac{3}{4} = \frac{3 \times 8}{4 \times \square} = \frac{\square}{32}$ **e** $\frac{4}{5} = \frac{4 \times \square}{5 \times \square} = \frac{40}{50}$ **f** $\frac{7}{8} = \frac{7 \times \square}{\square \times \square} = \frac{28}{32}$

6 Divide to find equal fractions:

a $\frac{6}{8}=\frac{6\div 2}{8\div\square}=\frac{3}{4}$ **b** $\frac{8}{10}=\frac{8\div\square}{10\div 2}=\frac{4}{\square}$ **c** $\frac{10}{15}=\frac{10\div 5}{15\div\square}=\frac{\square}{3}$

d $\frac{18}{21}=\frac{18\div 3}{21\div\square}=\frac{\square}{\square}$ **e** $\frac{15}{25}=\frac{\square\div 5}{25\div\square}=\frac{\square}{5}$ **f** $\frac{18}{20}=\frac{\square\div\square}{20\div\square}=\frac{9}{\square}$

7 Find $\square$ if:

a $\frac{\square}{3}=\frac{7}{21}$ **b** $\frac{\square}{5}=\frac{12}{15}$ **c** $\frac{\square}{11}=\frac{56}{77}$ **d** $\frac{15}{35}=\frac{\square}{7}$

e $\frac{27}{63}=\frac{\square}{7}$ **f** $\frac{27}{81}=\frac{\square}{3}$ **g** $\frac{\square}{13}=\frac{9}{39}$ **h** $\frac{48}{72}=\frac{\square}{12}$

8 Find $\triangle$ if:

a $\frac{4}{5}=\frac{16}{\triangle}$ **b** $\frac{5}{12}=\frac{50}{\triangle}$ **c** $\frac{6}{\triangle}=\frac{3}{4}$ **d** $\frac{15}{\triangle}=\frac{3}{5}$

e $\frac{7}{8}=\frac{35}{\triangle}$ **f** $\frac{63}{\triangle}=\frac{7}{9}$ **g** $\frac{21}{23}=\frac{63}{\triangle}$ **h** $\frac{48}{\triangle}=\frac{8}{11}$

D SIMPLIFYING FRACTIONS

In **Chapter 5** we saw how there is a proper order in which the operations in an expression should be performed. We called this order **BEDMAS**.

The division line of fractions behaves like a set of brackets. This means that the numerator and denominator must be found before doing the division.

Example 2 Self Tutor

Simplify: **a** $\frac{28-4}{3\times 4}$ **b** $\frac{17+3}{12-2\times 4}$

a $\frac{28-4}{3\times 4}$

$=\frac{24}{12}$

$=2$

b $\frac{17+3}{12-2\times 4}$

$=\frac{20}{12-8}$

$=\frac{20}{4}$

$=5$

EXERCISE 6D.1

1 Simplify:

a $\frac{36}{9-5}$ **b** $\frac{12+8}{2^2-2}$ **c** $\frac{6\times 6}{8+4}$

d $\frac{4+18\div 3}{5}$ **e** $\frac{15-3\times 2}{2+1}$ **f** $\frac{24}{2\times 6}$

g $\dfrac{4^2}{14 - 3 \times 2}$ **h** $\dfrac{3^2 + 2^3}{17}$ **i** $\dfrac{(5-2)^2 \times 4}{18 \div 2}$

LOWEST TERMS

We can also **simplify** a fraction by writing it as an equal fraction where the numerator and denominator are as small as possible.

For example, $\frac{20}{40} = \frac{1}{2}$ in simplest form.

To write a fraction in **simplest** or **lowest terms**, we need to remove the common factors from the numerator and denominator.

For example, 12 and 30 have HCF = 6. So $\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$.

Example 3 Self Tutor

Simplify to lowest terms: **a** $\frac{32}{72}$ **b** $\frac{175}{125}$

a $\frac{32}{72}$

$= \frac{32 \div 8}{72 \div 8}$ {8 is the HCF of 32 and 72}

$= \frac{4}{9}$

b $\frac{175}{125}$

$= \frac{175 \div 25}{125 \div 25}$ {25 is the HCF of 175 and 125}

$= \frac{7}{5}$

EXERCISE 6D.2

1 Simplify to lowest terms:

a $\frac{8}{10}$ **b** $\frac{9}{36}$ **c** $\frac{21}{28}$ **d** $\frac{15}{35}$ **e** $\frac{24}{42}$

f $\frac{55}{77}$ **g** $\frac{48}{84}$ **h** $\frac{6}{30}$ **i** $\frac{123}{300}$ **j** $\frac{625}{1000}$

2 Simplify to lowest terms:

a $\frac{12}{15}$ **b** $\frac{18}{20}$ **c** $\frac{72}{96}$ **d** $\frac{35}{49}$ **e** $\frac{49}{91}$

f $\frac{39}{52}$ **g** $\frac{60}{80}$ **h** $\frac{15}{55}$ **i** $\frac{246}{600}$ **j** $\frac{875}{1000}$

3 Simplify:

a $\frac{56}{77}$ **b** $\frac{45}{80}$ **c** $\frac{12}{20}$ **d** $\frac{15}{45}$ **e** $\frac{250}{1000}$

f $\frac{3}{51}$ **g** $\frac{24}{81}$ **h** $\frac{45}{180}$ **i** $\frac{24}{360}$ **j** $\frac{135}{360}$

4 Which of these fractions are in lowest terms?

a $\frac{15}{20}$ **b** $\frac{1}{3}$ **c** $\frac{13}{24}$ **d** $\frac{132}{144}$ **e** $\frac{6}{9}$ **f** $\frac{21}{28}$

g $\frac{22}{24}$ **h** $\frac{5}{6}$ **i** $\frac{75}{100}$ **j** $\frac{14}{15}$ **k** $\frac{9}{100}$ **l** $\frac{39}{52}$

E FRACTIONS OF QUANTITIES

In this section we see how fractions are applied to the real world. They can describe a part of a quantity or a group of objects.

When writing fractions that involve measurements it is important that we use the **same units** in the numerator and the denominator.

Example 4 Self Tutor

What fraction of 1 metre is 37 cm?

$$37 \text{ cm as a fraction of 1 metre} = \frac{37 \text{ cm}}{1 \text{ metre}}$$
$$= \frac{37 \text{ cm}}{100 \text{ cm}} \quad \{\text{the same units}\}$$
$$= \tfrac{37}{100}$$

Example 5 Self Tutor

Matthew was given a box of chocolates. 5 had red wrappers, 4 had blue, 4 had gold and 2 had green.

a What fraction of the chocolates had red wrappers?

b What fraction of the chocolates did not have gold wrappers?

a $$\text{Fraction with red wrappers} = \frac{\text{number with red wrappers}}{\text{total number of chocolates}}$$
$$= \tfrac{5}{15}$$
$$= \tfrac{1}{3}$$

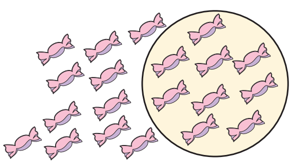
b 11 chocolates did not have gold wrappers.

$$\text{Fraction without gold wrappers} = \frac{\text{number without gold wrappers}}{\text{total number of chocolates}}$$
$$= \tfrac{11}{15}$$

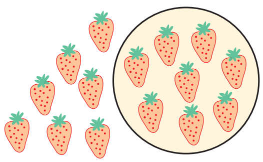
EXERCISE 6E

1 What fraction of each of the following different quantities has been circled?

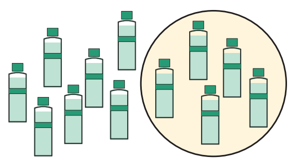
a



b

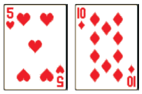


c

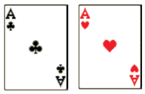


2 Use a full pack of 52 playing cards to work out the following questions. Click on the link if you need to see what all of the cards look like. Calculate what fraction of the full pack are:

DEMO

a red cards such as 

b spades such as 

c aces such as 

d picture cards such as 

e all the odd numbered cards

f all the even numbered black cards

3 In simplest form, state what fraction of:

a 1 metre is 20 cm
b 2 metres is 78 cm
c 1 kg is 500 g
d 1 week is 2 days
e 1 day is 5 hours
f November is two days
g a decade is one year
h 2 dollars is 27 cents

4 What fraction of one hour is:

a 30 minutes **b** 10 minutes **c** 45 minutes **d** 12 minutes ?

5 What fraction of one day is:

a 1 hour **b** 4 hours **c** 30 minutes **d** 1 minute ?

6 Gordon spent \$3 on a drink and \$4 on chocolates. What fraction of \$10 did he spend?

7 Jenny scored 27 correct answers in her test of 40 questions. What fraction of her answers were incorrect?

8 Linda had a bag of 9 apples. She ate 3 and she fed 2 others to her horse. What fraction of her apples remain?

9 James was travelling a journey of 420 km. His car broke down after 280 km. What fraction of his journey did he still have to travel?

10 What number is:

a $\frac{1}{2}$ of 10 **b** $\frac{1}{2}$ of 36 **c** $\frac{1}{3}$ of 12 **d** $\frac{1}{3}$ of 45
e $\frac{1}{4}$ of 20 **f** $\frac{1}{4}$ of 44 **g** $\frac{1}{5}$ of 30 **h** $\frac{1}{5}$ of 120
i $\frac{1}{6}$ of 30 **j** $\frac{1}{6}$ of 126 **k** $\frac{1}{8}$ of 48 **l** $\frac{1}{12}$ of 600?

11 Tran started his homework at 8.15 pm and completed it at 9.08 pm. If he had allowed one hour to do his homework, what fraction of that time did he use?

12 Vijay had 95 cm of rope. He cut 3 pieces from it, each 30 cm long. What fraction of the rope remained?

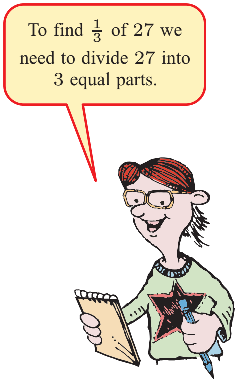
Example 6 **Self Tutor**

On the first day of school this year, $\frac{1}{3}$ of the 6th grade class were aged 12 years or older. If there were 27 students in the class, how many were 12 years or older?

The full number is 27.

So, $\frac{1}{3}$ is $27 \div 3 = 9$ students

There were 9 students aged 12 years or older.



13 Find:

a $\frac{1}{3}$ of 12 people **b** $\frac{1}{4}$ of 20 lollies **c** $\frac{1}{5}$ of 35 drinks

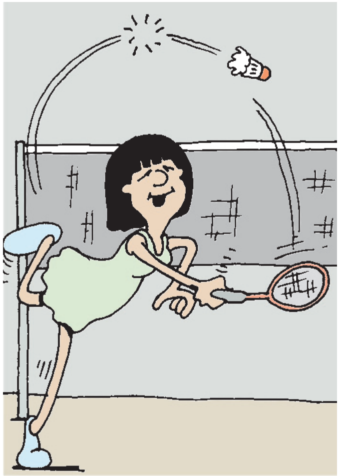
d $\frac{1}{10}$ of 650 g **e** $\frac{1}{2}$ of €38 **f** $\frac{1}{4}$ of 60 minutes.

14 Viktor only won one third of the games of tennis that he played for his school team. If he played 15 games, how many did he win?

15 One fifth of the students at a school were absent because of chicken pox. If there were 245 students in the school, how many were absent?

16 One sixth of the cars from an assembly line were painted white. If 222 cars came from the assembly line, how many were painted white?

17 Ling spent one third of her money on a new badminton racket. If she had 936 RMB before she bought the racket, how much did the racket cost?



18 While Evan was on holidays, one eighth of the tomato plants in his greenhouse died. If he had 96 plants alive when he went away, how many were still alive when he came home?

19 There are 360^o in 1 full revolution or turn.

a Find the number of degrees in:

i one quarter turn **ii** a half turn

iii three quarters of a turn

b What fraction of a revolution is:

i 30^o **ii** 60^o **iii** 240^o?

Example 7 **Self Tutor**

$\frac{2}{3}$ of the birds in my aviary are finches.

If there are 24 birds in my aviary, how many finches are there?

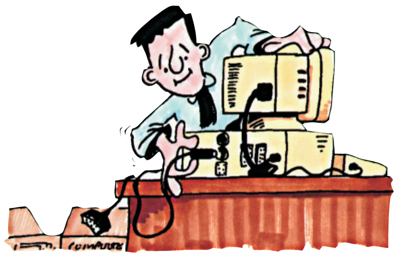
$\frac{1}{3}$ of 24 is $\frac{24}{3} = 24 \div 3 = 8$

So, $\frac{2}{3}$ of 24 must be $2 \times 8 = 16$

There are 16 finches in my aviary.

20 One morning two fifths of the passengers on my bus were school children. If there were 45 passengers, how many were school children?

21 Richard spent three quarters of his working day installing computers, and the remainder of the time travelling between jobs. If his working day was 8 hours, how much time did he spend travelling?



22 When Sasha played netball, she scored a goal with seven eighths of her shots for goal. If she shot for goal 16 times in a match, how many goals did she score?

23 A business hired a truck to transport boxes of equipment. The total weight of the equipment was 3000 kg, but the truck could only carry $\frac{5}{8}$ of the boxes in one load.

- **a** What weight did the truck carry in the first load?
- **b** If there were 80 boxes, how many did the truck carry in the first load?

24 Answer the questions in the **Opening Problem** on page **108**.

F COMPARING FRACTION SIZES

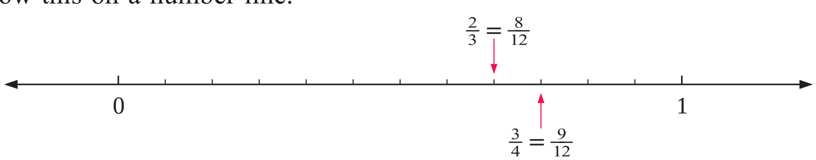
We often wish to compare the size of two fractions. For example, would you rather have $\frac{3}{4}$ or $\frac{2}{3}$ of a block of chocolate?

The sizes of two fractions are easily compared when they have the same denominator.

For example, $\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$ and $\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$.

Since $9 > 8$, $\frac{9}{12} > \frac{8}{12}$ and so $\frac{3}{4} > \frac{2}{3}$.

We can show this on a number line.



To compare fractions we first convert them to equal fractions with a common denominator which is the lowest common multiple of the original denominators. This denominator is called the **lowest common denominator** or **LCD**.

Example 8 **Self Tutor**

Find the LCD of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ by first finding the lowest common multiple of 2, 3, and 4.

The multiples of 2 are 2 4 6 8 10 **12** 14 16 18 20 22 **24**

The multiples of 3 are 3 6 9 **12** 15 18 21 **24**

The multiples of 4 are 4 8 **12** 16 20 **24**

$\therefore$ the common multiples of 2, 3 and 4 are: 12, 24, and so on.

$\therefore$ the lowest common multiple is 12.

$\therefore$ the LCD of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ is 12.

LCD is the abbreviation for **L**owest **C**ommon **D**enominator.

Example 9 **Self Tutor**

Write the fractions $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{3}{4}$ with the lowest common denominator or LCD. Hence write the original fractions in ascending order of size (smallest to largest).

The lowest common multiple of 3, 5 and 4 is 60.

So, the LCD of $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{3}{4}$ is 60.

$\frac{2}{3} = \frac{2\times 20}{3\times 20} = \frac{40}{60}$ $\quad$ $\frac{3}{5} = \frac{3\times 12}{5\times 12} = \frac{36}{60}$ $\quad$ $\frac{3}{4} = \frac{3\times 15}{4\times 15} = \frac{45}{60}$

Now $\frac{36}{60} < \frac{40}{60} < \frac{45}{60}$, so $\frac{3}{5} < \frac{2}{3} < \frac{3}{4}$.

EXERCISE 6F

1 Find the LCM of:

a 7, 3 **b** 5, 3 **c** 3, 6 **d** 12, 18
e 6, 8, 9 **f** 10, 5, 6 **g** 5, 6, 11 **h** 12, 4, 9

2 Write each set of fractions with the lowest common denominator and hence write the original fractions in ascending order (smallest to largest):

a $\frac{1}{2}, \frac{1}{4}$ **b** $\frac{2}{3}, \frac{3}{4}$ **c** $\frac{1}{2}, \frac{4}{7}$
d $\frac{5}{8}, \frac{3}{4}$ **e** $\frac{7}{10}, \frac{5}{6}$ **f** $\frac{7}{9}, \frac{3}{4}$
g $\frac{5}{8}, \frac{8}{10}$ **h** $\frac{8}{11}, \frac{5}{8}$ **i** $\frac{9}{25}, \frac{7}{20}, \frac{1}{4}$

Ascending means *going up*. **Descending** means *going down*.

3 By writing each set of fractions with the lowest common denominator, arrange the fractions in descending order:

a $\frac{1}{2}, \frac{2}{5}, \frac{7}{10}$ **b** $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$ **c** $\frac{1}{2}, \frac{7}{12}, \frac{4}{6}$

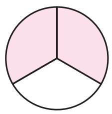
G IMPROPER FRACTIONS AND MIXED NUMBERS

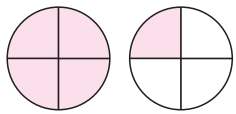
IMPROPER FRACTIONS

All the fractions we have looked at so far have had values between zero and one. This means that their numerators were less than their denominators.

A fraction which has numerator **less** than its denominator is called a **proper fraction**.

A fraction which has numerator **greater** than its denominator is called an **improper fraction**.

For example: $\frac{2}{3}$ is a proper fraction.  represents $\frac{2}{3}$.

$\frac{5}{4}$ is an improper fraction.  represents $\frac{5}{4}$.

To obtain five quarters or $\frac{5}{4}$ we take *two* wholes, divide both into quarters, then shade 5 quarters. We can see from the diagram that $\frac{5}{4}$ is the same as $1\frac{1}{4}$, or 1 and $\frac{1}{4}$.

MIXED NUMBERS

When an improper fraction is written as a whole number and a fraction, it is called a **mixed number**.

For example, $1\frac{1}{4}$ is a mixed number.

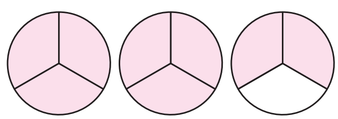
It is often necessary to change a number from an improper fraction to a mixed number and vice versa.

For example, $\frac{8}{3} = 8 \div 3 = 2$ wholes and 2 equal parts (thirds) left over.

So, $\frac{8}{3} = 2\frac{2}{3}$.

Another way of doing this is:

$$\begin{aligned}\frac{8}{3} &= \frac{6}{3} + \frac{2}{3}\\ &= 2 + \frac{2}{3}\\ &= 2\frac{2}{3}\end{aligned}$$



Example 10 | **Self Tutor**

Write as a whole number or a mixed number:

a $\frac{15}{5}$ **b** $\frac{21}{5}$

a $\frac{15}{5}$
$= 15 \div 5$
$= 3$

b $\frac{21}{5}$
$= \frac{20}{5} + \frac{1}{5}$
$= 4 + \frac{1}{5}$
$= 4\frac{1}{5}$

EXERCISE 6G

1 Write as a whole number:

a $\frac{16}{4}$ **b** $\frac{20}{5}$ **c** $\frac{18}{6}$ **d** $\frac{40}{8}$ **e** $\frac{30}{6}$ **f** $\frac{30}{3}$

g $\frac{30}{10}$ **h** $\frac{30}{1}$ **i** $\frac{30}{30}$ **j** $\frac{64}{8}$ **k** $\frac{125}{25}$ **l** $\frac{63}{7}$

2 Write as a mixed number:

a $\frac{5}{4}$ **b** $\frac{7}{6}$ **c** $\frac{18}{4}$ **d** $\frac{19}{6}$ **e** $\frac{15}{2}$ **f** $\frac{17}{3}$

g $\frac{16}{7}$ **h** $\frac{23}{8}$ **i** $\frac{22}{7}$ **j** $\frac{35}{9}$ **k** $\frac{41}{4}$ **l** $\frac{109}{12}$

Example 11 Self Tutor

Write $2\frac{4}{5}$ as an improper fraction.

$2\frac{4}{5}$

$= 2 + \frac{4}{5}$ {split the mixed number}

$= \frac{10}{5} + \frac{4}{5}$ {write with common denominator}

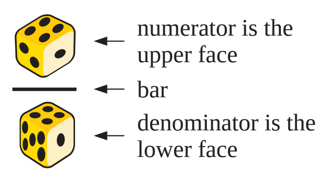
$= \frac{14}{5}$

3 Write as an improper fraction:

a $3\frac{1}{2}$ **b** $4\frac{2}{3}$ **c** $2\frac{3}{4}$ **d** $1\frac{2}{3}$ **e** $1\frac{1}{2}$ **f** $3\frac{3}{4}$

g $1\frac{4}{5}$ **h** $6\frac{1}{2}$ **i** $4\frac{5}{9}$ **j** $5\frac{7}{8}$ **k** $6\frac{6}{7}$ **l** $1\frac{11}{12}$

4 Suppose we have two dice. We roll one to give the numerator of a fraction and the other to give the denominator. Find:



a the smallest fraction it is possible to roll

b the largest *proper* fraction it is possible to roll

c the largest *improper* fraction which is not a whole number that it is possible to roll

d the number of different fractions it is possible to roll.

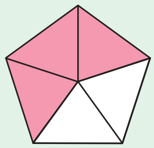
e List the different combinations that can be simplified to a whole number.

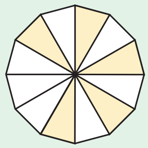
KEY WORDS USED IN THIS CHAPTER

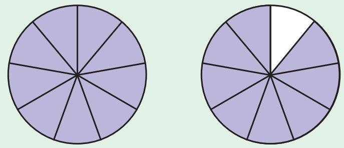
- common fraction
- equivalent fractions
- lowest common denominator
- lowest terms
- number line
- proper fraction
- denominator
- improper fraction
- lowest common multiple
- mixed number
- numerator
- simplest form

REVIEW SET 6A

1 What fraction is represented by the following?

a 

b 

c 

2 Express with denominator 12:

a $\frac{5}{6}$ **b** $\frac{3}{4}$

3 Find the lowest common multiple of:

a 8 and 12 **b** 15, 6 and 5

4 Write T for true and F for false:

a $\frac{3}{9} = \frac{15}{40}$ **b** $3\frac{4}{7} = \frac{24}{7}$ **c** $\frac{76}{8} = 9\frac{1}{2}$ **d** $\frac{375}{1000} = \frac{3}{8}$

5 Find:

a $\frac{1}{4}$ of €256 **b** $\frac{2}{5}$ of 100 g

6 In lowest terms, state what fraction of:

a one week is 3 days **b** one metre is 35 cm

7 Solve the following problems:

a Lex had a carton of 12 eggs. He used 3 of them to make a cake. What fraction of the eggs did he use?

b Dawn has 3 cats, 2 dogs and 5 fish. What fraction of her pets are cats?

c Sara went on a holiday for 20 days. It rained on a quarter of the days. On how many days did it rain?

8 By writing each set of fractions with a common denominator, arrange the fractions in ascending order (smallest to largest):

a $\frac{2}{9}, \frac{1}{4}$ **b** $\frac{5}{8}, \frac{7}{11}$

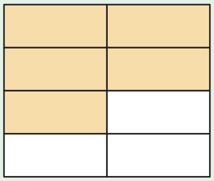
9 Which is the greater, $\frac{3}{7}$ or $\frac{4}{9}$?

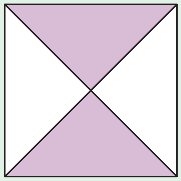
10 Write as a whole number or a mixed number:

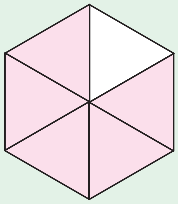
a $\frac{32}{6}$ **b** $\frac{27}{3}$

REVIEW SET 6B

1 What fraction is represented by the following?

a 

b 

c 

2 Express with denominator 24:

a $\frac{1}{8}$ **b** $\frac{5}{12}$ **c** $\frac{3}{4}$

3 Find $\square$ if:

a $\frac{\square}{11} = \frac{15}{55}$ **b** $\frac{45}{72} = \frac{5}{\square}$

4 **a** Convert $\frac{39}{8}$ to a mixed number.

b What fraction of £900 is £180?

c What fraction of 800 m is 200 m?

5 Express $\frac{2}{5}$, $\frac{3}{4}$ and $\frac{13}{20}$ with a lowest common denominator.
Hence write the original fractions in descending order of size.

6 **a** Find $\frac{3}{4}$ of 28.

b Find the values of $\square$ and $\triangle$ given that $\frac{3}{4} = \frac{\square}{20} = \frac{27}{\triangle}$.

7 Write T for true and F for false:

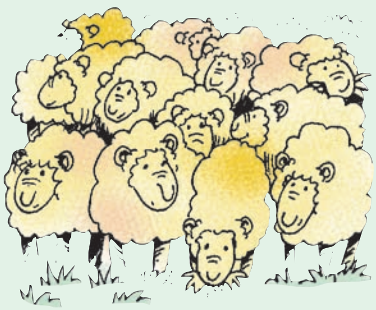
a $\frac{3}{7} = \frac{6}{14} = \frac{15}{35}$ **b** $\frac{675}{1000} = \frac{5}{8}$ **c** $5\frac{6}{7} = \frac{41}{6}$

8 Solve the following problems:

a A man who weighed 90 kg went on a diet and lost 10 kg. What fraction of his original weight did he lose?

b $\frac{1}{5}$ of a flock of sheep numbered 240. Find the size of the whole flock.

c $\frac{3}{7}$ of the students of a school attended a film night.
If there were 840 students in the school, how many attended the film night?



9 By writing each set of fractions with a common denominator, arrange the fractions in descending order (largest to smallest):

a $\frac{7}{10}$, $\frac{3}{4}$ **b** $\frac{8}{11}$, $\frac{7}{9}$

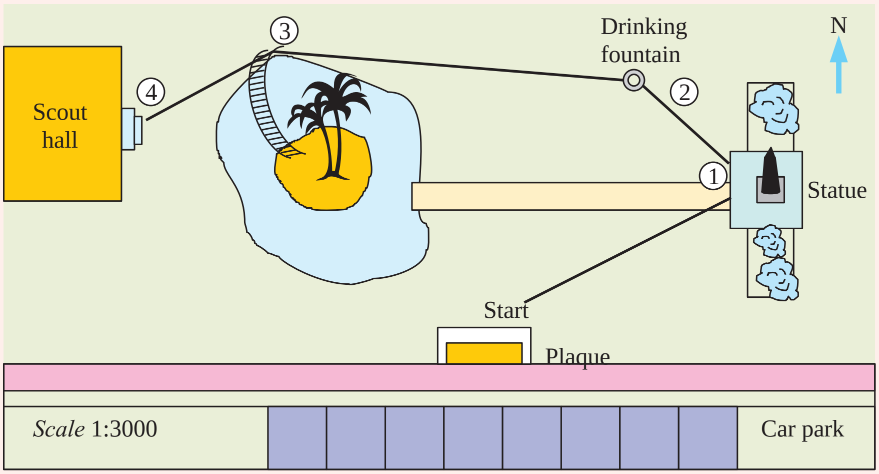
10 Write as an improper fraction:

a $2\frac{5}{6}$ **b** $4\frac{3}{7}$

ACTIVITY MAKING AN ORIENTEERING COURSE

What to do:

1. Obtain a map of your school and its grounds, or a local park or playground. Divide your class into small groups and design a simple orienteering course leading from one point or landmark to another. The landmarks might include a distinctive tree, the corner of a building, or a sign.
2. Choose 3 or 4 landmarks and draw the course on your map. Number the landmarks in the order you want them visited. Each landmark must be clearly visible from the previous one.



3. Go to your starting location and use a compass to measure the bearing of the first landmark.
   Measure the distances between landmarks using a trundle wheel, or if you do not have one you can estimate them by pacing them out.
   Use the same person to pace out each leg and measure the length of a pace several times to make your estimate as accurate as possible.
   Find the bearing of each landmark from the previous one on your course, and the distance between them.

4. Prepare a table of instructions that will enable others to follow your course.

Leg	*Landmark*	*Compass bearing*	*Distance*
start to 1			
1 to 2			
2 to 3			
3 to 4			

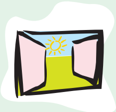
5. Swap your instructions with another group and test out each other's courses. Did you correctly identify each other's landmarks? How accurate were your bearings and distances?

Chapter 7

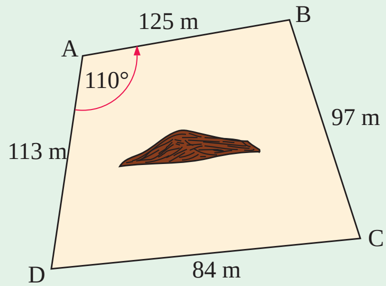
Polygons

Contents:

OPENING PROBLEM



There are four posts at the corners A, B, C and D of a paddock. In the middle is a raised mound, which means we cannot measure directly from A to C. However, the distances between the posts are easily measured and are shown on the figure.



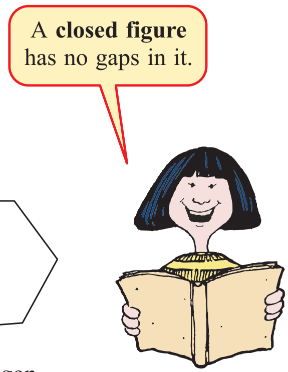
rough figure

The angle at A is measured to be 110^o.

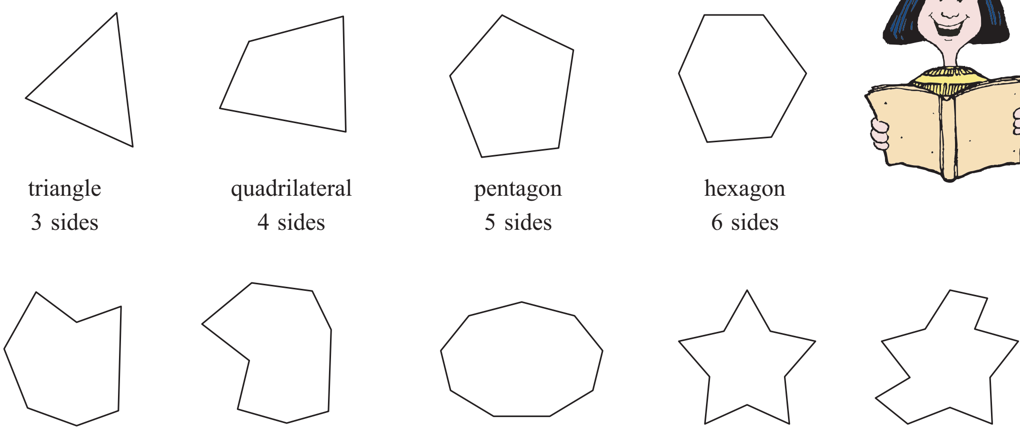
How can we find the distance from A to C to reasonable accuracy? Explain your answer.

A POLYGONS

A **polygon** is a straight-sided closed figure that does not cross itself and can only be drawn on a plane surface.

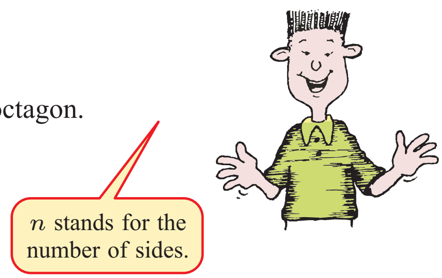


Here are some examples of polygons:

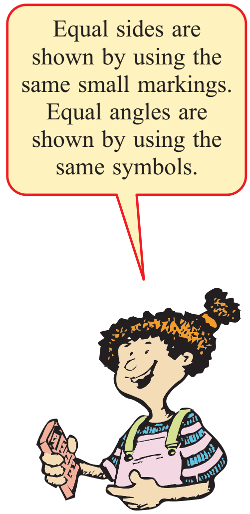


An n-sided polygon is sometimes called an n-**gon**.

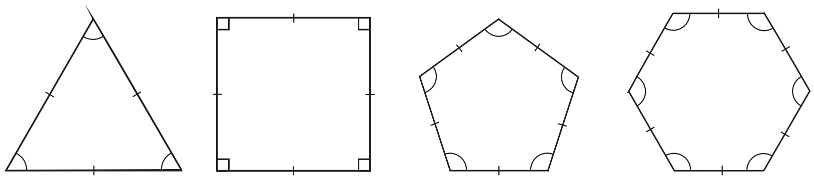
So, an 8-sided polygon may be called an 8-gon or an octagon.



A **regular polygon** is a polygon with all sides the same length and all angles the same size.



The polygons below are marked to show that they are regular:



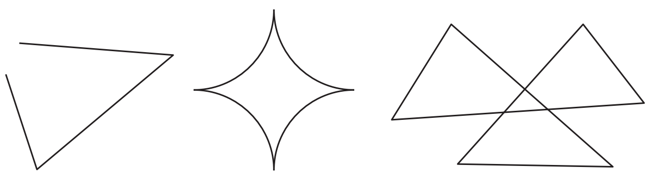
equilateral triangle	square	regular pentagon	regular hexagon
3 equal sides	4 equal sides	5 equal sides	6 equal sides
3 equal angles	4 equal angles	5 equal angles	6 equal angles

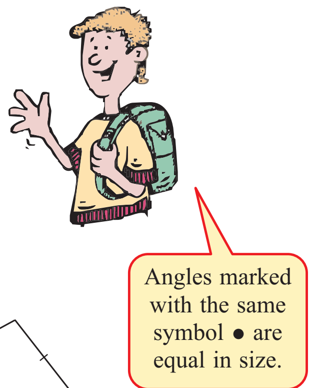
If a polygon is not regular, we say it is **irregular**.

EXERCISE 7A

1 Give one reason why these are not polygons:

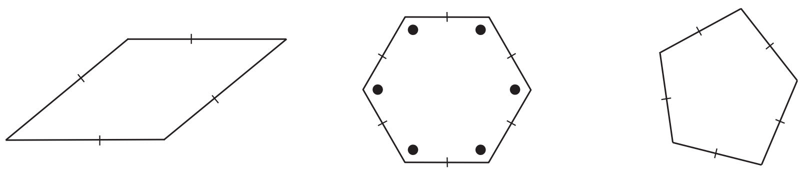
a **b** **c**





2 Which of the following are regular polygons?

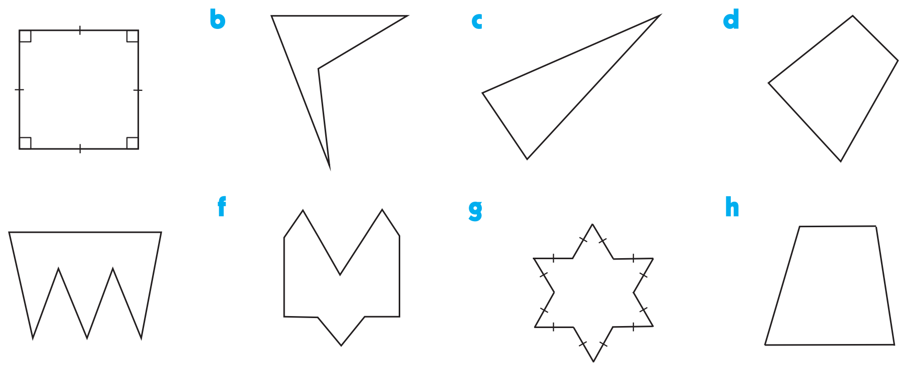
a **b** **c**

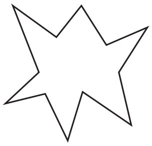


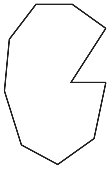
3 Name these polygons:

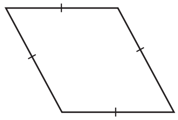
a **b** **c** **d**

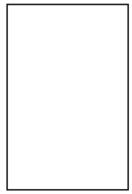
e **f** **g** **h**



i 

j 

k 

l 

4 Draw an example of:

a a quadrilateral **b** an equilateral triangle **c** a hexagon

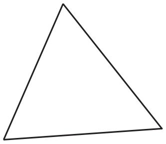
d a decagon **e** a regular pentagon **f** an octagon

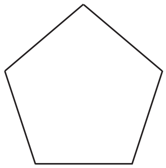
5 Draw and name polygons with the following descriptions:

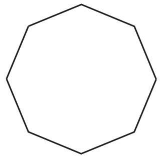
a six equal sides and six equal angles **b** three equal sides

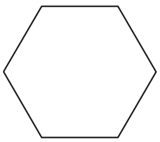
c five equal sides, but with unequal angles

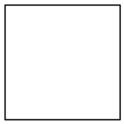
6 Using a ruler and protractor, classify the following as either *regular* or *irregular* polygons:

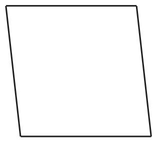
a 

b 

c 

d 

e 

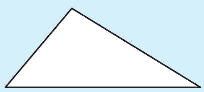
f 

B TRIANGLES

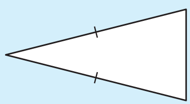
A **triangle** is a three-sided polygon.

There are 3 types of triangles which can be classified according to the number of sides which are equal in length. These are:

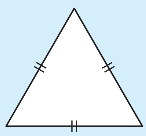
- **scalene** where the 3 sides all have different lengths



- **isosceles** where 2 sides have the same length



- **equilateral** where all 3 sides have the same length.

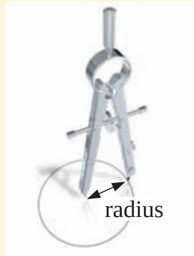


Notice that an equilateral triangle is also isosceles.

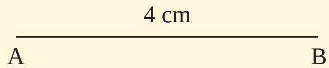
CONSTRUCTING A TRIANGLE

VIDEO CLIP

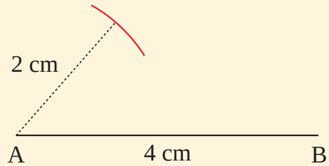
To accurately construct a triangle, we need a ruler and a **geometric compass**. The **radius** of a compass is the distance from the sharp point to the tip of your pencil. Construct a triangle with sides 4 cm, 3 cm and 2 cm long.



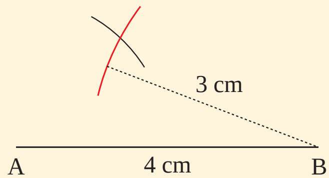
Step 1: Draw a line segment the length of one of the sides. It is often best to choose the longest side. We will call the line segment [AB] and use it as the base of the triangle.



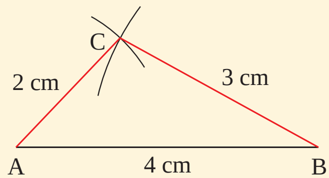
Step 2: Open your compass to a radius equal to the length of one of the other sides. Using this radius draw an arc from one end A of the base line.



Step 3: Now open the compass to a radius equal to the length of the other side. Draw another arc from B to intersect the first arc.



Step 4: The point of intersection of the two arcs is the third vertex C of triangle ABC. Construct line segments [AC] and [BC] to complete the triangle.

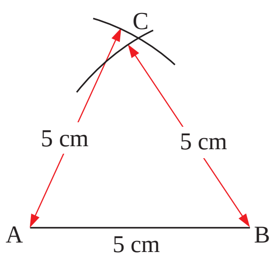


EXERCISE 7B.1

1 Accurately construct a triangle with sides:

a 4 cm, 5 cm and 6 cm **b** 3 cm, 6 cm and 7 cm.

2 Draw [AB] of length 5 cm.
Set the compass points 5 cm apart. With centre A, draw an arc of a circle above [AB].
With centre B draw an arc to intersect the other one.
Let C be the point where these arcs meet. Join [AC] and [BC].

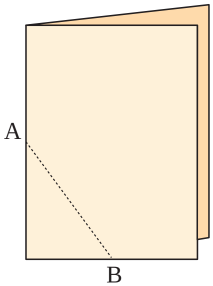


a What type of triangle is ABC? Explain your answer.

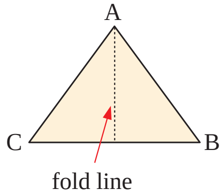
b Measure angles ABC, BCA, CAB using a protractor.

c Copy and complete: *"All angles of an equilateral triangle measure ....^{o}"*

3 Obtain a clean sheet of paper and fold it down the middle. Draw a straight line [AB] as shown. Then with the two sheets pressed tightly together, cut along [AB] through both sheets.



Keep the triangular piece of paper. When you unfold it, you should obtain the triangle ABC shown.



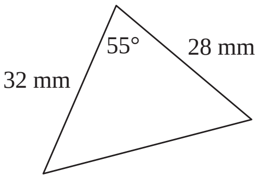
a Explain why triangle ABC is isosceles.

b Explain why the angles opposite the equal sides are equal in size.

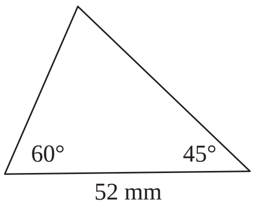
You should not have to use a ruler and protractor.

4 Accurately construct these triangles using a protractor, compass and ruler:

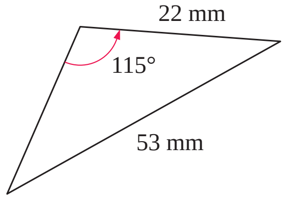
a



b

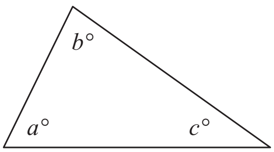
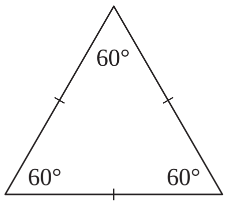
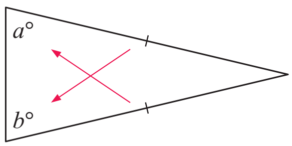


c



TRIANGLE PROPERTIES

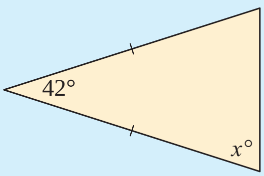
From **Chapter 3** and **Exercise 7B.1** you should have discovered that:

In any triangle the sum of the angles is 180°.  $a + b + c = 180$	All angles of an equilateral triangle measure 60°. 	The angles opposite the equal sides of an isosceles triangle are equal.  $a = b$

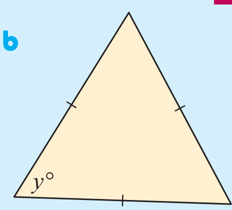
Example 1 Self Tutor

Find the value of the unknown in each figure:

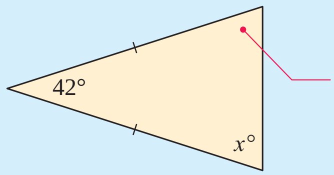
a



b



a



Since two sides of the triangle are equal, the triangle is isosceles. The angles opposite the equal sides must be equal in size.
So, this angle is x^o as well.

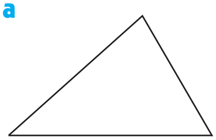
But $x + x + 42 = 180$ {angle sum of Δ}
$\therefore$ $x + x = 138$ {as $138 + 42 = 180$}
$\therefore$ $x = 69$ {as $69 + 69 = 138$}

b All angles of an equilateral triangle measure 60^o $\therefore$ $y = 60$.

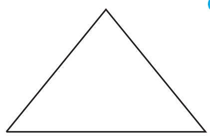
EXERCISE 7B.2

1 Measure the length of the sides of the triangles and use these measurements to classify each as equilateral, isosceles or scalene:

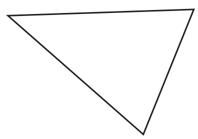
a



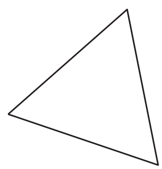
b



c

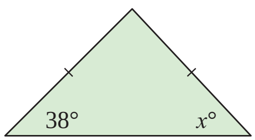


d

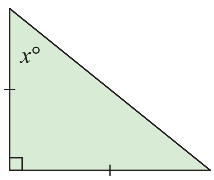


2 Find the unknowns in the following which are *not drawn to scale*:

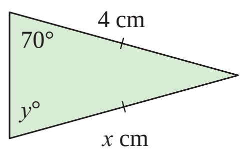
a



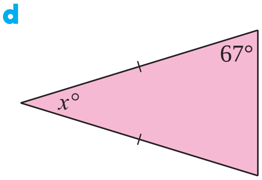
b



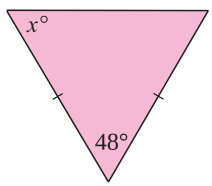
c



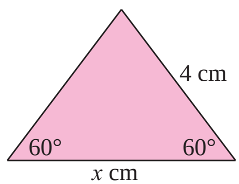
d



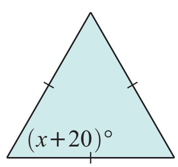
e



f



g



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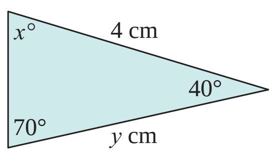

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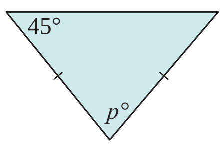
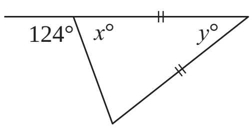
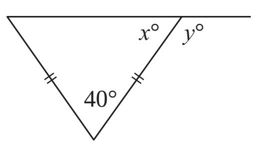


j



k



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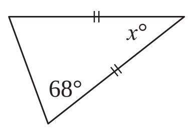
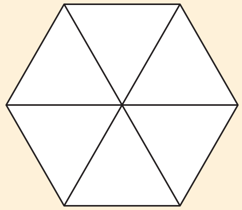


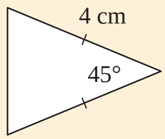
INVESTIGATION 1 THE ANGLES OF REGULAR POLYGONS

What to do:

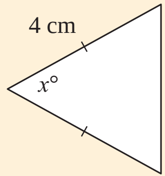


1 Six equilateral triangles are cut out and laid down to form a regular hexagon. Explain using this figure why:

- the angles of an equilateral triangle are 60^o
- the angles of a regular hexagon are 120^o.



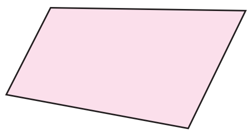
2 Make eight isosceles triangles like the one illustrated. Put them together to form a regular octagon. What is the size of an angle of a regular octagon?



3 What size should x be so that five triangles like the one shown can be put together to form a regular pentagon? What is the size of an angle of a regular pentagon?

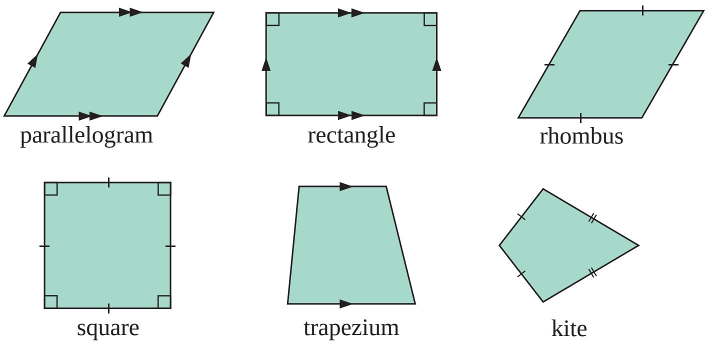
C QUADRILATERALS

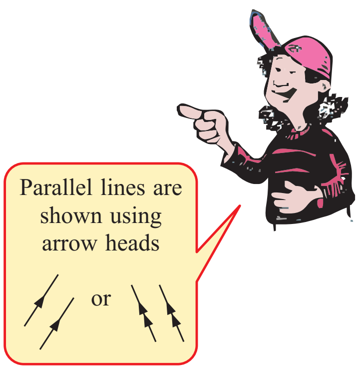
A **quadrilateral** is a polygon with four sides.



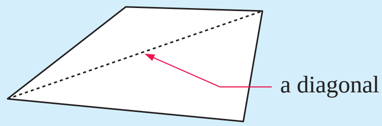
There are six special quadrilaterals:

- A **parallelogram** is a quadrilateral which has opposite sides parallel.
- A **rectangle** is a parallelogram with four equal angles of 90^o.
- A **rhombus** is a quadrilateral in which all sides are equal.
- A **square** is a rhombus with four equal angles of 90^o.
- A **trapezium** is a quadrilateral which has a pair of opposite sides parallel.
- A **kite** is a quadrilateral which has two pairs of adjacent sides equal.





A **diagonal** of a quadrilateral is a straight line segment which joins a pair of opposite vertices.



INVESTIGATION 2 PROPERTIES OF QUADRILATERALS

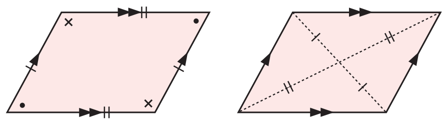
What to do:

PRINTABLE WORKSHEETS

1. Print the worksheets obtained by clicking on the icon.
2. For each **parallelogram**:
   - **a** measure the lengths of opposite sides and record them
   - **b** measure the sizes of opposite angles and record them
   - **c** draw the diagonals and measure the distances from each vertex to the point of intersection. Record your results.
3. For each **rectangle**:
   - **a** measure the lengths of the opposite sides and record them
   - **b** measure the lengths of the diagonals and record them.
   - **c** Copy and complete: The rectangle is the only parallelogram with diagonals that are: ....
4. For each **rhombus**:
   - **a** check that opposite sides are parallel
   - **b** measure the sizes of opposite angles and record them
   - **c** draw the diagonals and measure the distances from each vertex to the point of intersection. Record your results.
   - **d** At what angle do the diagonals intersect?
   - **e** Fold each rhombus along each diagonal. What do you notice about the angles formed?
5. For each **square**:
   - **a** check that opposite sides are parallel
   - **b** Fold each square along each diagonal. What do you notice about the angle where the diagonals intersect.
   - **c** What else do you notice about the diagonals?
6. For the **kite**:
   - **a** measure its opposite angles and record them
   - **b** Fold each kite about its diagonals and after taking measurements record any observations.

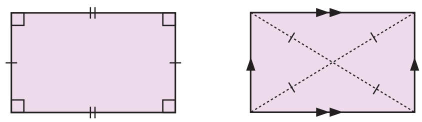
From the investigation, you should have discovered these properties of special quadrilaterals:

Parallelogram



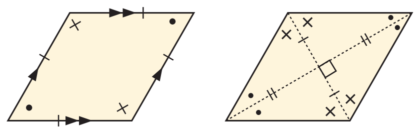
- opposite sides are equal
- opposite angles are equal
- diagonals bisect each other (divide each other in half).

Rectangle



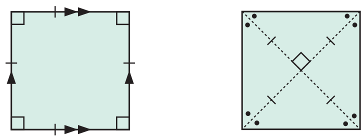
- opposite sides are equal in length
- diagonals are equal in length
- diagonals bisect each other.

Rhombus



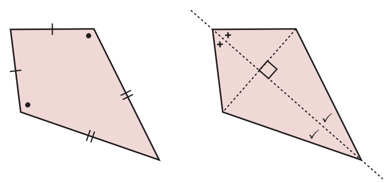
- opposite sides are parallel
- opposite angles are equal in size
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex.

Square



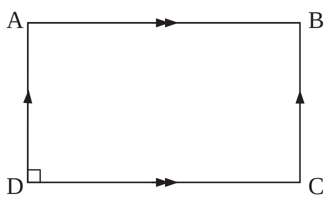
- opposite sides are parallel
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex.

Kite



- *one* pair of opposite angles is equal in size
- diagonals cut each other at right angles
- diagonals bisect *one* pair of angles at the vertices

PARALLEL AND PERPENDICULAR LINES



In the figure we notice that [AB] is parallel to [DC]. We write this as [AB] $\parallel$ [DC].

[AD] is at right angles or **perpendicular** to [DC].

We write this as [AD] $\perp$ [DC].

$\parallel$ reads *is parallel to.* $\perp$ reads *is perpendicular to.*

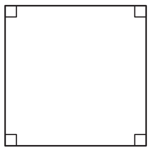
EXERCISE 7C.1

1 Draw a fully labelled sketch of:

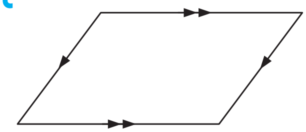
a a parallelogram **b** a rhombus **c** a kite.

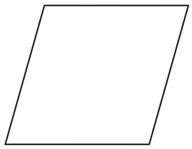
2 There are 3 special parallelograms. Name each of them.

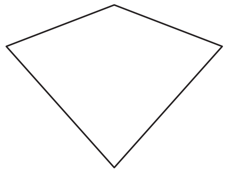
3 Use a ruler to help classify the following:

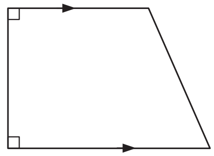
a


b


c


d


e


f


4 True or false?

a A parallelogram is a quadrilateral which has opposite sides parallel.

b A rectangle is a parallelogram with four equal angles of 90^o.

c A rhombus is a quadrilateral in which all sides are equal.

d A square is a rhombus with four equal angles of 90^o.

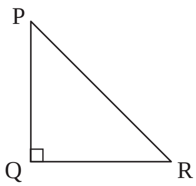
e A trapezium is a quadrilateral which has a pair of opposite sides parallel.

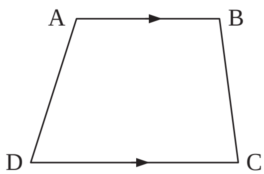
f A kite is a quadrilateral which has two pairs of adjacent sides equal.

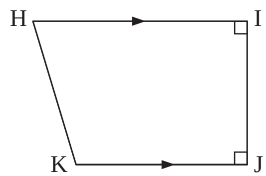
g A quadrilateral with one pair of opposite angles equal is a kite.

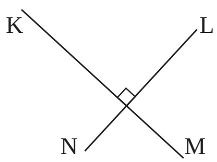
h The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.

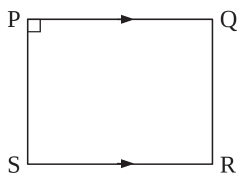
5 Using $\|$ and $\perp$, write statements about the following figures:

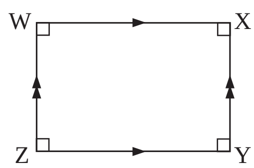
a


b


c


d


e


f


6 Draw the figures from these instructions. A freehand labelled sketch is needed in each case.

a [AB] is 4 cm long. [BC] is 3 cm long. [AB] $\perp$ [BC].

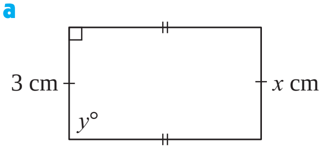
b [PQ] is 5 cm long. [RS] is 4 cm long. [RS] $\|$ [PQ] and [RS] is 3 cm from [PQ].

c ABCD is a quadrilateral in which [BC] $\|$ [AD] and [AB] $\perp$ [AD].

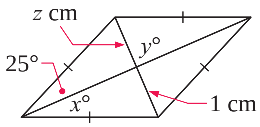
d ABCD is a quadrilateral where [AB] $\|$ [DC] and [AD] $\|$ [BC] and [AB] $\perp$ [BC].

7 Find the values of the variables in these figures:

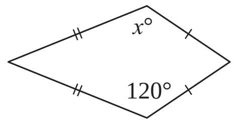
a



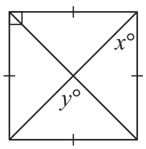
b



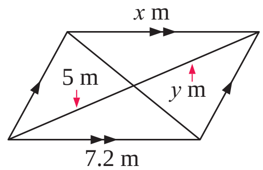
c



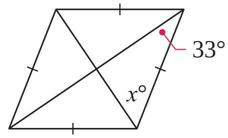
d



e



f



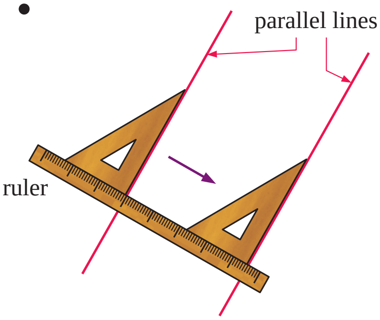
CONSTRUCTING QUADRILATERALS

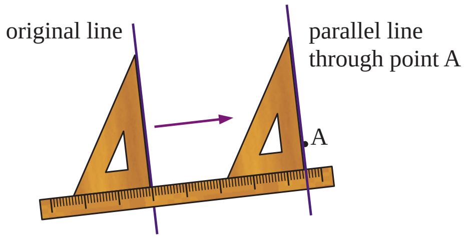
A set square and a ruler can be used to construct **parallel lines**.

DEMO

To do this, we slide the set square along the ruler.

For example:

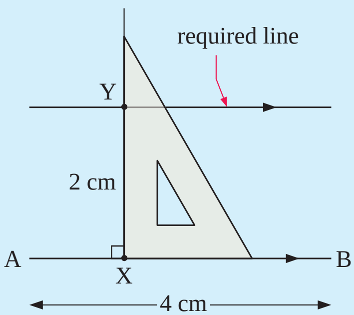
- 

- 

Here we see how to draw a parallel line through point A not on the original line.

Example 2 **Self Tutor**

[AB] is a line segment which is 4 cm long. Accurately construct a parallel line which is 2 cm from [AB].



Step 1: Use a ruler to draw [AB] exactly 4 cm long.

Step 2: Through a point X on [AB], use a set square to draw a line which is perpendicular to [AB].

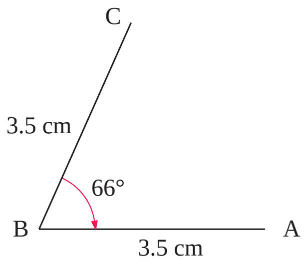
Step 3: Mark point Y on the perpendicular 2 cm from X.

Step 4: We replace the set square and use a ruler to slide it along to Y. We then construct a line through Y (parallel to [AB].)

EXERCISE 7C.2

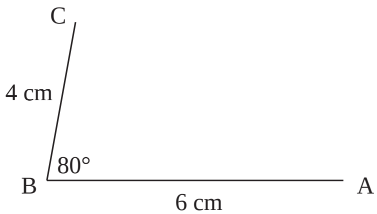
1 [AB] is 5 cm long. Construct [CD] parallel to [AB] and 25 mm from it.

2



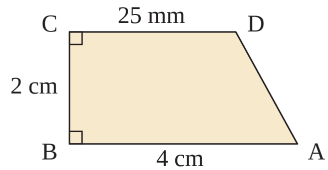
a Use a ruler and protractor to reproduce the given figure.
b By sliding a set square, construct through A a line parallel to [BC].
c Through C, construct a line parallel to [BA].
d If the lines from **b** to **c** meet at D, what type of quadrilateral is ABCD?
e What is the length of [AC] to the nearest mm?

3



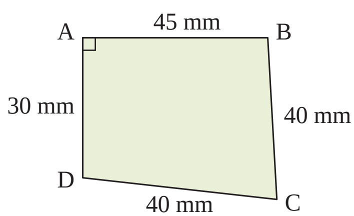
a Reproduce the given figure using a ruler and protractor.
b Construct a parallelogram ABCD.
c Find the lengths of the diagonals [AC] and [BD] to the nearest mm.

4 **a** Accurately construct a trapezium ABCD with the dimensions shown.
b Use your protractor to find the measure of angle ADC to the nearest degree.
c Find the length of [AD] to the nearest mm.

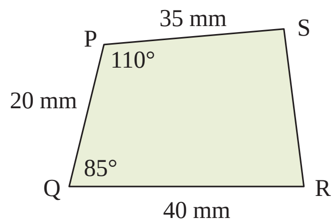


5 Construct these quadrilaterals:

a



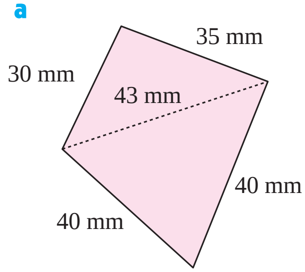
b



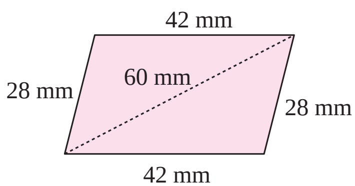
Hence find the lengths of [DB] and [RS].

6 Construct these quadrilaterals using only a ruler and compass:

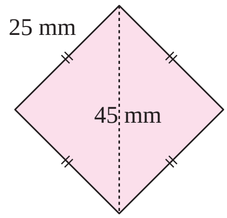
a



b



c

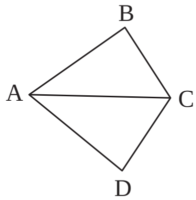


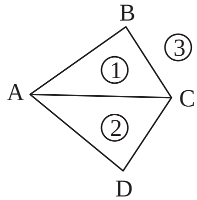
D EULER'S RULE FOR PLANE FIGURES

The figure alongside consists of 4 **vertices**: A, B, C and D.

It has 5 **edges** connecting the vertices: [AB], [BC], [AC], [AD], and [CD].

The edges divide the plane into 3 **regions**: inside triangle ABC, inside triangle ACD, and outside quadrilateral ABCD.

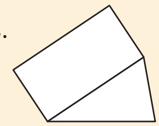




INVESTIGATION 3 VERTICES, EDGES AND REGIONS

In this investigation we seek a connection between the number of vertices, edges, and regions of any figure drawn in a plane.

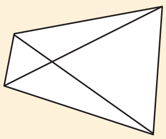
For example, this figure has 5 vertices, 3 regions, and 6 edges. Outside the figure counts as 1 region.



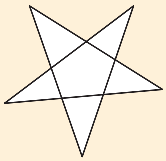
What to do:

1 Consider the following figures:

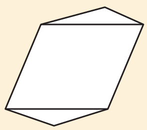
a



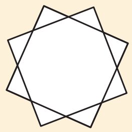
b



c



d



Copy and complete the following table. **e** to **h** are for four diagrams of plane figures like those above, but of your choice.

Figure	*Vertices* (V)	*Regions* (R)	*Edges* (E)	$V + R - 2$
Given example	5	3	6	6
a				
b				
c				
d				
e				
f				
g				
h				

2 Suggest a relationship or rule between V, R and E.

EULER'S RULE

From the previous **Investigation** you should have discovered **Euler's Rule**:

In any closed figure, the number of edges is always two less than the sum of the numbers of vertices and regions. $\boldsymbol{E = V + R - 2}$.

EXERCISE 7D

1 Using Euler's Rule, determine the number of:
- **a** edges for a figure with 5 vertices and 4 regions
- **b** edges for a figure with 6 vertices and 5 regions
- **c** vertices for a figure with 7 edges and 3 regions
- **d** vertices for a figure with 9 edges and 4 regions
- **e** regions for a figure with 10 edges and 8 vertices
- **f** regions for a figure with 12 edges and 7 vertices.

2 Draw a possible figure for each of the cases in **1**.

3 Draw two *different* figures which have 5 vertices and 7 edges.

4 Salvi has just drawn a plane figure. He says that the number of edges is 12, the number of vertices is 9, and the number of regions is 6. Can you draw Salvi's figure?

ACTIVITY 1 IDENTIFYING SHAPES

Look at the following photograph of a building.

What to do:

1 Make a list of all the different shapes you can see in the photograph.

2 Write sentences to describe where you see
- **a** parallel lines
- **b** perpendicular lines.

ACTIVITY 2 ISLAMIC ART

Alongside is an example of Islamic art:

What to do:

1 Collect pictures of Islamic art from magazines, books or from the internet.

2 List shapes used in the designs.

KEY WORDS USED IN THIS CHAPTER

- diagonal
- Euler's rule
- isosceles triangle
- parallel lines
- perpendicular lines
- rectangle
- rhombus
- trapezium
- edge
- hexagon
- kite
- parallelogram
- polygon
- region
- scalene triangle
- vertex
- equilateral triangle
- irregular polygon
- octagon
- pentagon
- quadrilateral
- regular polygon
- square

LINKS
click here

PROTECTING YOURSELF, THE OLD FASHIONED WAY

Areas of interaction:
Human ingenuity

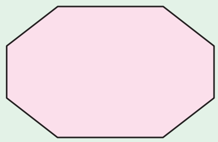
REVIEW SET 7A

1 Name the following polygons:

a

b

c

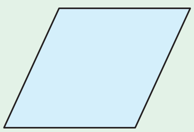


2 Draw the following polygons:

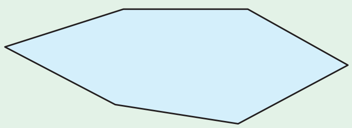
a isosceles triangle **b** regular hexagon **c** rhombus

3 Using a ruler and protractor, classify the following shapes as regular or irregular polygons:

a



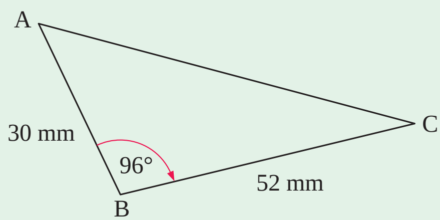
b



4 Using a compass and ruler only, construct a triangle with sides of length 3 cm, 4 cm and 6 cm.

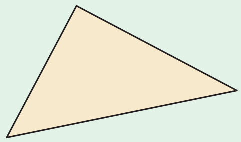
5 Using a protractor and ruler, accurately construct a triangle with the measurements shown:

What is the length of [AC]?

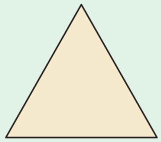


6 Classify the following triangles by measuring their sides:

a



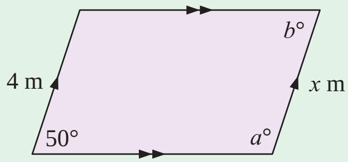
b



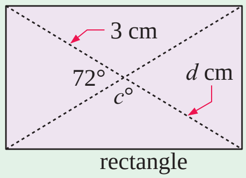
7 Draw a fully labelled sketch of a rectangle.

8 Find the values of the variables in these figures:

a

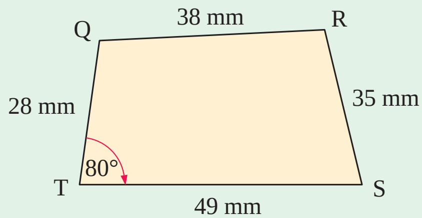


b



9 Construct a rectangle PQRS where [PQ] is 5 cm, [QR] is 3 cm, and [PQ] ⊥ [QR].

10 Using a compass, protractor and ruler, accurately construct a quadrilateral with the measurements shown.
Now measure the length of [RT] to the nearest mm.



11 Use Euler's rule $E = V + R - 2$ to determine the number of:

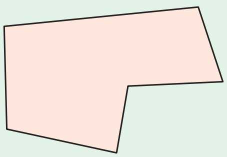
a edges in a plane figure with 11 vertices and 5 regions

b regions in a plane figure with 9 vertices and 17 edges.

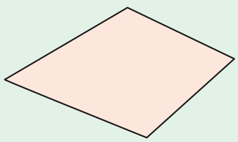
REVIEW SET 7B

1 Name the following polygons:

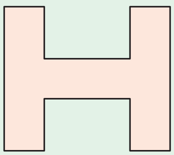
a



b



c



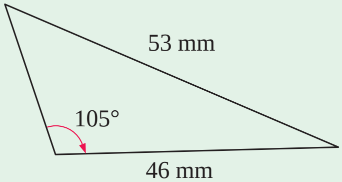
2 Draw and name polygons with the following descriptions:

a five equal sides and five equal angles

b four equal sides and opposite angles equal.

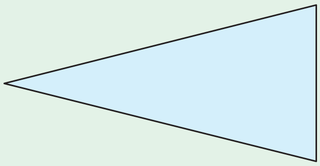
3 Using a compass and ruler only, construct an isosceles triangle with base length 5 cm and equal sides 4 cm.

4 Using a protractor, compass and ruler, accurately construct a triangle with the measurements shown.

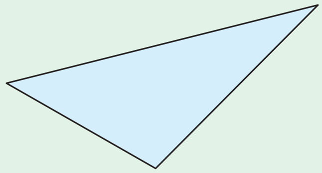


5 Classify the following triangles by measuring their sides:

a

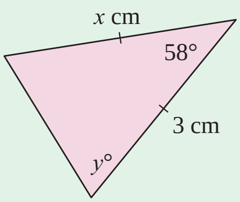


b

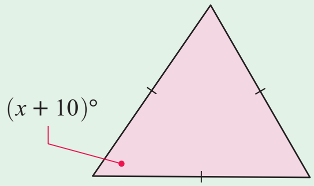


6 Find the variables in the following which are *not drawn to scale*:

a

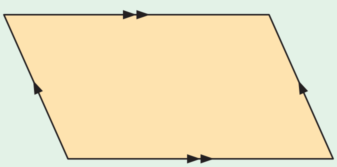


b

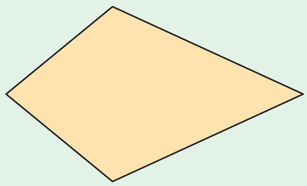


7 Use a ruler to help classify the following:

a

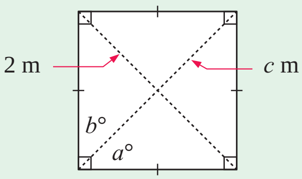


b

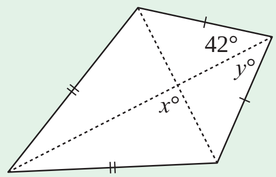


8 Find the values of the variables in these figures:

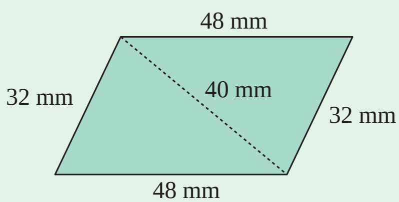
a



b



9 Using a compass and ruler only, make an accurately constructed quadrilateral with measurements as shown:



10 Use Euler's rule $E = V + R - 2$ to determine the number of:

a vertices if there are 7 regions and 14 edges

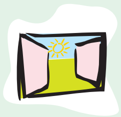
b regions if there are 12 edges and 8 vertices.

Chapter 8

Fraction operations

Contents:

OPENING PROBLEMS



Problem 1:

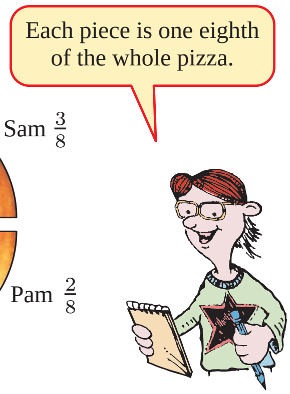
Three friends go shopping to buy a DVD player for a birthday present. The electrical store is having a special sale, and offer it for $\frac{1}{2}$ price. If the three friends share the cost equally, what fraction of the original price does each of them contribute?

Problem 2:

A family have a bag containing rice. One day they eat $\frac{1}{2}$ of it and on the next day they eat $\frac{1}{3}$ of it. What fraction of the bag remains for the third day?

A ADDING FRACTIONS

Mario serves pizzas cut into 8 pieces, so each piece is exactly $\frac{1}{8}$ of the pizza.



At one table Sam eats $\frac{3}{8}$ and Pam eats $\frac{2}{8}$ of their pizza.

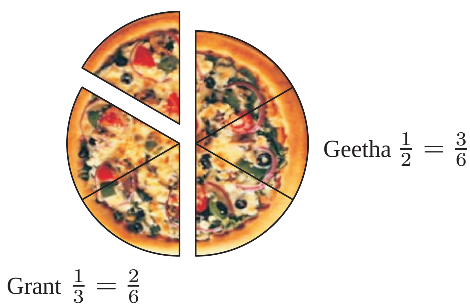
We can see from the diagram that together Sam and Pam have eaten $\frac{5}{8}$ of the pizza, so $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$.

At another table Mark and Ming eat $\frac{3}{8}$ and $\frac{1}{2}$ of their pizza respectively.

Since only one piece remains we know that $\frac{3}{8}+\frac{1}{2}=\frac{7}{8}$.

Notice that the $\frac{1}{2}$ can also be written as $\frac{4}{8}$.

$$\text{So,}\quad \frac{3}{8}+\frac{1}{2}=\frac{3}{8}+\frac{4}{8}=\frac{7}{8}.$$



At another Pizza shop, pizzas are cut into 6 equal portions. Geetha and Grant eat $\frac{1}{2}$ and $\frac{1}{3}$ of their pizza respectively. Since only one piece remains, we know that $\frac{5}{6}$ was eaten.

$$\text{So,}\quad \frac{1}{2}+\frac{1}{3}=\frac{5}{6}.$$

Geetha's $\frac{1}{2}$ can be written as $\frac{3}{6}$, while Grant's $\frac{1}{3}$ can be written as $\frac{2}{6}$.

$$\text{Hence,}\quad \frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}.$$

As you can see, fractions are easily added if they have the same **denominator**.

RULE FOR ADDITION OF FRACTIONS

To add fractions:

- If necessary, change the fractions to equal fractions with the lowest common denominator.
- Add the fractions by adding the new numerators. The denominator stays the same.

Example 1 Self Tutor

Find: **a** $\frac{2}{5}+\frac{1}{5}$ **b** $2+\frac{1}{7}+\frac{4}{7}$

a $\frac{2}{5}+\frac{1}{5}$
$=\frac{3}{5}$

b $2+\frac{1}{7}+\frac{4}{7}$
$=2+\frac{5}{7}$
$=2\frac{5}{7}$

Example 2 Self Tutor

Find: $\frac{1}{2}+\frac{2}{3}$

$\frac{1}{2}+\frac{2}{3}$ {LCD = 6}
$=\frac{1\times 3}{2\times 3}+\frac{2\times 2}{3\times 2}$ {converting to 6ths}
$=\frac{3}{6}+\frac{4}{6}$ {simplifying}
$=\frac{7}{6}$ {adding the numerators}
$=1\frac{1}{6}$ {converting to a mixed number}

EXERCISE 8A.1

1 Without showing any working, add the following:

a $\frac{1}{3}+\frac{2}{3}$ **b** $\frac{3}{4}+\frac{1}{4}$ **c** $\frac{1}{6}+\frac{4}{6}$
d $\frac{3}{11}+\frac{7}{11}$ **e** $\frac{3}{8}+\frac{10}{8}$ **f** $\frac{1}{8}+\frac{3}{8}+\frac{4}{8}$
g $3+\frac{1}{5}+\frac{2}{5}$ **h** $4+\frac{1}{3}+\frac{2}{3}$ **i** $\frac{3}{4}+\frac{5}{4}$
j $\frac{1}{7}+\frac{2}{7}+\frac{3}{7}$ **k** $2+\frac{3}{4}+\frac{5}{4}$ **l** $\frac{5}{9}+\frac{2}{9}+\frac{11}{9}$
m $\frac{3}{10}+\frac{1}{10}+1$ **n** $6+\frac{2}{3}+\frac{2}{3}$ **o** $\frac{1}{8}+\frac{2}{8}+\frac{3}{8}+\frac{4}{8}$

2 Find:

a $\frac{1}{2}+\frac{1}{4}$ **b** $\frac{1}{3}+\frac{1}{5}$ **c** $\frac{2}{3}+\frac{1}{4}$
d $\frac{3}{4}+\frac{1}{3}$ **e** $\frac{1}{10}+\frac{1}{5}$ **f** $\frac{3}{10}+\frac{2}{5}$
g $\frac{9}{10}+\frac{1}{4}$ **h** $\frac{3}{4}+\frac{1}{6}$ **i** $\frac{5}{6}+\frac{1}{3}$
j $\frac{3}{4}+\frac{1}{5}$ **k** $\frac{1}{3}+\frac{5}{7}$ **l** $\frac{3}{8}+\frac{4}{9}$

3 Kris spends $\frac{2}{5}$ of his money on food and $\frac{1}{3}$ of his money on bills. What fraction of his money has been used?

Example 3 Self Tutor

Find: $\frac{1}{3}+\frac{3}{4}+\frac{3}{8}$

$\frac{1}{3}+\frac{3}{4}+\frac{3}{8}$ {have LCD of 24}

$=\frac{1\times 8}{3\times 8}+\frac{3\times 6}{4\times 6}+\frac{3\times 3}{8\times 3}$ {converting to 24ths}

$=\frac{8}{24}+\frac{18}{24}+\frac{9}{24}$ {simplifying}

$=\frac{35}{24}$ {adding the numerators}

$=1\frac{11}{24}$ {converting to a mixed number}

4 Find:

a $\frac{1}{5}+\frac{1}{2}+\frac{1}{6}$ **b** $\frac{1}{2}+\frac{1}{4}+\frac{2}{5}$ **c** $\frac{1}{4}+\frac{1}{3}+\frac{1}{2}$

d $\frac{2}{3}+\frac{1}{6}+\frac{1}{2}$ **e** $\frac{2}{5}+\frac{3}{10}+\frac{1}{2}$ **f** $\frac{3}{4}+\frac{1}{2}+\frac{7}{12}$

5 Carly eats $\frac{1}{8}$ of a pizza, Su-Lin eats $\frac{2}{5}$, and Terri eats $\frac{1}{4}$. How much of the pizza has been eaten?

ADDITION OF MIXED NUMBERS

When we add mixed numbers together, there are **two possible methods** we can use.

Method 1: Convert to improper fractions, then add, then convert back to a mixed number.

Example 4 Self Tutor

Add $3\frac{7}{9}$ and $2\frac{2}{3}$.

$3\frac{7}{9}+2\frac{2}{3}$

$=\frac{34}{9}+\frac{8}{3}$ {convert to improper fractions}

$=\frac{34}{9}+\frac{8\times 3}{3\times 3}$ {converting to 9ths}

$=\frac{34}{9}+\frac{24}{9}$ {simplifying}

$=\frac{58}{9}$ {adding the numerators}

$=6\frac{4}{9}$

Method 2: Add the whole numbers and the fractions separately, then combine the results.

Example 5 Self Tutor

Add $3\frac{7}{9}$ and $2\frac{2}{3}$.

$3\frac{7}{9}+2\frac{2}{3}$	
$=5+\frac{7}{9}+\frac{2}{3}$	{adding whole numbers together first}
$=5+\frac{7}{9}+\frac{2\times 3}{3\times 3}$	{converting to 9ths}
$=5+\frac{7}{9}+\frac{6}{9}$	{simplifying}
$=5+\frac{13}{9}$	{adding the numerators}
$=5+1\frac{4}{9}$	{converting back to mixed number}
$=6\frac{4}{9}$	{simplifying}

EXERCISE 8A.2

1 Find:

a $1\frac{1}{6}+2\frac{1}{3}$ **b** $2\frac{1}{3}+\frac{7}{12}$ **c** $1\frac{1}{3}+3\frac{5}{6}$

d $1\frac{7}{8}+\frac{4}{5}$ **e** $2\frac{1}{4}+2\frac{3}{5}$ **f** $1\frac{1}{4}+3\frac{2}{3}$

g $3\frac{1}{2}+2\frac{2}{3}$ **h** $2\frac{2}{3}+4\frac{1}{5}$ **i** $5\frac{7}{8}+2\frac{1}{4}$

2 Sarah is an artist. She spends $3\frac{1}{2}$ hours on Saturday painting a portrait and a further $2\frac{1}{3}$ hours finishing it off on Sunday. How long did it take her to paint the portrait?

USING A CALCULATOR

When dealing with complicated fractions, a calculator may be used. Most calculators have a key for entering fractions which looks like [a b/c] .

Example 6 **Self Tutor**

Use a calculator to find: **a** $\frac{3}{7}+\frac{8}{19}$ **b** $2\frac{1}{6}+3\frac{2}{13}$

a $\frac{3}{7}+\frac{8}{19}$ would be keyed like this:

3 [a b/c] 7 [+] 8 [a b/c] 19 [=] *Answer:* $\frac{113}{133}$

b $2\frac{1}{6}+3\frac{2}{13}$ would be keyed like this:

2 [a b/c] 1 [a b/c] 6 [+] 3 [a b/c] 2 [a b/c] 13 [=] *Answer:* $5\frac{25}{78}$

3 Use a calculator to find:

a $\frac{3}{8}+\frac{5}{14}$ **b** $\frac{7}{9}+\frac{5}{16}$ **c** $3\frac{1}{12}+4\frac{2}{15}$

d $2\frac{1}{18}+5\frac{9}{11}$ **e** $\frac{15}{17}+\frac{23}{28}$ **f** $\frac{3}{13}+2\frac{5}{21}$

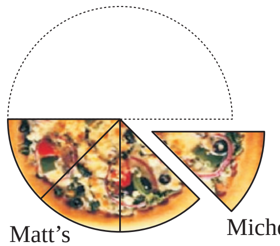
g $4\frac{2}{11}+\frac{21}{23}$ **h** $13\frac{3}{7}+12\frac{2}{5}$ **i** $\frac{1}{8}+\frac{7}{9}+3\frac{1}{10}$

B SUBTRACTING FRACTIONS

At Mario's pizza parlour a half of a pizza has been eaten.

Matt eats $\frac{3}{4}$ of what remains.

What fraction remains for Michelle?



Suppose the pizza was originally cut into 8 equal portions.

$\frac{1}{2} = \frac{4}{8}$ has been eaten, leaving $\frac{4}{8}$ for Matt and Michelle.

We are told that Matt eats $\frac{3}{4}$ of the remaining pieces, so this is 3 pieces or $\frac{3}{8}$ of the original pizza.

Michelle is left with one piece or $\frac{1}{8}$ of the original pizza,

so $\frac{1}{2} - \frac{3}{8} = \frac{4}{8} - \frac{3}{8} = \frac{1}{8}$.

Fractions are easily subtracted if they have a **common denominator**.

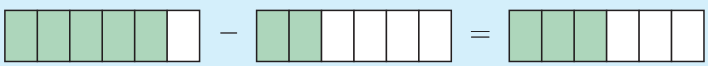
RULE FOR SUBTRACTION OF FRACTIONS

To subtract one fraction from another:

- If necessary, change the fractions to equal fractions with the lowest common denominator.
- Subtract the fractions by subtracting the new numerators. The denominator stays the same.

Example 7 Self Tutor

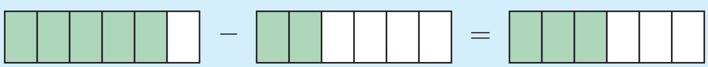
Given the following 'fraction diagram':



a Using 'sixths' write down the equation that the diagram represents.

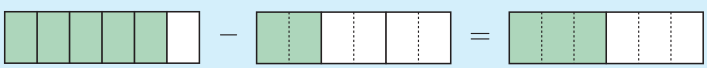
b Write the equation in simplest form.

a



$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

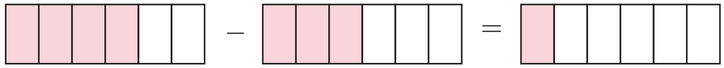
b



$\frac{5}{6} - \frac{1}{3} = \frac{1}{2}$

EXERCISE 8B

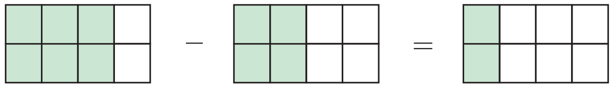
1 Consider the following ‘fraction diagram’:



a Using ‘sixths’ write down the equation that the diagram represents.

b Write the equation in simplest form.

2 Consider the following ‘fraction diagram’:



a Using ‘eighths’ write down the equation that the diagram represents.

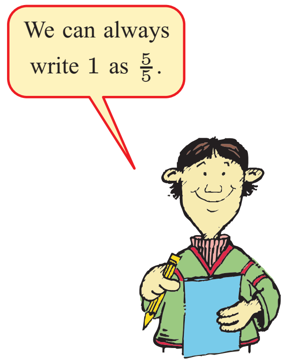
b Write the equation in simplest form.

Example 8 — Self Tutor

Find: **a** $\frac{6}{7} - \frac{4}{7}$ **b** $5 - \frac{3}{5}$

a $\frac{6}{7} - \frac{4}{7}$
$= \frac{2}{7}$

b $5 - \frac{3}{5}$
$= 4 + \frac{5}{5} - \frac{3}{5}$
$= 4\frac{2}{5}$



3 Find:

a $\frac{7}{8} - \frac{5}{8}$ **b** $\frac{5}{9} - \frac{2}{9}$ **c** $\frac{3}{4} - \frac{1}{4}$ **d** $1 - \frac{1}{3}$

e $1 - \frac{4}{5}$ **f** $1 - \frac{3}{8}$ **g** $1 - \frac{7}{11}$ **h** $2 - \frac{2}{5}$

i $3 - \frac{3}{7}$ **j** $4 - \frac{2}{13}$ **k** $1\frac{4}{5} - \frac{2}{5}$ **l** $2\frac{5}{8} - \frac{4}{8}$

Example 9 — Self Tutor

Find: **a** $\frac{3}{4} - \frac{1}{3}$ **b** $\frac{5}{6} - \frac{1}{3} - \frac{2}{9}$

a $\frac{3}{4} - \frac{1}{3}$ {LCD = 12}
$= \frac{3\times3}{4\times3} - \frac{1\times4}{3\times4}$ {converting to 12ths}
$= \frac{9}{12} - \frac{4}{12}$ {simplifying}
$= \frac{5}{12}$ {subtracting the numerators}

b $\frac{5}{6} - \frac{1}{3} - \frac{2}{9}$ {LCD = 18}
$= \frac{5\times3}{6\times3} - \frac{1\times6}{3\times6} - \frac{2\times2}{9\times2}$ {converting to 18ths}
$= \frac{15}{18} - \frac{6}{18} - \frac{4}{18}$ {simplifying}
$= \frac{5}{18}$ {subtracting the numerators}

4 Find:

a $\frac{2}{3}-\frac{1}{6}$ **b** $\frac{5}{6}-\frac{2}{3}$ **c** $\frac{3}{8}-\frac{1}{4}$

d $\frac{3}{4}-\frac{3}{8}$ **e** $\frac{7}{8}-\frac{3}{4}$ **f** $\frac{1}{3}-\frac{1}{4}$

g $\frac{4}{5}-\frac{1}{3}$ **h** $\frac{3}{4}-\frac{2}{3}$ **i** $\frac{4}{5}-\frac{1}{4}$

5 Find:

a $\frac{9}{10}-\frac{1}{5}-\frac{1}{2}$ **b** $\frac{5}{6}-\frac{1}{3}-\frac{1}{2}$ **c** $\frac{7}{8}-\frac{1}{4}-\frac{1}{2}$

d $1-\frac{1}{3}-\frac{1}{4}$ **e** $\frac{1}{4}+\frac{1}{6}-\frac{1}{8}$ **f** $\frac{3}{4}+\frac{5}{6}-\frac{2}{3}$

6 Shaggy leaves $\frac{1}{3}$ of his fortune to Scooby, $\frac{2}{5}$ to Josie, and the rest to Ian. What fraction does Ian get?

7 Answer the **Opening Problem** question **1** on page **146**.

8 Bob owns $\frac{3}{4}$ of a business, Kim owns $\frac{1}{6}$, and Mark owns the rest. What fraction does Mark own?

Example 10 **Self Tutor**

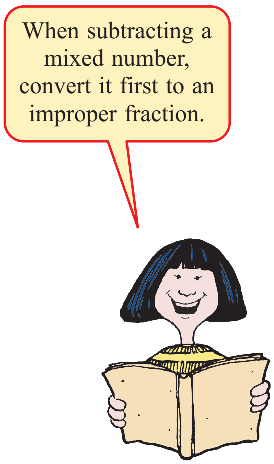
Find: **a** $2-\frac{2}{5}$ **b** $3\frac{1}{2}-2\frac{2}{5}$

a

$$2-\frac{2}{5}$$
$$=\frac{10}{5}-\frac{2}{5}$$
$$=\frac{8}{5}$$
$$=1\frac{3}{5}$$

b

$$3\frac{1}{2}-2\frac{2}{5}$$
$$=\frac{7}{2}-\frac{12}{5}$$
$$=\frac{7\times5}{2\times5}-\frac{12\times2}{5\times2}$$
$$=\frac{35}{10}-\frac{24}{10}$$
$$=\frac{11}{10} \text{ or } 1\frac{1}{10}$$



9 Find:

a $2\frac{1}{3}-1\frac{1}{3}$ **b** $2\frac{1}{3}-1\frac{1}{2}$ **c** $3-1\frac{2}{3}$ **d** $2\frac{1}{2}-\frac{3}{5}$

e $3\frac{1}{2}-1\frac{7}{8}$ **f** $2\frac{1}{3}-1\frac{3}{4}$ **g** $3\frac{1}{4}-\frac{2}{3}$ **h** $2\frac{5}{6}-\frac{11}{12}$

i $1\frac{2}{5}-\frac{3}{4}$ **j** $6-3\frac{2}{3}$ **k** $3\frac{1}{2}-1\frac{5}{7}$ **l** $7\frac{2}{3}-3\frac{1}{4}$

10 Pia estimates it will take her 8 hours to make a dress. She spends $5\frac{1}{4}$ hours on one day and finishes it in another $1\frac{2}{3}$ hours the next day. How much under her estimate was she?

11 Use a calculator to find:

a $\frac{9}{11}-\frac{3}{8}$ **b** $\frac{15}{19}-\frac{6}{13}$ **c** $\frac{23}{24}-\frac{2}{15}-\frac{1}{4}$

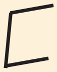
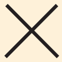
d $5\frac{3}{13}-2\frac{1}{17}$ **e** $3\frac{1}{4}-\frac{8}{21}$ **f** $5\frac{1}{18}-2\frac{1}{4}-1\frac{3}{22}$

INVESTIGATION — EGYPTIAN FRACTIONS

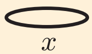
Part A

An important Egyptian archaeological find now called the **Rhind Papyrus** was written by **Ahmes** around 1650 BC. It contains the first recorded organised list of fractions.

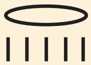
The Ancient Egyptians had special symbols for the common fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$, and $\frac{3}{4}$.

$\frac{1}{2} =$  $\frac{1}{4} =$  $\frac{2}{3} =$  $\frac{3}{4} =$ 

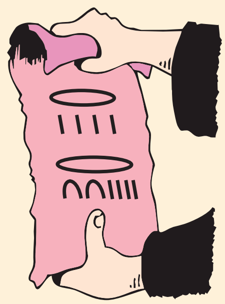
All other fractions were written as **unit fractions** with a numerator of one, or as sums of unit fractions.

The  symbol was used to represent a **reciprocal**. This means that if we have some number x then  means 1 divided by x, or $\frac{1}{x}$.

For example:

$\frac{1}{5} =$ , $\frac{1}{32} =$ , $\frac{7}{24} = \frac{6}{24} + \frac{1}{24} = \frac{1}{4} + \frac{1}{24} =$ 

What to do:



1 Can $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ ...... be written as sums of unit fractions?
Avoid repetition of one fraction, such as $\frac{1}{4} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$.

2 Can all unit fractions be written as a sum of two different unit fractions?
Hint: Examine the difference between unit fractions with consecutive denominators, for example $\frac{1}{3} - \frac{1}{4}$.

3 Can unit fractions be written as sums of three unit fractions?

4 Can you write these fractions as the sums of unit fractions:

a $\frac{2}{3}$ **b** $\frac{5}{6}$ **c** $\frac{4}{7}$ **d** $\frac{7}{9}$

Part B

The Egyptians recorded most of their fractions as sums of unit fractions. How the Ancient Egyptians worked out their unit fraction representations was of interest to **Fibonacci** (1202) and later **J G Sylvester** (in the 1800's).

Fibonacci proposed a method for representing fractions between 0 and 1.

Step 1: Find the largest unit fraction (the one with the smallest denominator) which is less than the given fraction. If we divide the numerator of the given fraction into its denominator, then add one to the quotient, this will be the denominator we need.

Step 2: Subtract the new fraction from the given fraction.

Step 3: Find the largest unit fraction less than the difference in *Step 2*.

Step 4: Subtract again and continue until the difference is a unit fraction.

For example, consider $\frac{5}{17}$:

Step 1: $17 \div 5 = 3 \text{ r } 2$ so the new denominator $= 3 + 1 = 4$

But $\frac{5}{17} - \frac{1}{4} = \frac{20-17}{68} = \frac{3}{68}$

$\therefore \frac{5}{17} = \frac{1}{4} + \frac{3}{68}$

$68 \div 3 = 22 \text{ r } 2$ so the new denominator $= 22 + 1 = 23$

But $\frac{3}{68} - \frac{1}{23} = \frac{69-68}{1564} = \frac{1}{1564}$

$\therefore \frac{5}{17} = \frac{1}{4} + \frac{1}{23} + \frac{1}{1564}$

What to do:

1 Use Fibonacci's method on the following fractions:

a $\frac{2}{35}$ **b** $\frac{4}{5}$ **c** $\frac{13}{20}$ **d** $\frac{4}{13}$ **e** $\frac{2}{9}$ **f** $\frac{2}{43}$

2 How do you think the Ancient Egyptians added their fractions?

C MULTIPLYING FRACTIONS

We can demonstrate the multiplication of fractions by dividing up a rectangle.

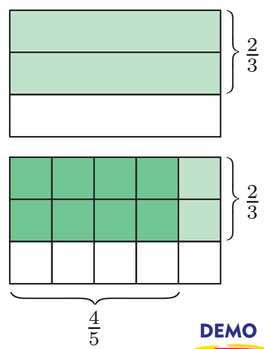
Marni's father has some money in his wallet. He gives $\frac{2}{3}$ of it to Marni. She keeps $\frac{4}{5}$ of the money and spends the rest. Marni keeps $\frac{4}{5}$ of $\frac{2}{3}$, which means she keeps $\frac{4}{5} \times \frac{2}{3}$ of the money.

We start with a rectangle which represents all of the money in Marni's father's wallet. Marni receives $\frac{2}{3}$ of her father's money, so we divide the rectangle into 3 and shade 2 of the 3 strips.

Marni keeps $\frac{4}{5}$ of the money given to her, so we divide the rectangle cross-wise into 5 then shade $\frac{4}{5}$ of the $\frac{2}{3}$.

Overall we have 15 equal sections of which 8 are shaded dark green.

So, Marni keeps $\frac{8}{15}$ of the money, and $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$.



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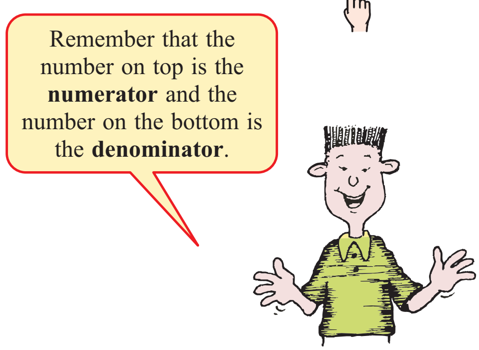


RULE FOR MULTIPLYING FRACTIONS

To multiply two fractions, we multiply the two numerators to get the new numerator, and multiply the two denominators to get the new denominator.

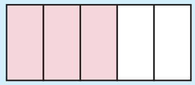
$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Example 11 — Self Tutor

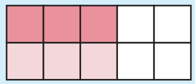
a Use a rectangle diagram to find $\frac{1}{2}$ of $\frac{3}{5}$.

b Check your answer to **a** by finding $\frac{1}{2} \times \frac{3}{5}$ using the rule for multiplying.

a We shade $\frac{3}{5}$ of a rectangle.



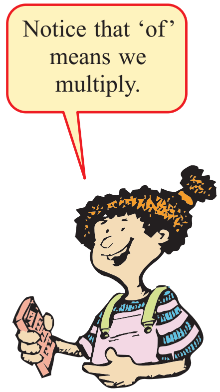
Now we divide the shaded rectangle into halves and shade $\frac{1}{2}$ of the $\frac{3}{5}$.



$\therefore$ $\frac{1}{2}$ of $\frac{3}{5} = \frac{3}{10}$ is shaded.

b $\frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5}$

$= \frac{3}{10}$



EXERCISE 8C.1

1 **a** Use a rectangle diagram to find $\frac{1}{2}$ of $\frac{3}{4}$.

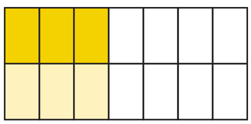
b Check your answer to **a** by finding $\frac{1}{2} \times \frac{3}{4}$ using the rule for multiplying.

2 **a** Use a rectangle diagram to find $\frac{2}{3}$ of $\frac{2}{3}$.

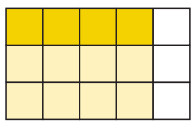
b Check your answer to **a** by finding $\frac{2}{3} \times \frac{2}{3}$ using the rule for multiplying.

3 Write down the fraction multiplication and the answer for the following shaded rectangles:

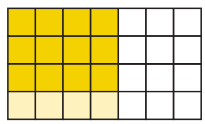
a



b



c



Example 12 — Self Tutor

Find: **a** $\frac{3}{4} \times \frac{1}{5}$ **b** $\left(\frac{3}{5}\right)^2$

a $\frac{3}{4} \times \frac{1}{5}$

$= \frac{3 \times 1}{4 \times 5}$

$= \frac{3}{20}$

b $\left(\frac{3}{5}\right)^2$

$= \frac{3}{5} \times \frac{3}{5}$

$= \frac{9}{25}$

4 Find:

a $\frac{3}{4} \times \frac{1}{5}$ **b** $\frac{2}{3} \times \frac{5}{7}$ **c** $\left(\frac{3}{4}\right)^2$ **d** $\left(\frac{4}{5}\right)^2$

e $\left(\frac{1}{2}\right)^2$ **f** $\left(\frac{2}{3}\right)^2$ **g** $\frac{2}{3} \times \frac{4}{5} \times \frac{2}{7}$ **h** $\frac{4}{11} \times \frac{2}{5} \times \frac{6}{7}$

Example 13 — Self Tutor

Find: $\frac{2}{3} \times 1\frac{4}{5}$

$\frac{2}{3} \times 1\frac{4}{5}$

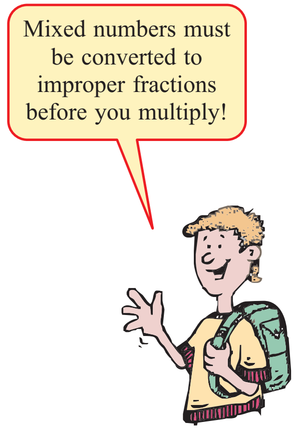
$= \frac{2}{3} \times \frac{9}{5}$ {converting to improper fraction}

$= \frac{18}{15}$ {using the rule for multiplying}

$= \frac{18 \div 3}{15 \div 3}$ {HCF = 3}

$= \frac{6}{5}$ {simplifying}

$= 1\frac{1}{5}$ {converting to a mixed number}



5 Find the following products, giving your answers in simplest form:

a $\frac{1}{2}$ of $1\frac{3}{5}$ **b** $\frac{1}{4} \times 3\frac{1}{3}$ **c** $\frac{3}{5} \times 2$ **d** $\frac{5}{4} \times 1\frac{1}{5}$

e $\frac{1}{3} \times 2\frac{1}{2}$ **f** $1\frac{1}{2} \times 2\frac{1}{4}$ **g** $(1\frac{1}{4})^2$ **h** $2\frac{1}{2} \times 3\frac{1}{3}$

CANCELLATION

In the questions above, some of your answers needed to be simplified. This means that in the original fractions being multiplied, there were **common factors** in the numerator of one fraction and the denominator of the other fraction.

These common factors can be **cancelled before multiplication**. This keeps the numbers smaller and easier to handle, and removes the need to simplify at the end.

For example: $\frac{8}{9} \times \frac{3}{4} = \frac{\cancel{4}^{1} \times 2}{{}_{1}\cancel{3} \times 3} \times \frac{\cancel{3}^{1}}{\cancel{4}_{1}} = \frac{2}{3}$

Example 14 — Self Tutor

Find: **a** $\frac{2}{3} \times \frac{5}{6}$ **b** $\frac{2}{7}$ of 210 **c** $\frac{2}{3} \times 1\frac{1}{4}$

a $\frac{2}{3} \times \frac{5}{6}$

$= \frac{{}^{1}\cancel{2}}{3} \times \frac{5}{\cancel{6}_{3}}$

$= \frac{5}{9}$

b $\frac{2}{7} \times 210$

$= \frac{2}{{}_{1}\cancel{7}} \times \frac{\cancel{210}^{30}}{1}$

$= 60$

c $\frac{2}{3} \times 1\frac{1}{4}$

$= \frac{{}^{1}\cancel{2}}{3} \times \frac{5}{\cancel{4}_{2}}$

$= \frac{5}{6}$

EXERCISE 8C.2

1 Find:

a $\frac{1}{3} \times \frac{6}{7}$ **b** $\frac{3}{4} \times \frac{1}{6}$ **c** $\frac{2}{3}$ of $\frac{3}{4}$ **d** $\frac{1}{2}$ of $\frac{4}{3}$

e $\frac{3}{4} \times 24$ **f** $\frac{2}{5}$ of 30 **g** $\frac{1}{2} \times 4$ **h** $\frac{2}{3}$ of 12

i $5 \times \frac{2}{3}$ **j** $15 \times \frac{3}{5}$ **k** $\frac{3}{7}$ of 35 **l** $2 \times \frac{1}{4}$

m $3 \times \frac{11}{3}$ **n** $1\frac{1}{4} \times 8$ **o** $\frac{4}{5}$ of 25 **p** $20 \times \frac{3}{4}$

q $\frac{5}{8} \times 24$ **r** $64 \times \frac{3}{8}$ **s** $\frac{7}{10}$ of 30 **t** $\frac{5}{12}$ of 600

2 Francesca drinks $\frac{1}{4}$ of a 600 mL cola. How much does she drink?

3 Suzi needs 4 pieces of wood that are all $2\frac{3}{5}$ m long. What is the total length required?

4 Amanda eats $\frac{3}{4}$ of half a cake. What fraction of the total does she eat?

5 Use your calculator to evaluate:

a $\frac{3}{14} \times \frac{5}{16}$ **b** $\frac{4}{17} \times \frac{1}{12}$ **c** $2\frac{1}{5} \times \frac{3}{40}$ **d** $\frac{2}{23} \times 4\frac{1}{2}$

e $1\frac{1}{2} \times \frac{5}{18}$ **f** $(2\frac{1}{3})^2$ **g** $(1\frac{1}{2})^3$ **h** $2\frac{1}{3} \times 3\frac{3}{40}$

D RECIPROCALS

Two numbers are **reciprocals** of each other if their product is one.

For example, $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals since $\frac{2}{3} \times \frac{3}{2} = 1$.

Whole numbers have reciprocals also.

For example, 2 and $\frac{1}{2}$ are reciprocals since $2 \times \frac{1}{2} = 1$.

So, the reciprocal of a whole number is 1 divided by the number. This was the definition of reciprocal used by the Ancient Egyptians which we saw on page **153**.

More generally, $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals since $\frac{a}{b} \times \frac{b}{a} = 1$.

EXERCISE 8D

1 Find $\square$ if: **a** $\frac{2}{3} \times \square = 1$ **b** $3 \times \square = 1$ **c** $\square \times \frac{4}{3} = 1$ **d** $\square \times 5 = 1$

2 Find the reciprocal of: **a** $\frac{3}{4}$ **b** $\frac{5}{4}$ **c** $\frac{1}{7}$ **d** 5 **e** $2\frac{1}{3}$

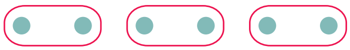
3 Find, without showing any working:

a $\frac{3}{4} \times \frac{4}{3}$ **b** $3 \times \frac{2}{5} \times \frac{5}{2}$ **c** $\frac{5}{7} \times 100 \times \frac{7}{5}$

d $\frac{3}{8} \times 87 \times \frac{8}{3}$ **e** $913 \times 8 \times \frac{1}{8}$ **f** $\frac{4}{11} \times 400 \times \frac{11}{4}$

E DIVIDING FRACTIONS

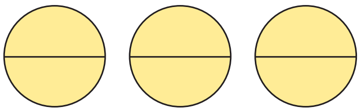
To find $6 \div 2$ we ask the question: *How many twos are there in six*?



The answer is 3, so $6 \div 2 = 3$.

We can divide fractions in the same way.

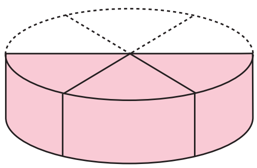
For example, $3 \div \frac{1}{2}$ may be interpreted: *How many halves are there in 3*?



The answer is clearly 6,

so $3 \div \frac{1}{2} = 6$.

However, we know that $3 \times 2 = 6$ also, which suggests that dividing by $\frac{1}{2}$ is equivalent to multiplying by its *reciprocal*, 2.



Now consider dividing half a cheese equally between 3 people.

Each person would get $\frac{1}{6}$ of the whole,

so $\frac{1}{2} \div 3 = \frac{1}{6}$

But $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ also.

DEMO

This also suggests that dividing by a number is equivalent to multiplying by its reciprocal.

RULE FOR DIVIDING FRACTIONS

To **divide** by a number, we multiply by its reciprocal.

Example 15 Self Tutor

Find:

a $\frac{2}{3} \div \frac{5}{7}$ **b** $\frac{3}{4} \div 5$

a $\frac{2}{3} \div \frac{5}{7}$

$= \frac{2}{3} \times \frac{7}{5}$ {multiplying by the reciprocal}

$= \frac{14}{15}$

b $\frac{3}{4} \div 5$

$= \frac{3}{4} \div \frac{5}{1}$

$= \frac{3}{4} \times \frac{1}{5}$

$= \frac{3}{20}$

EXERCISE 8E

1 Find:

a $\frac{3}{4} \div \frac{2}{3}$	**b** $\frac{1}{3} \div \frac{2}{3}$	**c** $\frac{1}{4} \div \frac{1}{2}$	**d** $\frac{1}{2} \div \frac{1}{3}$
e $\frac{1}{2} \div 2$	**f** $\frac{2}{3} \div 4$	**g** $\frac{1}{2} \div 3$	**h** $\frac{1}{5} \div 2$
i $6 \div \frac{2}{3}$	**j** $1 \div \frac{1}{4}$	**k** $10 \div \frac{1}{7}$	**l** $\frac{1}{7} \div 10$
m $3 \div \frac{1}{10}$	**n** $\frac{1}{10} \div 3$	**o** $\frac{1}{5} \div 100$	**p** $100 \div \frac{1}{5}$

Example 16 — Self Tutor

Find: **a** $\frac{1}{4} \div 1\frac{2}{3}$ **b** $1\frac{3}{4} \div 2\frac{1}{2}$

a
$$\frac{1}{4} \div 1\frac{2}{3} = \frac{1}{4} \div \frac{5}{3} = \frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$$

b
$$1\frac{3}{4} \div 2\frac{1}{2}$$
$$= \frac{7}{4} \div \frac{5}{2} \quad \{\text{converting to an improper fraction}\}$$
$$= \frac{7}{\cancel{4}_2} \times \frac{\cancel{2}^1}{5} \quad \{\text{multiplying by the reciprocal and cancel}\}$$
$$= \frac{7}{10}$$

2 Find:

a $\frac{1}{3} \div 3\frac{1}{3}$	**b** $1\frac{2}{3} \div 2\frac{1}{2}$	**c** $2\frac{1}{2} \div 1\frac{1}{3}$	**d** $3\frac{1}{5} \div 1\frac{1}{2}$
e $1\frac{1}{2} \div 3\frac{1}{5}$	**f** $3\frac{3}{4} \div \frac{7}{12}$	**g** $2\frac{7}{12} \div \frac{3}{4}$	**h** $\frac{1}{5} \div 2\frac{1}{3}$

3 Roger takes $\frac{1}{5}$ of an hour to jog around the park.
How many laps of the park can he complete in $1\frac{1}{2}$ hours?

4 Kylie's stride length is $1\frac{1}{3}$ m. How many strides does it take her to walk 24 m?

F PROBLEM SOLVING

Many realistic problems involve fractions in their solution.

If the answer involves a fraction, give your answer in **simplest form** by *cancelling any common factors* from the numerator and denominator.

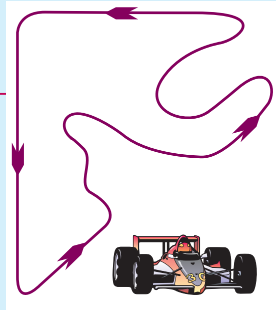
Example 17 — Self Tutor

Sally's sister gave her $\frac{2}{3}$ of a pie. Sally gave $\frac{1}{4}$ of this amount to her daughter. What fraction of the original pie did her daughter receive?

She received $\frac{1}{4}$ of $\frac{2}{3}$
$$= \frac{1}{\cancel{4}_2} \times \frac{\cancel{2}^1}{3} \quad \{\text{of is replaced by } \times\}$$
$$= \frac{1}{6}$$

Example 18 — Self Tutor

A Grand Prix is run on a circuit of $6\frac{1}{4}$ km for a total distance of 200 km. How many laps are needed for the race?



The number of laps is $200 \div 6\frac{1}{4}$

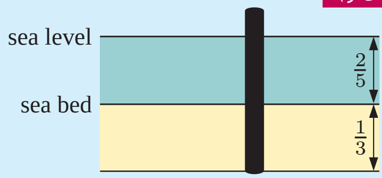
$= \frac{200}{1} \div \frac{25}{4}$

$= \frac{\cancel{200}^{8}}{1} \times \frac{4}{\cancel{25}_{1}}$

$= 32$

Example 19 — Self Tutor

A jetty pylon has $\frac{1}{3}$ of its length below ground and $\frac{2}{5}$ of its length in the water. What fraction is above the water?



The fraction above water is $1 - \frac{1}{3} - \frac{2}{5}$

$= \frac{15}{15} - \frac{1\times 5}{3\times 5} - \frac{2\times 3}{5\times 3}$

$= \frac{15}{15} - \frac{5}{15} - \frac{6}{15}$

$= \frac{4}{15}$

EXERCISE 8F

1 Find the sum of $\frac{2}{3}$ and $\frac{3}{4}$.

2 Find $\frac{7}{12}$ of my investment of €180 000.

3 What number must $\frac{3}{4}$ be multiplied by to get an answer of 15? **Hint:** Find $15 \div \frac{3}{4}$.

4 By how much does $\frac{4}{5}$ exceed $\frac{7}{12}$?

5 In a pig pen containing 36 piglets, 16 are female. What fraction are male?

6 Which is the better score in a mathematics test, 17 out of 20 or 21 out of 25?

7 Find $\frac{2}{5}$ of £245.

8 How many $2\frac{1}{3}$ m lengths of rope can be cut from a rope of length 21 m?

9 Five pieces of material are required to make some curtains, each of length $3\frac{3}{4}$ m. Find the total length required.

10 On consecutive days you eat $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ of a lasagne.

a What fraction has been eaten? **b** What fraction remains?

11 What is the difference between $\frac{3}{7}$ and $\frac{2}{5}$?

12 $\frac{2}{5}$ of a cake remains and is shared equally by 4 children. What fraction of the original cake does each child get?

13 A race track is $3\frac{3}{4}$ km long. How many circuits are needed to complete a 100 km race?

14 Abel leaves $\frac{1}{3}$ of his money to his son, $\frac{3}{8}$ of it to his wife, and the rest is donated to fund cancer research. What fraction is left to cancer research?

15 A marathon swimmer swims $\frac{3}{7}$ of the race distance in the first hour and $\frac{2}{5}$ in the second hour. What fraction of the race has the swimmer left to swim?

16 If I used $\frac{3}{5}$ of a 4 litre can of petrol and $\frac{3}{4}$ of a 10 litre can, how much petrol did I use altogether?

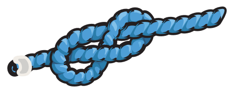
17 A man has \$480 to take home each week. He banks $\frac{1}{8}$ of it, gives $\frac{1}{3}$ of it to his wife, and pays \$100 rent out of what remains. How much of his weekly pay is left?

18 Joel owns $\frac{2}{3}$ of a business and Pam owns $\frac{1}{4}$. Fred owns the remainder.

a What fraction does Fred own?

b If Joel and Pam are to have equal shares, what fraction of the business must Joel give to Pam?

19 As many $\frac{3}{5}$ m lengths as possible are cut from a 16 m length of rope. What length remains?



KEY WORDS USED IN THIS CHAPTER

- common factor
- equal fractions
- lowest common denominator
- numerator
- denominator
- improper fraction
- mixed number
- reciprocal

REVIEW SET 8A

1 **a** Find $2 + \frac{3}{9} + \frac{5}{9}$ without showing any working.

b Add $4\frac{3}{8}$ and $2\frac{3}{4}$.

c Use a calculator to find $\frac{13}{19} + \frac{7}{11}$.

d State the reciprocal of $\frac{2}{3}$.

e Simplify $(1\frac{1}{3})^2$.

f If $\square \times \frac{7}{11} = 1$, find $\square$.

g Show using a diagram how to obtain $\frac{2}{3}$ of $\frac{3}{5}$.

h Find $\frac{1}{3} \div 2\frac{1}{2}$.

2 Find: **a** $\frac{3}{8} + \frac{2}{3}$ **b** $\frac{5}{6} + 1\frac{3}{4}$ **c** $\frac{3}{4} - \frac{3}{8} + \frac{5}{12}$

3 Find:

a $2\frac{1}{4} \times 1\frac{3}{4}$ **b** $2\frac{3}{4} \div 1\frac{1}{4}$ **c** $\frac{7}{8}$ of 56

4 Find without any working:

a $\frac{5}{4} \times \frac{4}{5}$ **b** $126 \times \frac{3}{8} \times \frac{8}{3}$

5 Solve the following problems:

a Twelve pieces of wire mesh, each $4\frac{2}{3}$ metres long, are required to go around a tennis court. Find the total length to be purchased.

b An athlete runs $\frac{2}{5}$ of a race in the first hour and $\frac{1}{3}$ in the second hour.
What fraction of the race does he have left to run?

c Jon bought 27 m of metal piping and cut it into $\frac{3}{4}$ m lengths.
How many lengths did he obtain?

6 If a greengrocer sells $\frac{2}{3}$ of his apples on Monday and $\frac{1}{2}$ of the remainder on Tuesday, what fraction of the apples are still unsold?

7 Simplify:

a $2\frac{5}{8} - 1\frac{3}{4}$ **b** $3\frac{1}{2} + 5\frac{3}{4}$ **c** $\frac{3}{5} \times 15$

REVIEW SET 8B

1 **a** Find $3 + \frac{2}{7} + \frac{4}{7}$ without showing any working.

b Use a calculator to find $\frac{4}{7} + \frac{11}{16}$.

c Find $5 - 1\frac{1}{3}$.

d Use a diagram to find $\frac{1}{3}$ of $\frac{2}{5}$.

e Find $\square$ if $\frac{8}{9} \times \square = 1$.

f Find $\frac{2}{5} \times 37 \times \frac{5}{2}$ without showing any working.

g Find $2\frac{3}{4} \div \frac{5}{12}$.

h What is the difference between $\frac{2}{9}$ and $\frac{4}{7}$?

2 Find:

a $2\frac{2}{3} - \frac{3}{4}$ **b** $3\frac{1}{3} + 1\frac{3}{8}$ **c** $\frac{3}{4} - \frac{2}{5}$

3 Find:

a $2\frac{1}{2} \times 3\frac{1}{2} \times \frac{4}{7}$ **b** $5\frac{1}{3} \div 1\frac{1}{3}$ **c** $3 \div \frac{3}{4}$

4 Find:

a $\frac{7}{12}$ of 8400 **b** $8 - 5\frac{2}{3}$

5 Find:

a $\frac{4}{5} \times \frac{1}{2} \times 2\frac{1}{2}$ **b** $\frac{5}{8}$ of 400 **c** $(3\frac{1}{2})^2$

6 Solve the following problems:

a How many $3\frac{1}{2}$ m lengths of rope can be cut from a length of 35 m?

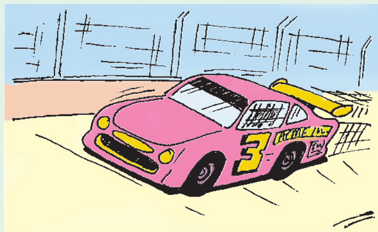
b I give $\frac{5}{8}$ of my fortune to my wife. If she divides this amount equally between our six children, what fraction of my fortune will each child receive?

c A cyclist completes $\frac{2}{5}$ of her training ride in the first hour and $\frac{3}{7}$ in the second hour. What fraction of her ride still remains?

7 Find:

a $\frac{3}{4} + \frac{1}{8} - \frac{1}{2}$ **b** $3\frac{1}{4} \div 1\frac{1}{3}$ **c** $\frac{3}{10}$ of 40

8 Solve the following problems:



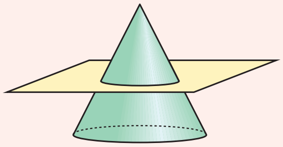
a A race track is $2\frac{1}{4}$ km long. How many laps are needed to complete a 90 km race?

b Davinia Liew earned $45 000 last year. She lost $\frac{1}{3}$ of the amount in tax and $\frac{2}{5}$ of the remainder was needed to pay her home loan. How much did Davinia have left?

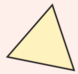
ACTIVITY — CUTTING THROUGH SOLIDS

Match the shape of each cut surface to the solid that it comes from:

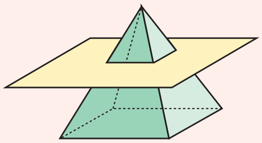
a



A



b



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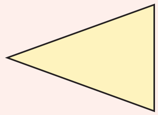

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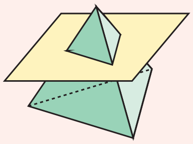

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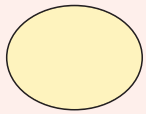

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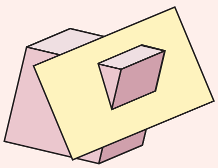

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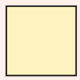

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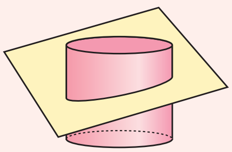
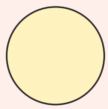
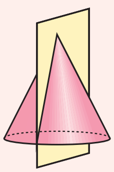


E



f



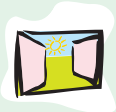
F

Chapter 9

Decimals

Contents:

OPENING PROBLEM



Arrange these amounts of money in ascending order:

$56.65, $56.05, $50.65, $55.50, $56.50

Which is largest?

DECIMAL NUMBERS ARE EVERYWHERE

A CONSTRUCTING DECIMAL NUMBERS

We have seen previously how whole numbers are constructed by placing digits in different **place values**.

For example, 384 has place value table

3	8	4
hundreds	tens	units

since $384 = 3 \times 100 + 8 \times 10 + 4 \times 1$.

Numbers like 0.37 and 4.569 are called **decimal numbers**. We use them to represent numbers *between* the whole numbers. The **decimal point** or dot separates the whole number part to the left of the dot, from the fractional part to the right of the dot.

If the whole number part is zero, we write a zero in front of the decimal point. So, we write 0.37 instead of .37.

0.37 is a short way of writing $\frac{3}{10}+\frac{7}{100}$, and 4.569 is really $4+\frac{5}{10}+\frac{6}{100}+\frac{9}{1000}$

So, the **place value** table for 0.37 and 4.569 is:

Decimal number	units		tenths	hundredths	thousandths	*Expanded form*
0.37	0	.	3	7		$\frac{3}{10}+\frac{7}{100}$
4.569	4	.	5	6	9	$4+\frac{5}{10}+\frac{6}{100}+\frac{9}{1000}$

Example 1 Self Tutor

Express in written or oral form:

a 0.9 **b** 3.06 **c** 11.407

a 0.9 is 'zero point nine'.
b 3.06 is 'three point zero six'.
c 11.407 is 'eleven point four zero seven'.

Oral form means how you would say it.

Example 2 Self Tutor

Write in a place value table:

a 7 hundredths **b** $23+\frac{4}{10}+\frac{9}{1000}$

	Number	tens	units		tenths	hundredths	thousandths	*Written Numeral*
a	7 hundredths			.	0	7		0.07
b	$23+\frac{4}{10}+\frac{9}{1000}$	2	3	.	4	0	9	23.409

EXERCISE 9A

1 Express the following in written or oral form:

a 0.6 **b** 0.45 **c** 0.908 **d** 8.3 **e** 6.08
f 96.02 **g** 5.864 **h** 34.003 **i** 7.581 **j** 60.264

2 Convert into decimal form:

a eight point three seven **b** twenty one point zero five
c nine point zero zero four **d** thirty eight point two zero six

3 Write in a place value table and then as a decimal number:

PRINTABLE WORKSHEET

a $\frac{8}{10}+\frac{3}{100}$

b $4+\frac{1}{10}+\frac{2}{100}+\frac{8}{1000}$

c $9+\frac{4}{1000}$

d $\frac{5}{100}+\frac{6}{1000}$

e $28+\frac{6}{10}+\frac{9}{100}+\frac{9}{1000}$

f $139+\frac{7}{100}+\frac{7}{1000}$

Number	thousands	hundreds	tens	units		tenths	hundredths	thousandths	*Written Numeral*
					.				

4 Write in a place value table and then as a decimal number:

a 8 tenths
b 3 thousandths
c 7 tens and 8 tenths
d 9 thousands and 2 thousandths
e 2 hundreds, 9 units and 4 hundredths
f 8 thousands, 4 tenths and 2 thousandths
g 5 thousands, 20 units and 3 tenths
h 9 hundreds, 8 tens and 34 thousandths
i 6 tens, 8 tenths and 9 hundredths
j 36 units and 42 hundredths

If a word for a digit ends in **ths** then the number follows the decimal point.

Example 3 Self Tutor

Express 5.706 in expanded form:

$$5.706=5+\frac{7}{10}+\frac{0}{100}+\frac{6}{1000}$$
$$=5+\frac{7}{10}+\frac{6}{1000}$$

5 Express the following in expanded form:

a 5.4 **b** 14.9 **c** 2.03 **d** 32.86
e 2.264 **f** 1.308 **g** 3.002 **h** 0.952
i 4.024 **j** 2.973 **k** 20.816 **l** 7.777
m 9.008 **n** 154.451 **o** 808.808 **p** 0.064

6 Write the following in decimal form:

a $\frac{6}{10}$ **b** $\frac{9}{100}$ **c** $\frac{4}{10}+\frac{3}{100}$

d $\frac{8}{10}+\frac{9}{1000}$ **e** $\frac{7}{1000}$ **f** $\frac{5}{100}+\frac{2}{1000}$

g $\frac{5}{10}+\frac{6}{100}+\frac{8}{1000}$ **h** $\frac{2}{1000}+\frac{3}{10\,000}$ **i** $\frac{9}{100}+\frac{4}{1000}$

j $\frac{1}{10}+\frac{1}{1000}$ **k** $4+\frac{3}{10}+\frac{8}{100}+\frac{7}{1000}$ **l** $\frac{3}{100}+\frac{8}{10\,000}$

m $\frac{3}{10}+\frac{3}{1000}+\frac{3}{10\,000}$ **n** $\frac{2}{10}+\frac{5}{100\,000}$ **o** $5+\frac{5}{10}+\frac{5}{100}+\frac{5}{1000}$

Example 4 Self Tutor

State the value of the digit 6 in the following: 0.3964

0.3964 So, the 6 stands for $\frac{6}{1000}$.

$\frac{1}{10}$s $\frac{1}{100}$s $\frac{1}{1000}$s

7 State the value of the digit 3 in the following:

a 4325.9 **b** 6.374 **c** 32.098 **d** 150.953

e 43.4444 **f** 82.7384 **g** 24.8403 **h** 3874.941

8 State the value of the digit 5 in the following:

a 18.945 **b** 596.08 **c** 4.5972 **d** 94.8573

e 75 948.264 **f** 275.183 **g** 358 946.843 **h** 0.0005

Example 5 Self Tutor

Write $\frac{39}{1000}$ in decimal form.

$$\begin{aligned}\frac{39}{1000} &= \frac{30}{1000} + \frac{9}{1000} \\ &= \frac{3}{100} + \frac{9}{1000} \\ &= 0.03 + 0.009 \\ &= 0.039\end{aligned}$$

9 Write in decimal form:

a $\frac{23}{100}$ **b** $\frac{79}{100}$ **c** $\frac{30}{100}$ **d** $\frac{117}{1000}$ **e** $\frac{469}{100}$

f $\frac{703}{1000}$ **g** $\frac{600}{1000}$ **h** $\frac{540}{1000}$ **i** $\frac{4672}{10\,000}$ **j** $\frac{3600}{10\,000}$

10 Convert the following to decimal form:

a seventeen and four hundred and sixty five thousandths

b twelve and ninety six thousandths

c three and six hundred and ninety four thousandths

d four and twenty two hundredths

e 9 hundreds, 8 tens and 34 thousandths

f 36 units and 42 hundredths

11 State the value of the digit 2 in the following:

a $4\frac{324}{1000}$ **b** $62\frac{47}{100}$ **c** $946\frac{42}{100}$ **d** $24\frac{695}{1000}$

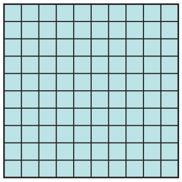
e $1\frac{3652}{10\,000}$ **f** $254\frac{8}{10}$ **g** $57\frac{2}{10}$ **h** $5\frac{652}{1000}$

B REPRESENTING DECIMAL NUMBERS

DECIMAL GRIDS

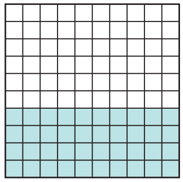
Decimals can also be represented on 2-dimensional grids.

Suppose this grid represents one whole unit.

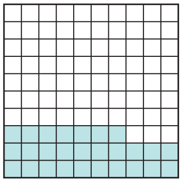


This shaded part is $\frac{4}{10}$ or $\frac{40}{100}$ of the whole unit.

This is 0.4 or 0.40 .

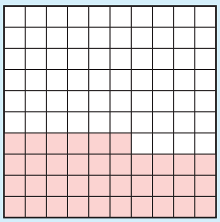


This shaded part is $\frac{27}{100}$ or 0.27 of the whole unit.



Example 6 **Self Tutor**

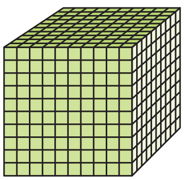
What decimal number is represented by:



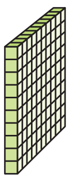
0	.	3	6
units		tenths	hundredths

MULTI ATTRIBUTE BLOCKS

Multi Attribute Blocks or MABs are a practical 3-dimensional way to represent decimals.



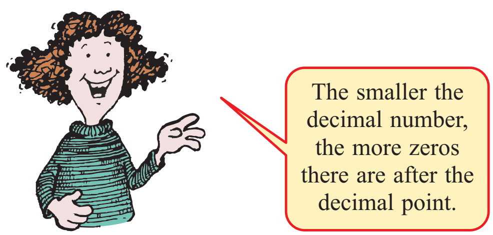
represents a unit or whole amount.



represents a tenth or 0.1 of the whole.

represents a hundredth or 0.01 of the whole.

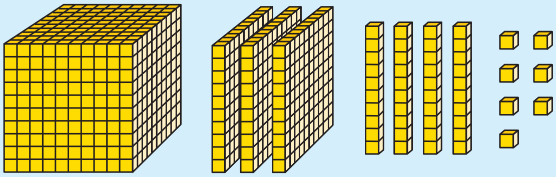
and  represents a thousandth of 0.001 of the whole.



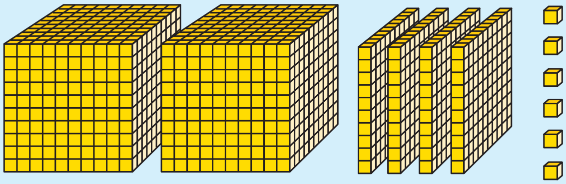
Example 7 **Self Tutor**

Write the decimal value represented by the MABs if the largest block represents one unit.

a



b

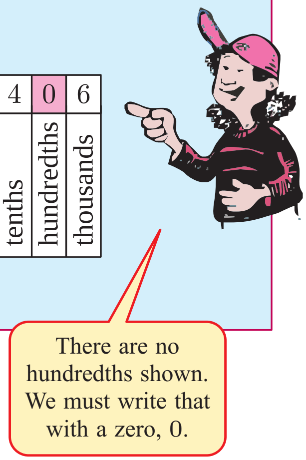


a

1	.	3	4	7
units		tenths	hundredths	thousands

b

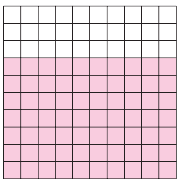
2	.	4	0	6
units		tenths	hundredths	thousands



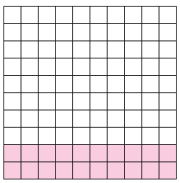
EXERCISE 9B

1 Write the decimal that represents the shaded area:

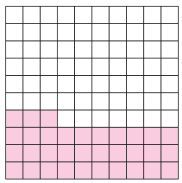
a



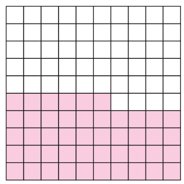
b



c

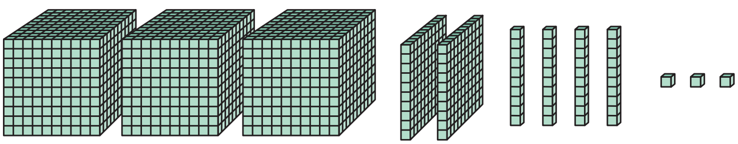


d

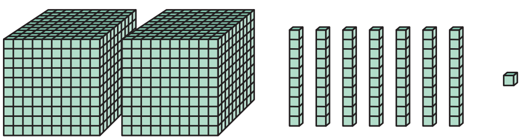


2 Write the decimal value represented by the MABs if the largest block represents one unit:

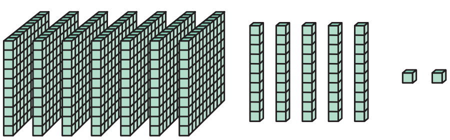
a



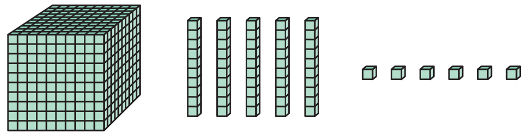
b



c



d



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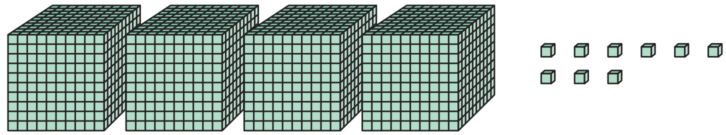
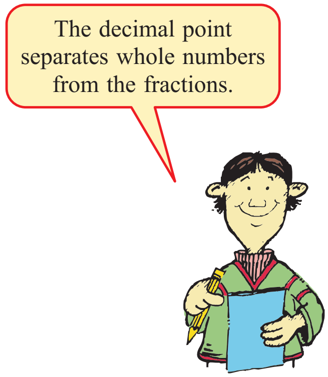


C DECIMAL CURRENCY

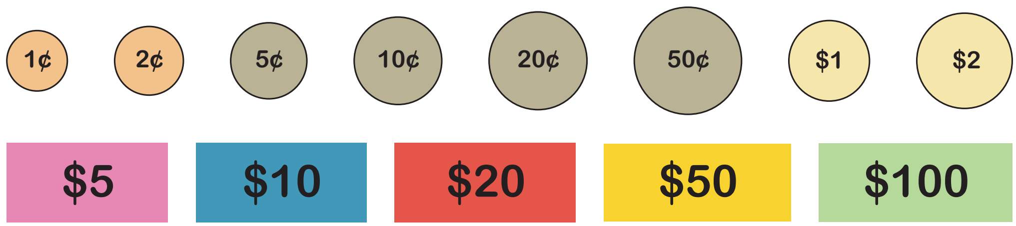
Decimal currency is one of the most practical ways to bring meaning to decimals.

When talking about and using money we are also using decimal numbers.

For example:
€27.35 is €27 plus $\frac{35}{100}$ of one €.



Suppose a country has the following coins and banknotes:



The currency is called **decimal** because it uses the base 10 system.

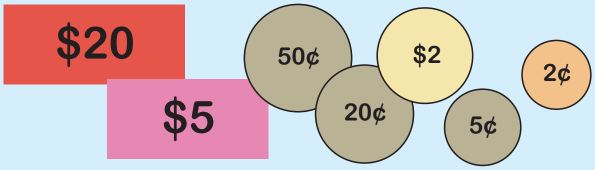
1¢ is $\frac{1}{100}$ or 0.01 of $1 | 2¢ is $\frac{2}{100}$ or 0.02 of $1

5¢ is $\frac{5}{100}$ or 0.05 of $1 | 10¢ is $\frac{10}{100}$ or 0.10 of $1

20¢ is $\frac{20}{100}$ or 0.20 of $1 | 50¢ is $\frac{50}{100}$ or 0.50 of $1

Example 8 **Self Tutor**

How much money is shown?



We have $20 + 5 + 2 = 27$ whole dollars
and $50 + 20 + 5 + 2 = 77$ cents

So, we have $27.77 altogether.

Example 9 **Self Tutor**

Using one euro (€) as the unit, change to a decimal value:

a seven euros, 45 euro cents **b** 275 euro cents

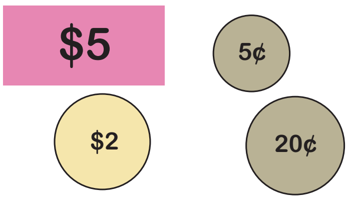
a €7.45

b 275 euro cents
= 200 euro cents + 75 euro cents
= €2 + €0.75
= €2.75

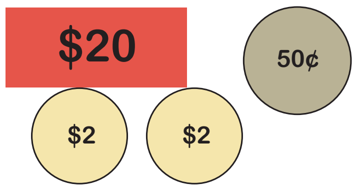
EXERCISE 9C

1 Change these currency values to decimals of one dollar:

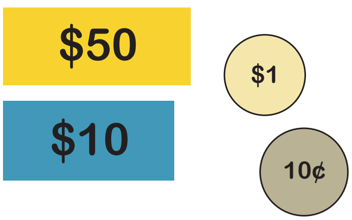
a



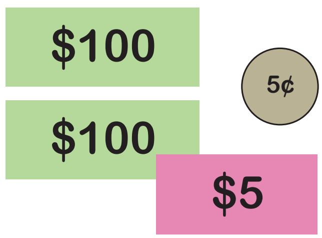
b



c



d



e

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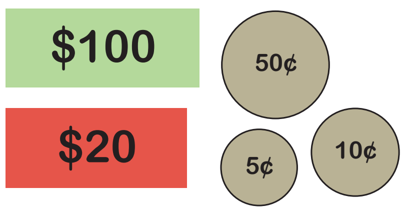


2 Write each amount as dollars using a decimal point:

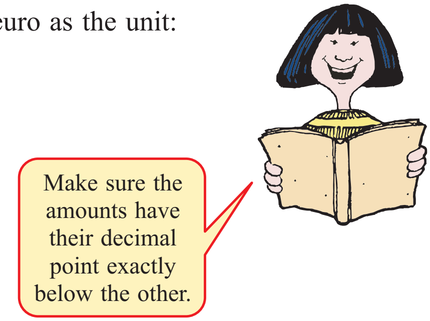
a 4 dollars 47 cents
b 15 dollars 97 cents
c seven dollars fifty five cents
d 36 dollars
e 150 dollars
f thirty two dollars eighty cents
g 85 dollars 5 cents
h 30 dollars 3 cents

3 **a** Change these amounts to decimals using the euro as the unit:

i 35 cents
ii 5 cents
iii 405 cents
iv 3000 cents
v 487 cents
vi 295 cents
vii 3875 cents
viii 638 475 cents

b Starting with the top row, what is the sum of each row above in euros?

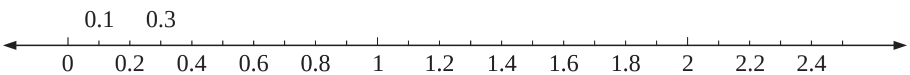
c What is the sum of each column above in euros?



D USING A NUMBER LINE

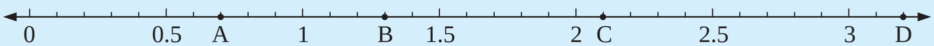
Just as whole numbers can be marked on a number line, we can do the same with decimal numbers. Consider the following number line where each whole number shown has ten equal divisions.

Each division on this number line represents $\frac{1}{10}$ or 0.1



Example 10 **Self Tutor**

Find the decimal values of A, B, C and D marked on the number line shown.

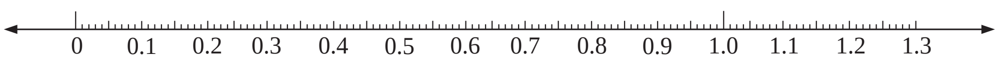


Each division on the number line represents 0.1

So, A is 0.7, B is 1.3, C is 2.1 and D is 3.2

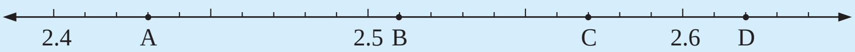
We can divide our number line into smaller parts than tenths.

Suppose we divide each of the parts which represent $\frac{1}{10}$ into 10 equal parts. Each unit is now divided into 100 equal parts and each division is $\frac{1}{100}$ or 0.01 of the unit.



Example 11 | **Self Tutor**

Find the decimal values of A, B, C and D marked on the number line shown.

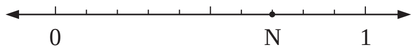


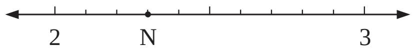
Each division on the number line represents 0.01 .

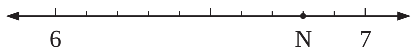
So, A is 2.43, B is 2.51, C is 2.57 and D is 2.62

EXERCISE 9D

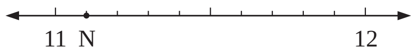
1 Write down the value of the number at N on the following number lines.

a


b


c


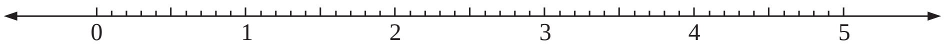
d


e


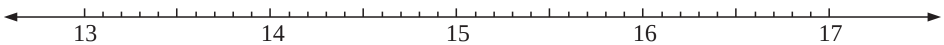
f


2 Copy the number lines given and mark the following numbers on them.

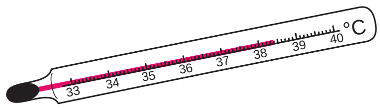
a $A = 1.6$, $B = 2.5$, $C = 2.9$, $D = 4.1$



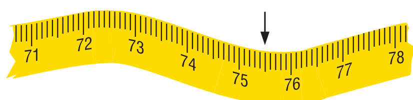
b $E = 13.7$, $F = 14.2$, $G = 15.3$, $H = 16.5$



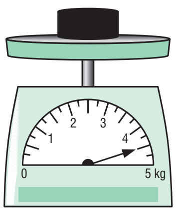
3 Read the temperature on the thermometer shown.



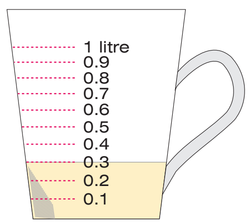
4 Read the length of Christina's skirt from the tape measure.



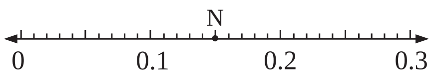
5 **a** What weight is shown on the scales?

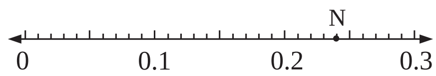


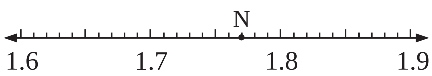
b How much milk is in the jug?

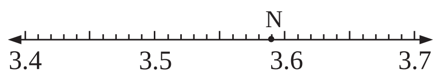


6 Write down the value of the number at N on the following number lines.

a


b


c


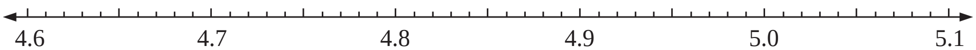
d


e

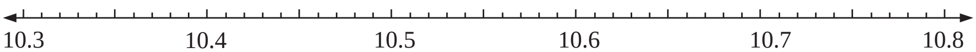

f


7 Copy the number lines given and mark the following numbers on them.

a A = 4.61, B = 4.78, C = 4.83, D = 4.97



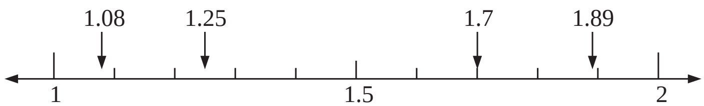
b E = 10.35, F = 10.46, G = 10.62, H = 10.79



E ORDERING DECIMALS

We can use a number line to help compare the sizes of decimal numbers.

For example, consider the following number line:



As we go from left to right, the numbers are increasing.

So, $1.08 < 1.25 < 1.7 < 1.89$

To compare decimal numbers without having to construct a number line, we place zeros on the end so each number has the same number of decimal places.

We can do this because adding zeros on the end does not affect the place values of the other digits.

Example 12 **Self Tutor**

Put the correct sign >, < or = , in the box to make the statement true:

a 0.305 ☐ 0.35

b 0.88 ☐ 0.808

We start by writing the numbers with the same number of decimal places.

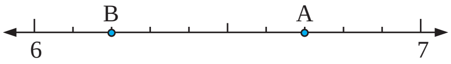
a 0.305 ☐ 0.350
So, $0.305 < 0.350$

b 0.880 ☐ 0.808
So, $0.880 > 0.808$

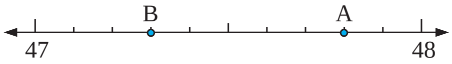
EXERCISE 9E

1 Write down the values of the numbers A and B on the following number line, and determine whether A > B or A < B:

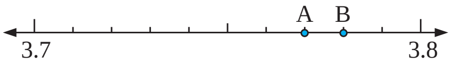
a



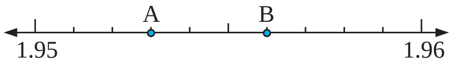
b



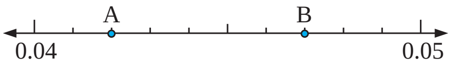
c



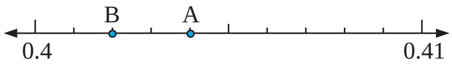
d



e



f



2 Insert the correct sign >, < or = to make the statement true:

a 0.7 □ 0.8 **b** 0.06 □ 0.05 **c** 0.2 □ 0.19
d 4.01 □ 4.1 **e** 0.81 □ 0.803 **f** 2.5 □ 2.50
g 0.304 □ 0.34 **h** 0.03 □ 0.2 **i** 6.05 □ 60.50
j 0.29 □ 0.290 **k** 5.01 □ 5.016 **l** 1.15 □ 1.035
m 21.021 □ 21.210 **n** 8.09 □ 8.090 **o** 0.904 □ 0.94

3 Arrange in ascending order (lowest to highest):

a 0.8, 0.4, 0.6
b 0.4, 0.1, 0.9
c 0.14, 0.09, 0.06
d 0.46, 0.5, 0.51
e 1.06, 1.59, 1.61
f 2.6, 2.06, 0.206
g 0.095, 0.905, 0.0905
h 15.5, 15.05, 15.55

4 Arrange in descending order (highest to lowest):

a 0.9, 0.4, 0.3, 0.8
b 0.51, 0.49, 0.5, 0.47
c 0.6, 0.596, 0.61, 0.609
d 0.02, 0.04, 0.42, 0.24
e 6.27, 6.271, 6.027, 6.277
f 0.31, 0.031, 0.301, 0.311
g 8.088, 8.008, 8.080, 8.880
h 7.61, 7.061, 7.01, 7.06

5 Continue the number patterns by writing the next three terms:

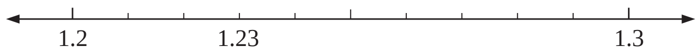
a 0.1, 0.2, 0.3, ...
b 0.9, 0.8, 0.7, ...
c 0.2, 0.4, 0.6, ...
d 0.05, 0.07, 0.09, ...
e 0.7, 0.65, 0.6, ...
f 2.17, 2.13, 2.09, ...
g 7.2, 6.4, 5.6, ...
h 0.25, 0.50, 0.75, ...
i 1.111, 1.123, 1.135, ...
j 0, 0.125, 0.250, ...

F ROUNDING DECIMAL NUMBERS

We are often given measurements as decimal numbers. For example, my bathroom scales tell me I weigh 59.4 kg. In reality I do not weigh *exactly* 59.4 kg, but this is an *approximation* of my actual weight. Measuring my weight to greater accuracy is not important.

We round off decimal numbers in the same way we do whole numbers. We look at values on the number line either side of our number, and work out which is closer.

For example, consider 1.23.



1.23 is closer to 1.2 than it is to 1.3, so we round down.

1.23 is approximately 1.2.

Consider	5716		Likewise,	0.5716	
	≈ 5720	(to the nearest 10)		≈ 0.572	(to 3 decimal places)
	≈ 5700	(to the nearest 100)		≈ 0.57	(to 2 decimal places)
	≈ 6000	(to the nearest 1000)		≈ 0.6	(to 1 decimal place)

RULES FOR ROUNDING OFF DECIMAL NUMBERS

- If the digit after the one being rounded is **less than 5**, i.e., 0, 1, 2, 3 or 4, then we round **down**.
- If the digit after the one being rounded is **5 or more**, i.e., 5, 6, 7, 8 or 9, then we round **up**.

Example 13 Self Tutor

Round: **a** 3.26 to 1 decimal place **b** 5.273 to 2 decimal places **c** 4.985 to 2 decimal places

a 3.26 is closer to 3.3 than to 3.2, so we round up.
So, $3.26 \approx 3.3$.

b 5.273 is closer to 5.27 than to 5.28, so we round down.
So, $5.273 \approx 5.27$.

c 4.985 lies halfway between 4.98 and 4.99, so we round up.
So, $4.985 \approx 4.99$.

EXERCISE 9F

1 Write these numbers correct to 1 decimal place:

a 2.43 **b** 3.57 **c** 4.92 **d** 6.38 **e** 4.275

2 Write these numbers correct to 2 decimal places:

a 4.236 **b** 2.731 **c** 5.625 **d** 4.377 **e** 6.5237

3 Write 0.486 correct to:
  a 1 decimal place **b** 2 decimal places.

4 Write 3.789 correct to:
  a 1 decimal place **b** 2 decimal places.

5 Write 0.183 75 correct to:
  a 1 decimal place **b** 2 decimal places **c** 3 decimal places
  d 4 decimal places.

6 Find decimal approximations for:
  a 3.87 to the nearest tenth
  b 4.3 to the nearest integer
  c 6.09 to one decimal place
  d 0.4617 to 3 decimal places
  e 2.946 to 2 decimal places
  f 0.175 61 to 4 decimal places.

G CONVERTING DECIMALS TO FRACTIONS

Decimal numbers can be easily written as fractions with powers of 10 as their denominators.

Example 14 **Self Tutor**

Write as a fraction or as a mixed number:

a 0.7 **b** 0.79 **c** 2.013

a $0.7 = \frac{7}{10}$

b $0.79 = \frac{79}{100}$

c $2.013 = 2 + \frac{13}{1000} = 2\frac{13}{1000}$

We have seen previously how some fractions can be converted to **simplest form** or **lowest terms** by dividing both the numerator and denominator by their **highest common factor**.

Example 15 **Self Tutor**

Write as a fraction in simplest form:

a 0.4 **b** 0.72 **c** 0.275

a $0.4 = \frac{4}{10} = \frac{4\div 2}{10\div 2} = \frac{2}{5}$

b $0.72 = \frac{72}{100} = \frac{72\div 4}{100\div 4} = \frac{18}{25}$

c $0.275 = \frac{275}{1000} = \frac{275\div 25}{1000\div 25} = \frac{11}{40}$

EXERCISE 9G

1 Write the following as fractions in simplest form:

a 0.1	**b** 0.7	**c** 1.5	**d** 2.2
e 3.9	**f** 4.6	**g** 0.19	**h** 1.25
i 0.18	**j** 0.65	**k** 0.05	**l** 0.07
m 2.75	**n** 1.025	**o** 0.04	**p** 2.375

2 Write the following as fractions in simplest form:

a 0.8	**b** 0.88	**c** 0.888	**d** 3.5
e 0.49	**f** 0.25	**g** 5.06	**h** 3.32
i 0.085	**j** 3.72	**k** 1.096	**l** 4.56
m 0.064	**n** 0.625	**o** 0.115	**p** 2.22

Example 16 Self Tutor

Write as a fraction: **a** 0.45 kg **b** 3.40 m

a 0.45 kg
$= \frac{45}{100}$ kg
$= \frac{45 \div 5}{100 \div 5}$ kg
$= \frac{9}{20}$ kg

b 3.40 m
$= 3 \text{ m} + \frac{40 \div 20}{100 \div 20} \text{ m}$
$= 3 \text{ m} + \frac{2}{5} \text{ m}$
$= 3\frac{2}{5}$ m

3 Write these amounts as fractions or mixed numbers in simplest form:

a 0.20 kg	**b** 0.25 hours	**c** 0.85 kg	**d** 1.50 km
e 1.75 g	**f** 2.74 m	**g** 4.88 tonnes	**h** 6.28 L
i €1.25	**j** €1.76	**k** €3.65	**l** €4.21
m €8.40	**n** €5.125	**o** £3.08	**p** £4.11
q £18.88	**r** £52.25		

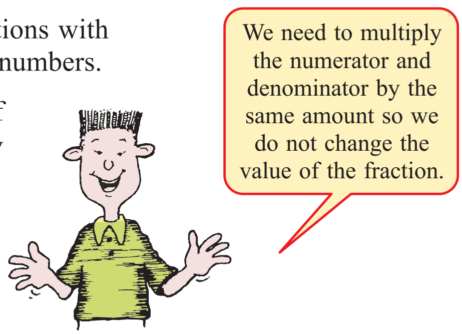
H CONVERTING FRACTIONS TO DECIMALS

We have already seen that it is easy to convert fractions with denominators 10, 100, 1000, and so on into decimal numbers.

Sometimes we can make the denominator a power of 10 by multiplying the numerator and denominator by the same numbers.

For example, $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$

$\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 0.28$



Example 17 — Self Tutor

Convert to decimal numbers:

a $\frac{3}{4}$ **b** $\frac{7}{20}$ **c** $\frac{23}{125}$

a $\frac{3}{4} = \frac{3\times 25}{4\times 25} = \frac{75}{100} = 0.75$

b $\frac{7}{20} = \frac{7\times 5}{20\times 5} = \frac{35}{100} = 0.35$

c $\frac{23}{125} = \frac{23\times 8}{125\times 8} = \frac{184}{1000} = 0.184$

EXERCISE 9H

1 By what whole number would you multiply the following, to obtain a power of 10?

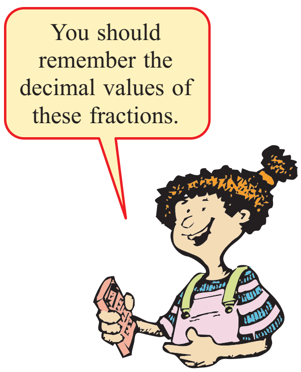
a 2 **b** 5 **c** 4 **d** 8 **e** 20 **f** 25
g 50 **h** 125 **i** 40 **j** 250 **k** 500 **l** 400

2 Convert to decimal numbers:

a $\frac{3}{20}$ **b** $\frac{17}{20}$ **c** $\frac{9}{25}$ **d** $\frac{21}{25}$ **e** $1\frac{1}{2}$ **f** $2\frac{1}{5}$
g $\frac{13}{50}$ **h** $\frac{138}{500}$ **i** $\frac{6}{250}$ **j** $\frac{91}{250}$ **k** $\frac{1}{4}$ **l** $\frac{9}{125}$
m $\frac{68}{125}$ **n** $\frac{117}{125}$ **o** $\frac{11}{500}$ **p** $\frac{51}{250}$ **q** $\frac{3}{8}$ **r** $\frac{71}{400}$

3 Copy and complete these conversions to decimals:

a $\frac{1}{2} = ...$
b $\frac{1}{5} = ...,\ \frac{2}{5} = ...,\ \frac{3}{5} = ...,\ \frac{4}{5} = ...,$
c $\frac{1}{4} = ...,\ \frac{2}{4} = ...,\ \frac{3}{4} = ...$
d $\frac{1}{8} = ...,\ \frac{2}{8} = ...,\ \frac{3}{8} = ...,\ \frac{4}{8} = ...,$
$\frac{5}{8} = ...,\ \frac{6}{8} = ...,\ \frac{7}{8} = ...$



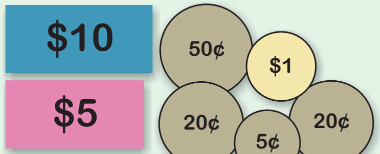
KEY WORDS USED IN THIS CHAPTER

- decimal
- fraction
- mixed numbers
- simplest form
- decimal currency
- highest common factor
- place value
- tenth
- decimal point
- hundredth
- round off
- thousandth

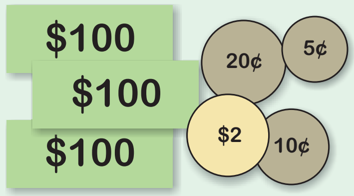
REVIEW SET 9A

1 If the dollar represents the unit, what are the decimal values of the following?

a

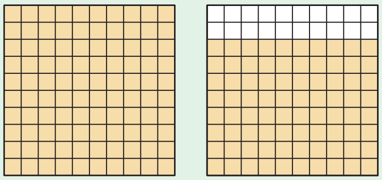


b

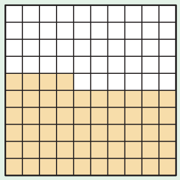


2 If each 10×10 grid represents one unit, what decimals are represented by the following grids?

a



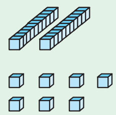
b



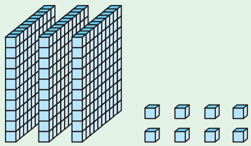
3 If  represents one thousandth, write the decimal numbers for:

a

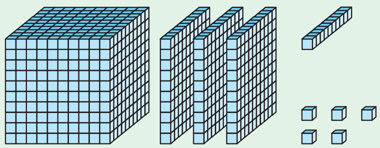
b



c



d



4 Write as a decimal number

a $\frac{7}{10} + \frac{3}{100}$ **b** $\frac{1}{10} + \frac{7}{1000}$ **c** $5 + \frac{6}{100} + \frac{9}{1000}$

5 Write:

a 25 euros and 35 euro cents as a decimal number

b $107 and 85 cents as a decimal number

c 'five and twenty nine thousandths' in decimal form

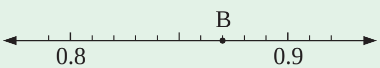
d 4.36 in two different fractional forms

e 2.049 in expanded form

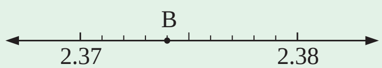
f the meaning of the 2 digit in 51.932.

6 Write down the value of the number at B on these number lines:

a



b



7 Which number is greater, 3.2 or 3.1978?

8 Round 3.8551 to:

a one decimal place **b** two decimal places.

9 Convert to a fraction in simplest form:

a 0.8 **b** 0.75 **c** 0.375 **d** 0.68

10 Convert to a decimal number:

a $\frac{4}{5}$ **b** $\frac{3}{20}$ **c** $\frac{5}{8}$ **d** $\frac{11}{40}$

11 Write in ascending order: 0.216, 0.621, 0.062, 0.206, 0.026

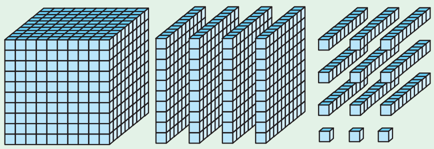
12 Continue this number pattern for 3 more terms:

0.81, 0.78, 0.75, 0.72, ...

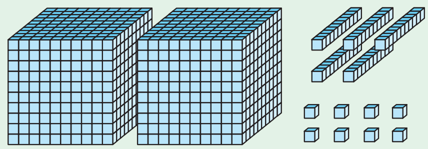
REVIEW SET 9B

1 If  represents one thousandth, write the decimal values of:

a

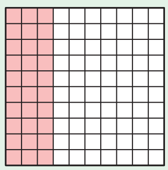


b



2 If each 10×10 grid represents one unit, what decimal numbers are represented by:

a



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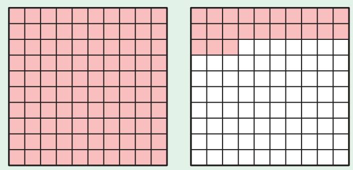


3 Write as a decimal number:

a $\frac{4}{10} + \frac{4}{100}$ **b** $\frac{3}{100} + \frac{3}{1000}$ **c** $1 + \frac{2}{1000}$ **d** $4 + \frac{1}{10} + \frac{5}{1000}$

4 **a** Convert 'sixteen point five seven four' to decimal form.

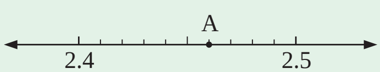
b Write 0.921 in two different non-decimal forms.

c State the value of the digit 3 in 41.039.

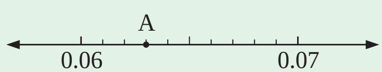
d Write £12 and 35 pence as a decimal.

5 Write down the value of the number at A on the following number lines:

a



b



6 Write the following decimal numbers in descending order:

0.444, 4.04, 4.44, 4.044, 4.404

7 Continue the number pattern by writing the next three terms: 0.3, 0.7, 1.1, ...

8 Which number is smaller, 2.3275 or 2.3199?

9 Round 3.995 to:

a 1 decimal place **b** 2 decimal places

10 Convert to a fraction in simplest form:

a 0.62 **b** 0.45 **c** 0.875 **d** 10.4

11 Convert to a decimal number:

a $\frac{3}{50}$ **b** $1\frac{1}{5}$ **c** $\frac{17}{25}$ **d** $\frac{1}{8}$

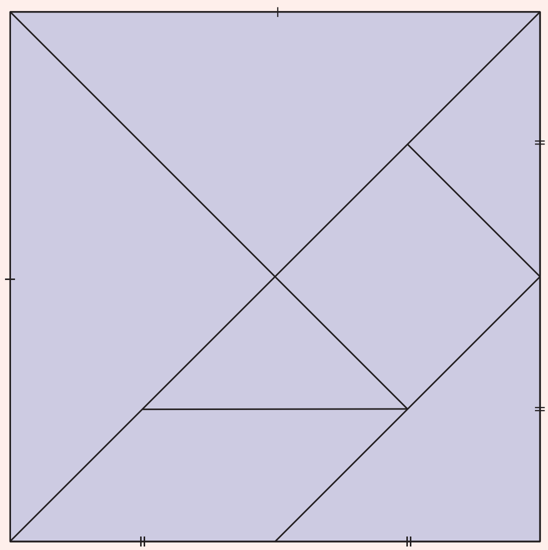
12 Copy and complete: $\frac{1}{8} = 0.125$, $\frac{2}{8} = ...$, $\frac{3}{8} = ...$, $\frac{4}{8} = ...$, $\frac{5}{8} = ...$.

ACTIVITY TANGRAMS

What to do:

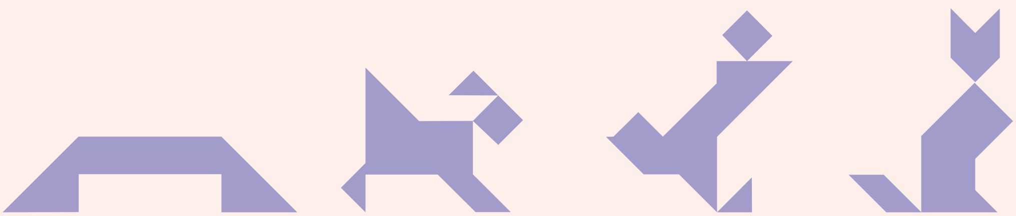
1 On a piece of card mark out a 20 cm by 20 cm square. Then copy the following lines onto it and cut along each line. You should have seven different pieces.

PRINTABLE TEMPLATE



2 Each of the following shapes can be made using all seven pieces of your tangram. See how many you can complete.

a bridge **b** puppy **c** person running **d** cat

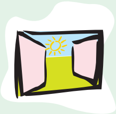


Chapter 10

Problem solving

Contents:

OPENING PROBLEM



Suppose a group of people have gathered for a conference, and each person shakes hands with every other person.

Things to think about:

a How many handshakes will take place if the group consists of:

- **i** 2 people
- **ii** 3 people
- **iii** 4 people
- **iv** 5 people?

b Can you find a pattern in your answers to **a**?

c Can you find a rule connecting the *number of people* P with the *number of handshakes* H?

d How many possible handshakes take place when all 192 representatives of the United Nations countries shake hands with each other?

You need to become familiar with a variety of problem solving techniques so you can feel confident when confronted by new problem situations.

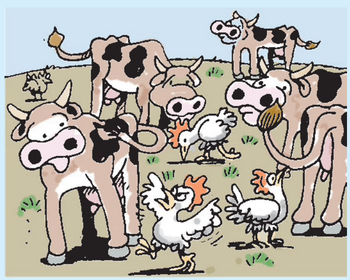
You need to be aware that any one problem can be solved in a variety of ways. The following examples show possible problem solving techniques.

A TRIAL AND ERROR

This is the most common method used by students at your level. It is self-explanatory in its name, but when you use trial and error you must always remember to **check** your answer.

Example 1 Self Tutor

On a farm there are some chickens and some cows. An observer counts 19 heads and 62 feet. Assuming each creature has only one head, cows have 4 feet and chickens have 2 feet, how many chickens and how many cows are on the farm?



We first guess the number of chickens to be 5. In this case the number of cows must be 14 since $5 + 14 = 19$.

With 5 chickens and 14 cows, the total number of feet is: $5 \times 2 + 14 \times 4 = 10 + 56 = 66$.

This does not equal the required 62 so another guess is needed. Since we require less feet, more chickens are needed and fewer cows.

So, we guess the number of chickens to be 7. The number of cows must be 12 since $7 + 12 = 19$.

The total number of feet is: $7 \times 2 + 12 \times 4 = 62$.

This is the required number of feet, so there are 7 chickens and 12 cows on the farm.

EXERCISE 10A

Solve these problems using the trial and error method. Write your final answers in clear sentences.

1 Find consecutive whole numbers that add up to 51.

2 In a jar there are some spiders and beetles. If there are 13 creatures in total and the total number of legs is 86, how many of each creature are in the jar?

3 Using the digits 2, 3, 4 and 5 in that order, and the symbols $\times$, $-$, $+$ in any order, write a mathematical expression that equals 9. You may need to use brackets.

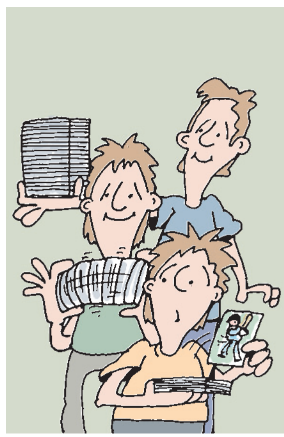
4 Hera paid for a €69 picture frame with coins she had saved. She used only €2 and €1 coins and noticed she was able to pay using the same number of each coin. How many €1 coins did she use to pay for her picture frame?

5 The sum on the right is not correct. Change *one* of the digits to make it correct.

$$\begin{array}{r} 386 \\ +\ \ 125 \\ \hline 521 \end{array}$$

6 If $a \times b = 24$, $b \times c = 12$ and $c \times a = 18$, find whole number values for a, b, and c.

7 Brothers Stephen, Kevin and Neil each own a collection of baseball cards. Kevin owns twice as many cards as Stephen, and Neil owns three times as many cards as Stephen. They own 72 cards between them. How many cards does Stephen own?



8 When a positive whole number is squared, the result is 90 more than the original number. Find the original number.

9 Jelena has three times as many brothers as sisters. Her brother Milan has two more brothers than he has sisters. How many boys and how many girls are there in the family?

10 Kristina is two years older than Fredrik, who is 6 years younger than Frida. Together their ages total 41 years. How old is each child?

B MAKING A LIST

Many problems involve finding the number of possibilities which satisfy a certain condition, or finding the number of possible ways to achieve a certain outcome.

As long as the number of possibilities is not too large, we can list them and then find the size of the list.

It is important to list the possibilities in a systematic order to ensure that all the possibilities are found.

Example 2 Self Tutor

How many two digit numbers contain a 7?

The two digit numbers which contain a 7 are, in order:

17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 87, 97

So, there are 18 two digit numbers which contain a 7.

Example 3 Self Tutor

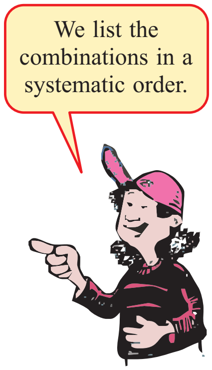
A captain and vice captain are to be chosen from a squad of 5 players. In how many ways can this be done?

Suppose the 5 players are A, B, C, D and E.

We let AB denote selecting A as the captain and B as the vice-captain.

The possible combinations are:
AB, BA, AC, CA, AD, DA, AE, EA, BC, CB,
BD, DB, BE, EB, CD, DC, CE, EC, DE, ED

So, the captain and vice captain can be chosen in 20 different ways.



EXERCISE 10B

Solve these problems by listing the possibilities:

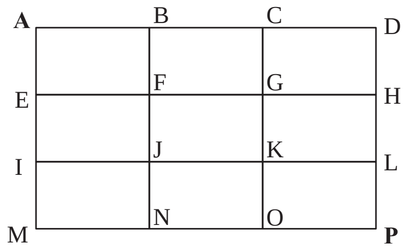
1 How many two digit numbers contain a 1 or a 2?

2 As part of their Physical Education course, year 6 students must select one indoor sport and one outdoor sport from the list given. How many possible combinations of sports are there to choose from?

Indoor	*Outdoor*
Basketball	Tennis
Squash	Football
Netball	Rugby
	Hockey

3 An icecream van offers four flavours of icecream: chocolate, vanilla, strawberry, and lemon. Donna is allowed to pick two different flavours for her icecream. How many possible flavour combinations could she choose?

4 Alex is travelling from A to his friend Peter's house at P. How many different routes are there from A to P, assuming Alex always moves towards P?

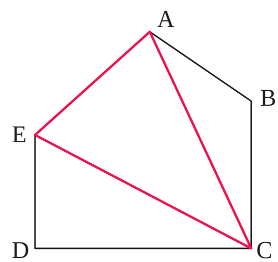


5 In how many ways can four friends Amy, Beth, Christine, and Deb sit in a row if Amy and Christine insist on sitting next to each other?

6 In his pocket Wei has a 5 cent coin, a 10 cent coin, a 20 cent coin and a 50 cent coin. How many different sums of money can he make using these four coins?

7 How many two digit numbers are there in which the tens digit is less than the ones digit?

8 How many triangles can be formed by joining three of the vertices of a pentagon? One such triangle is illustrated.



9 How many two digit numbers can be made using the digits 5, 6, 7, 8 and 9 at most once each?

10 Scott, Elizabeth and Richard are all trying to remember the 4 digit combinations for their school lockers.

- **a** Scott remembers that his combination contains a 4, 5, 7 and 9, but cannot remember the order. How many possible combinations will he need to try?
- **b** Elizabeth remembers that her combination contains two 5s, a 7, and an 8. How many possible combinations will she need to try?
- **c** Richard remembers that his combination contains three 2s and a 6. How many possible combinations will he need to try?

C MODELLING OR DRAWING A PICTURE

To solve some problems it is useful to act out the problem, draw pictures, or use equipment to model the situation.

Example 4 Self Tutor

Masumi has three different coloured tops: red, green, and white. She has two different coloured skirts, yellow and pink which she can wear with them. How many different combinations of skirts and tops can she wear?

We can model this situation by pairing up each of the

tops    with each of the skirts  

The possible pairings are:

So, there are 6 different combinations Masumi can wear.

EXERCISE 10C

Solve these problems by modelling or drawing pictures.

1 A square table has four seats around it. In how many different ways can four people sit around the table?

2 Year 6 and 7 students are doing the same orienteering course. A year 6 student leaves every 6 minutes and a year 7 student leaves every 3.5 minutes. The event begins at 9 am with one student from each year group leaving together. When is the next time a student from each year level will leave together?

3 Minh numbered the pages of his art folder using a packet of stickers. On each sticker there was one digit from 0 to 9. He started with page 1. When he finished he noticed he had used 77 stickers. How many pages were in his art folder?

4 What is the largest number of pieces you can cut a round pizza into using four straight cuts?

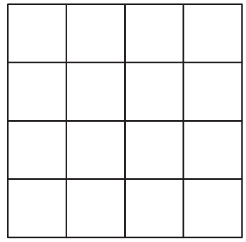
5 When Eduardo and Elvira went to a basketball game they had tickets for seats 93 and 94. They saw that the seat numbers followed the pattern:

Row 3	11	12	13	14	15	16	17	18
Row 2		5	6	7	8	9	10	
Row 1			1	2	3	4		

In what row were their seats?

6 When Phil put 10 counters in his bag it was $\frac{1}{3}$ full.
When James put 13 in his bag it was $\frac{1}{2}$ full.
When Blair put 7 in his bag it was $\frac{1}{4}$ full.
Who had the biggest bag?

7 How many squares are present in the figure alongside?



8 Runners A, B, C, D and E competed in a cross-country race. You are given the following details about the race:

E finished ahead of D, but behind C.
A finished ahead of B, but behind E.
D finished 4th.

Find the order in which the runners completed the race.

9 Lisa, Martin, Natalie, Owen, and Patel decided to paint the rooms of the house they share. The colours to be used were red, white, blue, green, and yellow. Each person was given a different colour to paint with.

The lounge was painted by Lisa, Martin and Owen, and was painted yellow, green and blue.

The kitchen was painted by Martin and Patel, and was painted blue and white.

The bathroom was painted by Lisa and Natalie, and was painted red and yellow.

The bedroom was painted by Natalie, Owen and Patel, and was painted white, red and green.

Match each person with the colour they were using.

10 Benita needs to measure 4 litres of water for cooking pasta. However, she only has a 3 litre bowl and a 5 litre bowl, and they have no measurements on their sides. Explain how Benita can use these two bowls to measure exactly 4 litres of water.

D MAKING A TABLE AND LOOKING FOR A PATTERN

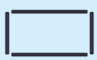
A good way to solve problems which involve large numbers is to consider simpler versions of the problem using smaller numbers.

We put the results of these simpler problems into a table, and try to find a pattern. The pattern is then used to predict the answer to the original problem.

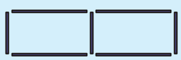
Example 5 **Self Tutor**

How many fence sections are required to enclose 30 square yards side by side?

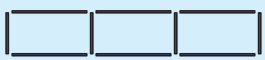
To enclose 1 yard, 4 fence sections are required.



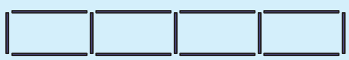
To enclose 2 yards, 7 fence sections are required.



To enclose 3 yards, 10 fence sections are required.



To enclose 4 yards, 13 fence sections are required.



We summarise the results in a table.

Yards	1	2	3	4
Fence Sections	4	7	10	13

+3 +3 +3

The pattern is *the number of fence sections increases by* 3 *for each additional yard*. We can use this pattern to continue the table up to 30 yards:

Yards	1	2	3	4	5	6	......	28	29	30
Fence Sections	4	7	10	13	16	19	......	85	88	91

+3 +3 +3 +3 +3 +3 +3

So, we require 91 fence sections to enclose 30 yards.

Alternatively, we notice that
$4 = 3 \times 1 + 1$ {for 1 yard}
$7 = 3 \times 2 + 1$ {for 2 yards}
$10 = 3 \times 3 + 1$ {for 3 yards}
$13 = 3 \times 4 + 1$ {for 4 yards}

In general, the *number of fence sections* $= 3 \times$ (*number of yards*) $+ 1$

So, for 30 yards, the number of fence sections $= 3 \times 30 + 1$
$= 91$

Example 6 **Self Tutor**

A captain and vice-captain are to be chosen from a squad of 20 players.
In how many ways can this be done?

We could list the possibilities as in **Example 3**, but this would take a very long time as there are now 20 players to choose from. We will instead consider smaller cases and look for a pattern.

For 2 players A and B, there are 2 ways: AB or BA.

For 3 players A, B and C, there are 6 ways: AB, BA, AC, CA, BC, or CB.

For 4 players A, B, C and D, there are 12 ways: AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, or DC.

For 5 players we know from **Example 3** that there are 20 ways.

We summarise the results in a table.

Number of Players	2	3	4	5
Ways	2	6	12	20

+4 +6 +8

The pattern is *each time we increase the number of players, we increase the number of ways by two more than the previous increase.*

Number of Players	2	3	4	5	6	7	......	18	19	20
Ways	2	6	12	20	30	42	......	306	342	380

+4 +6 +8 +10 +12 +36 +38

So, the captain and vice-captain can be chosen in 380 different ways.

Alternatively, notice that
$2 = 2 \times 1$ {for 2 players}
$6 = 3 \times 2$ {for 3 players}
$12 = 4 \times 3$ {for 4 players}
$20 = 5 \times 4$ {for 5 players}

In general, the *number of ways* = *(number of players)* × *(number of players − 1)*

So, for 20 players, the number of ways $= 20 \times 19$
$= 380$

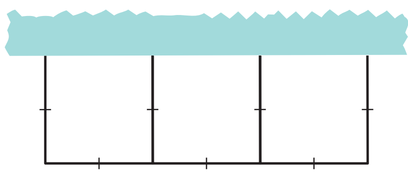
EXERCISE 10D

Solve these problems by solving simpler problems and then looking for a pattern.

1 How many different two course meals can I make given a choice of eight main courses and 16 desserts?
Hint: Try with a choice of two mains and one dessert, then two mains and two desserts, and so on. Make a table of your findings and look for a pattern.

2 A club rugby team has five different shirt colours and four shorts colours. How many different uniforms are possible?

3 Fences are used to divide land into square blocks as shown. One side of each block faces the river and is unfenced. How many fence lengths are required to make 45 blocks?

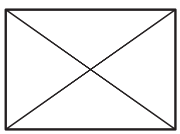


4 How many top and bottom rails would be needed to complete a straight fence with 55 posts?

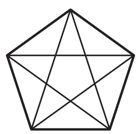
5 How many 27 digit numbers have digits which sum to 2?

6 A rectangular piece of paper is folded in half 7 times. When the paper is unfolded, how many sections will it be divided into?

7 How many diagonals does a 12-sided polygon have? Remember that a diagonal is a straight line that joins two vertices of a polygon but is not a side of the polygon.
For example:



A rectangle has 2 diagonals.



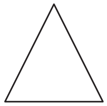
A pentagon has 5 diagonals.

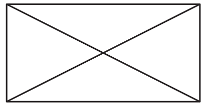
8 The pizza problem in **Exercise 10C** question **4** can also be solved by this method of patterns. Use this method to find how many pieces you can cut a pizza into with ten straight cuts. Each piece does not have to be the same size!

9 Answer the questions in the **Opening Problem** on page **186**.

10 **a** How many lines are required to join each vertex of a 30-sided polygon to every other vertex?

For example:





For a triangle 3 lines are needed. For a rectangle 6 lines are needed.

b Explain the connection between the problem in part **a** and the **Opening Problem**.

E WORKING BACKWARDS

In many problems we are given the final result, and need to work out the initial situation. In cases like this we need to work backwards through the problem.

Example 7 **Self Tutor**

I think of a number, add three to it, multiply this result by 2, subtract 4, and then divide by 7. The number I end up with is 2.

What was the number I first thought of?

To do this problem we start with the result and work in reverse order.

We start with the final result of 2. The last step was to divide by 7.
Since $14 \div 7 = 2$, the previous result was 14.

Before this we subtracted 4.
Since $18 - 4 = 14$, the previous result was 18.

Before this we multiplied by 2.
Since $9 \times 2 = 18$, the previous result was 9.

The first step was to add 3.
Since $6 + 3 = 9$, the original number must be 6.

Check this result by starting with 6 and ending up with 2.

Example 8 **Self Tutor**

Sally cycles $\frac{1}{3}$ of the way to school and then is driven the remaining 6 km by a friend's family. How far does she live from school?

She cycles $\frac{1}{3}$ and so is driven $\frac{2}{3}$ of the way.

$\therefore$ 6 km is $\frac{2}{3}$ of the way

$\therefore$ 3 km is $\frac{1}{3}$ of the way.

$\therefore$ 9 km is all of the way.

Sally lives 9 km from school.

EXERCISE 10E

Solve these problems using the working backwards method.

1 The number of rabbits in my rabbit farm doubles each month. At the end of last month there were 24 000 rabbits. When were there 1500 rabbits?

2 I think of a number and multiply it by 2, then subtract 8. This result is then divided by 4. I end up with 4. What number did I think of originally?

3 I bought some apples from the supermarket. I used $\frac{1}{3}$ of them to make a pie, and then my brother ate three of them. There are now 7 left. How many apples did I buy originally?

4 Ying started the day with a certain amount of money in her purse. She spent $30 on a shirt, and then half the money remaining in her purse was spent on lunch. She borrowed $40 from a friend, and spent half the money remaining in her purse on a book. Ying now has $30 left in her purse. How much did she have in her purse at the start of the day?

5 Nima and Kelly each own a collection of marbles. While playing during the day, Nima lost 3 marbles and Kelly lost 4. At the end of the day, Nima gave a third of her remaining marbles to Kelly, so that each of them had 22 marbles. How many marbles did each girl own at the start of the day?

6 Todd is trying to lose weight. For 6 weeks he keeps a record of his weight loss or gain for the week.
At the end of the 6 weeks, Todd weighs 85 kg. Find:
  a Todd's weight at the end of week 3
  b Todd's initial weight.

Week	*Result*
1	lose 2 kg
2	lose 1 kg
3	gain 2 kg
4	lose 2 kg
5	gain 1 kg
6	lose 2 kg

7 Two years ago, my mother was 7 times older than me. In one year's time, my father will be 5 times older than me. My father is currently 39 years old. How old is my mother now?

8 Nikora left home at a certain time. He rode his bike for 20 minutes, then walked for a further 15 minutes. He rested there for half an hour before continuing on to Sam's house, which was a further 25 minutes walk away. Nikora played at Sam's house for 45 minutes before moving on to his grandmother's home which took him another 20 minutes. He arrived at his grandmother's home, at noon, just in time for lunch. What was the time he left his home that morning?

KEY WORDS USED IN THIS CHAPTER

- list
- table
- model
- trial and error
- pattern

REVIEW SET 10A

1. Daniel has only £10 notes and £20 notes in his wallet. In total he has 11 notes with value £150. How many of each note does Daniel have in his wallet?

2. Claire has 12 oranges. She wants to divide them into 3 piles so that each pile contains a different number of oranges. In how many ways can this be done?
   Note: 5, 4, 3 is the same as 5, 3, 4.

3. **a** How many games are played in a knockout tennis tournament with 16 players?
   Hint: Draw a diagram and count the number of games played.
   b Explain a quicker method you could have used to solve **a**.
   c How many games are played in a knockout tennis tournament with 256 players?

4. Sergio does weights training once every 5 days and fitness training once every 7 days. If he did weights training on January 1st and fitness training on January 3rd, when will Sergio next have weights training and fitness training on the same day?

5. I think of a number, add 7, divide the result by 3, then subtract 4.
   The result is 1. Find the original number.

REVIEW SET 10B

1. A quadrilateral is formed by joining 4 of the vertices of a hexagon. How many quadrilaterals can be formed?

2. There are 4 trees in my garden. The pine tree is 3 m taller than the apple tree. The crepe myrtle tree is 3 m shorter than the oak tree. The oak tree is twice as tall as the apple tree. If the crepe myrtle tree is 7 m tall, how tall is the pine tree?

3. Start with a 2 digit number. The next number in the sequence is found by multiplying the digits together. This process continues until a single digit number is reached.

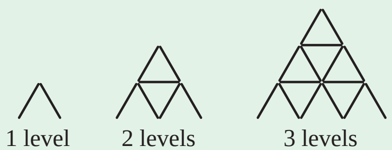
   For example, starting with 68: $68 \xrightarrow{6\times 8} 48 \xrightarrow{4\times 8} 32 \xrightarrow{3\times 2} 6$

   or starting with 53: $53 \xrightarrow{5\times 3} 15 \xrightarrow{1\times 5} 5.$

   Notice that starting with 68 produces a sequence of length 4, and starting with 53 produces a sequence of length 3.
   There is only one 2 digit number which produces a sequence of length 5. Which number is it?

4. A chairperson and secretary are to be chosen from a committee of 4 men and 3 women. In how many ways can this be done if the chairperson and secretary must be of different genders?

5. A house of cards is formed by balancing playing cards on top of one another:



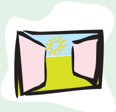
   How many cards are needed to produce a house of cards 10 levels high?

Chapter 11

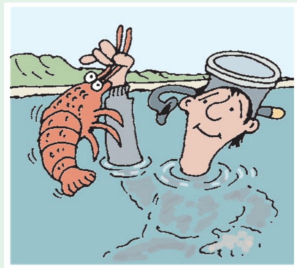
Operations with decimals

Contents:

OPENING PROBLEM



Andy caught 5 lobsters when scuba diving. They weighed 0.76 kg, 1.23 kg, 0.85 kg, 0.97 kg and 1.19 kg.



Thinks to think about:

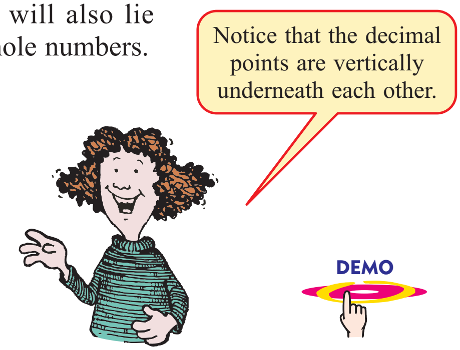
a What is the total weight of Andy's catch?

b If Andy caught this weight of lobster each day for a week, what would be the total weight of his catch?

A ADDING AND SUBTRACTING DECIMALS

When **adding** or **subtracting** decimal numbers, we write the numbers under one another so the decimal points are vertically underneath each other.

When this is done, the digits in each place value will also lie under one another. We then add or subtract as for whole numbers.



DEMO

Example 1 Self Tutor

Find $3.84 + 0.372$

$$\begin{array}{rr} & 3.840 \\ + & 0.372 \\ \hline & 4.212 \end{array}$$

(carries: 1 1)

Example 2 Self Tutor

Find: **a** $3.652 - 2.584$ **b** $6 - 0.637$

a
$$\begin{array}{rr} & 3.\overset{5}{\not6}\,\overset{14}{\not5}\,\overset{12}{\not2} \\ - & 2.5\,8\,4 \\ \hline & 1.0\,6\,8 \end{array}$$

Place the decimal points vertically under one another and subtract as for whole numbers.

b
$$\begin{array}{rr} & \overset{5}{\not6}.\overset{9}{\not0}\,\overset{9}{\not0}\,\overset{10}{\not0} \\ - & 0.6\,3\,7 \\ \hline & 5.3\,6\,3 \end{array}$$

We insert .000 so we have the same number of decimal places in both numbers.

EXERCISE 11A

1 Find:

a $0.4 + 0.5$ **b** $0.6 + 2.7$ **c** $0.9 + 0.23$

d $0.17 + 0.96$ **e** $23.04 + 4.78$ **f** $15.79 + 2.64$

g $0.4 + 0.8 + 4$ **h** $0.009 + 0.435$ **i** $0.95 + 1.23 + 8.74$

j $30 + 0.007 + 2.948$ **k** $0.0036 + 0.697$ **l** $0.071 + 0.677 + 4$

2 Find:

a $1.7 - 0.9$ **b** $2.3 - 0.8$ **c** $4.2 - 3.8$
d $2 - 0.6$ **e** $4 - 1.7$ **f** $3 - 0.74$
g $4.5 - 1.83$ **h** $1 - 0.99$ **i** $10 - 0.98$
j $5.6 - 0.007$ **k** $1 - 0.999$ **l** $0.18 + 0.072 - 0.251$

3 **a** Add 2.094 to the following:
i 36.918 **ii** 0.04 **iii** 0.982 **iv** 5.906
b Subtract 1.306 from the following:
i 2.407 **ii** 1.405 **iii** 13.06 **iv** 24

4 Add:

a 31.704, 8.097, 24.2 and 0.891
b 3.56, 4.575, 18.109 and 1.249
c 1.001, 0.101, 0.011, 10.101 and 1
d 3.0975, 1.904, 0.003 and 16.2874
e 4, 4.004, 0.044 and 400.44
f 0.76, 10.4, 198.4352 and 0.149.

5 Subtract:

a 29.712 from 35.693
b 6.089 from 7.1
c 19 from 23.481
d 3.7 from 171.048
e 9.674 from 68.3
f 8.0096 from 11.11
g 3.333 from 22.2
h $38.018 + 17.2$ from 63
i $(47.64 - 18.79)$ from 33.108
j \$109.75 from \$115.05
k €24.13 from €30.10
l £38.45 and £16.95 from £60.

6 Add:

a three point seven nine four two, eleven point zero five zero nine, thirty six point eight five nine four, and three point four one three eight
b seventeen and four hundred and twenty five thousandths, twelve and eighty five hundredths, three and nine hundred and seven thousandths, and eight and eighty four thousandths
c thirteen hundredths and twenty seven thousandths, and one and four hundredths
d fourteen dollars seventy eight, three dollars forty, six dollars eighty seven, and ninety three dollars and five cents.

7 **a** By how much is forty three point nine five four greater than twenty eight point zero eight seven?
b How much less than five and thirty eight hundredths is two and six hundred and forty nine thousandths?
c What is the difference between nine and seventy two hundredths and nine and thirty nine thousandths?
d How much remains from my sixty four pounds seventy five if I spend fifty seven pounds ninety?

8 John gets €5.40 pocket money, Pat gets €3.85, and Jill €7.85. How much pocket money do they get altogether?

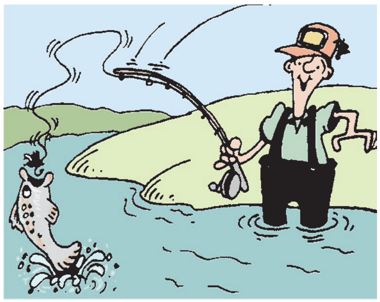
9 Helena is 1.75 m tall and Fred is 1.38 m tall. How much taller is Helena than Fred?

10 I weigh myself every week. At the beginning of the month I weighed 68.4 kg. In the first week I put on 1.2 kg, while in the second week I lost 1.6 kg. Unfortunately I put on another 1.4 kg in the third week. How much did I weigh at the end of the three week period?

11 At a golf tournament two players hit the same ball, one after the other. First Jeff hit the ball 132.6 m. Janet then hit the ball a further 104.8 m. How far did the ball travel altogether?

12 Shin is trying to save \$62.50 for a computer game. He had \$16.40 in his bank to start with and earned the following amounts doing odd jobs: \$2.45, \$6.35, \$19.50, \$14.35. Does he have enough money? If he does not, how much more does he need to earn?

13 Our class went trout fishing and caught five fish weighing the following amounts: 10.6 kg, 3.45 kg, 6.23 kg, 1.83 kg and 5.84 kg. What was the total weight of all five fish?



14 In a fish shop, four large fish weigh 4.72, 3.96, 3.09 and 4.85 kg. If a customer wants a minimum of 20 kg of fish, what extra weight is needed?

15 Find the total length of these three pieces of timber: 2.755 m, 3.084 m and 7.240 m.

16 How much change from €100 is left after I buy items for €10.85, €37.65, €19.05 and €24.35?

B MULTIPLYING AND DIVIDING BY POWERS OF 10

MULTIPLICATION

Consider multiplying 3.57 by 100: $3.57 \times 100 = \frac{357}{\cancel{100}_{1}} \times \frac{\cancel{100}^{1}}{1}$

$= 357$

and by 1000: $3.57 \times 1000 = \frac{357}{\cancel{100}_{1}} \times \frac{\cancel{1000}^{10}}{1}$

$= 357 \times 10$

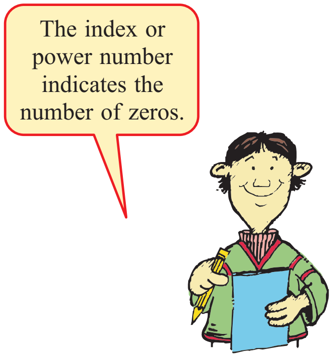
$= 3570$

When we multiply by 100, the decimal point of 3.57 shifts 2 places to the **right**.
3.57 becomes 357.

When we multiply by 1000, the decimal point shifts 3 places to the **right**.
3.570 becomes 3570.

> When multiplying by 10^n we shift the decimal point n places to the **right**.
> The number becomes 10^n times **larger** than it was originally.

Remember $10^1 = 10$
$10^2 = 100$
$10^3 = 1000$
$10^4 = 10\,000$
$\vdots$



Example 3 **Self Tutor**

Find: **a** 8.3×10 **b** 0.0932×100 **c** $4.32 \times 10\,000$

a 8.3×10
$= 8.3 \times 10^1$ {$10 = 10^1$, so shift the decimal point 1 place right}
$= 83$

b 0.0932×100
$= 0.0932 \times 10^2$ {$100 = 10^2$, so shift the decimal point 2 places right}
$= 9.32$

c $4.32 \times 10\,000$
$= 4.3200 \times 10^4$ {$10\,000 = 10^4$, so shift the decimal point 4 places right}
$= 43\,200$

EXERCISE 11B.1

1 Multiply the numbers to complete the table:

	Number	$\times 10$	$\times 100$	$\times 1000$	$\times 10^4$	$\times 10^6$
a	0.0943					
b	4.0837					
c	0.0008					
d	24.6801					
e	$57.85					

2 Find:

a 43×10
b 8×1000
c 5×10^6
d 0.6×10
e 4.6×10
f 0.58×100
g 3.09×100
h 2.5×100
i 0.8×100
j 3.24×100
k 0.9×1000
l 0.845×1000
m 0.24×1000
n 2.085×10^2
o 8.94×10^3
p 0.053×1000
q 0.0094×10^1
r $0.718 \times 100\,000$

3 Write the multiplier to complete the equation:

a $5.3 \times \square = 530$ **b** $0.89 \times \square = 890$ **c** $0.04 \times \square = 400$
d $38.094 \times \square = 3809.4$ **e** $70.4 \times \square = 704$ **f** $38.69 \times \square = 386.9$
g $65.871 \times \square = 6587.1$ **h** $0.0006 \times \square = 600$ **i** $0.003\,934 \times \square = 3.934$

DIVISION

Consider dividing 5.2 by 100:

$$\begin{aligned} 5.2 \div 100 &= \tfrac{52}{10} \div \tfrac{100}{1} \\ &= \tfrac{52}{10} \times \tfrac{1}{100} \\ &= \tfrac{52}{1000} \\ &= 0.052 \end{aligned}$$

and by 1000:

$$\begin{aligned} 5.2 \div 1000 &= \tfrac{52}{10} \times \tfrac{1}{1000} \\ &= \tfrac{52}{10\,000} \\ &= 0.0052 \end{aligned}$$

When we divide by 100, the decimal point in 5.2 shifts 2 places to the **left**.
005.2 becomes 0.052.

When we divide by 1000, the decimal point in 5.2 shifts 3 places to the **left**.
0005.2 becomes 0.0052.

When dividing by 10^n we shift the decimal point n places to the **left**.
The number becomes 10^n times **smaller** than it was originally.

Example 4 Self Tutor

Find: **a** $0.6 \div 10$ **b** $0.37 \div 1000$

a $0.6 \div 10$
$= 0.6 \div 10^1$ $\{10 = 10^1$, so shift the decimal point 1 place left$\}$
$= 0.06$

b $0.37 \div 1000$
$= 000.37 \div 10^3$ $\{1000 = 10^3$, so shift the decimal point 3 places left$\}$
$= 0.000\,37$

EXERCISE 11B.2

1 Divide the numbers to complete the table:

	Number	$\div 10$	$\div 100$	$\div 1000$	$\div 10^5$
a	647.352				
b	93 082.6				
c	42 870				
d	10.94				

2 Find:

a $2.3 \div 10$	**b** $3.6 \div 100$	**c** $42.6 \div 100$
d $3 \div 10$	**e** $58 \div 10$	**f** $58 \div 100$
g $394 \div 10$	**h** $7 \div 100$	**i** $45.8 \div 100$
j $8.007 \div 10$	**k** $24.05 \div 1000$	**l** $632 \div 10\,000$
m $579 \div 100$	**n** $579 \div 1000$	**o** $579 \div 10\,000$
p $0.03 \div 10$	**q** $0.03 \div 100$	**r** $0.046 \div 1000$

3 Write the divisor to complete the equation:

a $9.6 \div \square = 0.96$	**b** $38.96 \div \square = 0.3896$
c $6.3 \div \square = 0.063$	**d** $5.8 \div \square = 0.0058$
e $15.95 \div \square = 1.595$	**f** $386 \div \square = 0.0386$
g $3016.4 \div \square = 30.164$	**h** $874.86 \div \square = 0.847\,86$

C LARGE DECIMAL NUMBERS

Very large numbers are often shortened using letters and decimals.

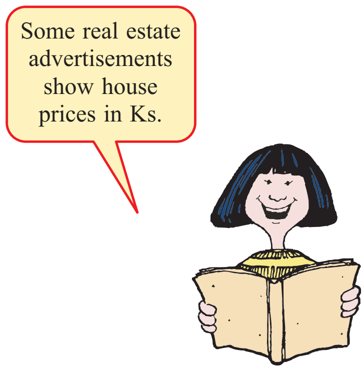
THOUSANDS

Older computers often have memory chips which hold thousands of bits or bytes of information.

The letters k or K are used to represent thousands.

For example, 512 kb is approximately 512 000 bytes of information.

In the employment section of most newspapers you will find annual salaries offered in thousands of dollars.



If Justin is paid an annual salary of €46.2K then he is paid $46.2 \times 1000 =$ €46 200 each year.

Example 5 — Self Tutor

Explain what is meant by a salary of $27.5 K - $29.6 K.

When discussing a salary, K represents 1000.

So, $27.5 K = $27.5 × 1000 = $27 500
and $29.6K = $29.6 × 1000 = $29 600.

The dash, -, indicates a range of salaries between the lowest and highest.

∴ $27.5 K - $29.6 K means a salary between $27 500 and $29 600.

MILLIONS

The letters m or M are used to shorten amounts to decimals of a **million**.

Example 6 — Self Tutor

Round off \$2 378 425 to 2 decimals of a million.

$$\begin{aligned}\$2\,378\,425 &= \$(2\,378\,425 \div 1\,000\,000)\text{ m}\\ &= \$2.378\,425\text{ m}\\ &\approx \$2.38\text{ m} \quad \{\text{rounded to 2 decimal places}\}\end{aligned}$$

BILLIONS

Large companies often give their profits or losses in decimals of billions of dollars.

Distances in space, world population, insect, animal and plague numbers, crops, and human body cells are some of the large numbers that are presented in decimals of a billion.

We use the letters bn to represent one billion.

Example 7 — Self Tutor

Round 37 425 679 420 to 2 decimal places of a billion.

$$\begin{aligned}37\,425\,679\,420 &= (37\,425\,679\,420 \div 1\,000\,000\,000)\text{ bn}\\ &= 37.425\,679\,420\text{ bn}\\ &\approx 37.43\text{ bn} \quad \{\text{rounded to 2 decimal places}\}\end{aligned}$$

EXERCISE 11C

1 Write these salary ranges in thousands of dollars, correct to 1 decimal place:

- **a** \$56 345 - \$61 840
- **b** \$32 475 - \$34 885
- **c** \$23 159 - \$24 386
- **d** \$70 839 - \$73 195
- **e** \$158 650 - \$165 749
- **f** \$327 890 - \$348 359

2 Explain what is meant by a salary of:

- **a** \$38.7 K - \$39.9 K
- **b** \$43.2 K - \$44.5 K
- **c** \$95.5 K - \$98.9 K

3 Round these figures to 2 decimals of a million:

- **a** 3 179 486
- **b** 91 734 598
- **c** 23 456 654
- **d** 1 489 701
- **e** 30 081 896
- **f** 9 475 962

4 Expand these to whole numbers:

- **a** 21.65 million
- **b** 1.93 million
- **c** 16.03 million

5 Expand the following to whole numbers:

- **a** 3.86 bn
- **b** 375.09 bn
- **c** 21.95 bn
- **d** 4.13 bn

6 Round these figures to 2 decimals of a billion:

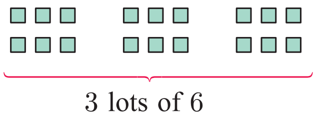
- **a** 3 867 900 000
- **b** 2 713 964 784
- **c** 97 055 843 899
- **d** 2 019 438 421

D MULTIPLYING DECIMAL NUMBERS

We have previously used shaded diagrams to help understand the multiplication of fractions. In this section we do the same thing to help understand the multiplication of decimals.

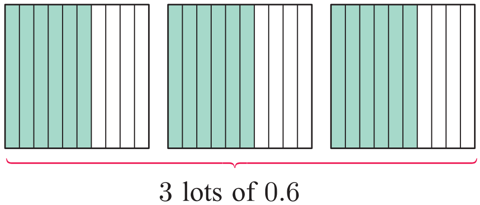
Consider the following multiplications:

- 3×6 looks like



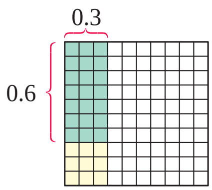
which is 18 units. So, $3 \times 6 = 18$.

- 3×0.6 looks like



which is 18 shaded tenths, $\frac{18}{10}$ or 1.8. So, $3 \times 0.6 = 1.8$

- 0.3×0.6 looks like



which is 18 hundredths of the whole shaded, or 0.18. So, $0.3 \times 0.6 = 0.18$

From these examples we see that:

$$3 \times 6 = 18$$
$$3 \times 0.6 = 1.8$$
$$0.3 \times 0.6 = 0.18$$

DEMO

We observe that:

- if we multiply two numbers that are both *greater than* 1, the result will be greater than the two original numbers.
- if we multiply two numbers that are both *less than* 1, the result will be less than the two original numbers.
- the number of decimal places in the original numbers affect the number of decimal places in the final answer.

ACTIVITY ESTIMATING DECIMAL PRODUCTS

Before we learn to multiply decimals, it is important to learn what sort of answers to expect. In particular, an estimate can warn us of an error we may have made using our calculator.

For example, $8.19 \times 4.87 \approx 8 \times 5$, so we expect the actual answer to be somewhere near 40.

What to do:

1 Choose the correct answer and then **check** using your calculator:

a	4.387×6	**i** 263.22		**ii** 26.322		**iii** 2.6322		**iv** 2632.2
b	18.71×19	**i** 355.49		**ii** 35.549		**iii** 35 549		**iv** 3554.9
c	0.028×11	**i** 3.080		**ii** 0.0308		**iii** 0.308		**iv** 30.800

2 Estimate the following using 1 figure approximations.
For example, $19.8 \times 41.89 \approx 20 \times 40 \approx 800$

a 8.6×5.1 **b** 9.8×13.2 **c** 12.2×11.9
d 1.96×3.09 **e** 15.39×8.109 **f** 39.04×2.08

Find the actual answers using your calculator.

INVESTIGATION DECIMAL PLACES IN THE PRODUCT

In this investigation we will look at the number of decimal places in a product like 0.67×0.8 and the number of decimal places in the final answer. You may use a calculator to do the multiplication.

What to do:

1 Write the number of decimal places in each of the following:

a 36.42 **b** 12.8 **c** 0.095 **d** 1.805 **e** 29.0908

2 Copy and complete the following table.

	Product	Number of decimal places in question	Estimate of product	Calculator answer	Number of decimal places in product
	2.91×3.04	$2 + 2 = 4$	$3 \times 3 = 9$	8.8464	4
a	42.8×2.16				
b	5.072×1.9				
c	69.1×20.05				
d	0.87×0.96				
e	9.84×3.092				
f	6.094×2.837				

3 Write the number of decimal places you would expect in the following products:

a 0.8×0.9 **b** 2.07×1.93 **c** 0.3×0.04
d 9×0.45 **e** 0.6×0.06 **f** 0.857×3
g 2.5×4.03 **h** $2 \times 0.2 \times 0.02$ **i** $0.5 \times 0.05 \times 0.005$

Find the actual answers using your calculator.

From the **Investigation** we notice that:

When **multiplying by decimals**, the number of decimal places in the question equals the number of decimal places in the answer.

We can show why this is so using fractions:

$$\begin{aligned} 0.3 \times 0.4 &= \tfrac{3}{10} \times \tfrac{4}{10} \\ &= \tfrac{12}{100} \\ &= 0.12 \end{aligned}$$

This suggests that to find 0.3×0.4 we multiply 3×4 and then divide the result by 10^2 to account for the decimal places.

Dividing by 10^2 involves shifting the decimal point two places to the left.

Example 8 — Self Tutor

Find: **a** 3×0.6 **b** 0.5×0.07 **c** 0.05×0.08

a 3×0.6
$= 3 \times \frac{6}{10}$
$= \frac{18}{10}$
$= 1.8.$
$= 1.8$

b 0.5×0.07
$= \frac{5}{10} \times \frac{7}{100}$
$= \frac{35}{10^3}$
$= 0.035.$
$= 0.035$

c 0.05×0.08
$= \frac{5}{100} \times \frac{8}{100}$
$= \frac{40}{10^4}$
$= 0.0040.$
$= 0.004$

EXERCISE 11D

1 Find these products:

a 0.2×4 **b** 0.3×8 **c** 5×0.7
d 6×0.8 **e** 0.4×0.7 **f** 0.4×0.5
g 0.03×0.6 **h** 0.02×0.9 **i** 0.03×11
j 15×0.04 **k** 0.07×0.09 **l** 0.006×0.05

2 Find the value of:

a 2.4×3 **b** 6.5×4 **c** 2.7×5
d 7×0.005 **e** 1.2×0.12 **f** 2.03×0.04
g $(0.6)^2$ **h** $(0.04)^2$ **i** $0.4 \times 0.3 \times 0.2$

3 Given that $34 \times 28 = 952$, find the value of the following:

a 34×2.8 **b** 3.4×2.8 **c** 34×0.028
d 0.34×2.8 **e** 0.034×28 **f** 0.34×0.28
g 0.034×2.8 **h** 0.034×0.028 **i** 340×0.0028

4 Given that $57 \times 235 = 13\,395$, find the value of the following:

a 5.7×235 **b** 5.7×23.5 **c** 5.7×2.35
d 5.7×0.235 **e** 57×0.235 **f** 0.57×2.35
g 0.57×0.235 **h** $5.7 \times 0.000\,235$ **i** 570×0.235

5 Find the value of:

a 0.4×6 **b** 0.11×8 **c** 0.5×5.0
d 0.03×9 **e** 0.03×90 **f** 3.8×4

g 0.9×0.8 h 0.007×0.9 i 0.04×0.04
j 0.16×0.5 k $(0.2)^2$ l $(0.03)^2$
m 1.2×0.06 n $(1.1)^2$ o 2.5×0.004

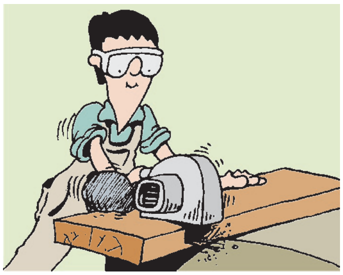
6 a Find the cost of 45 litres of petrol at 87.8 pence per litre.
b Find the cost of 9.6 metres of pipe at $3.85 per metre.
c Find the total capacity of 6 dozen 1.25 litre bottles.

7 Jennifer bought 6 iceblocks each costing 55 cents. How much did she pay for them in total?

8 Large clay pavers weighing 5.6 kg each are used to pave a driveway. If Duncan was able to push 8 of these pavers in his wheelbarrow at a time, how much weight can he push?

9 Matthias is concreting a small fish pond. He needs 0.4 m^3 of concrete which costs £72.50 per m^3. How much will the concrete for the pond cost Matthias?

10 I need 4.5 m of hose to water my garden. If hose costs me $3.40 per metre, how much will it cost me to buy my hose?



11 Manuel needs at least 25 metres of timber. He has found 6 pieces of timber in his shed, each 3.9 m long. Does he have enough? How much extra does he have, or how much more does he still need to find?

12 A caterer orders 5700 pies and 3600 pasties to sell at a football match. The pies and pasties each have a mass of 0.26 kg. What is the total mass of the:
a pies b pasties c pies and pasties?
d How many heated vans, each capable of carrying 500 kg, are needed to deliver the pies and pasties?
e If the caterer makes a profit of 29.7 cents on each pie or pasty and she sells them all, what is her total profit?

E DIVIDING DECIMALS BY WHOLE NUMBERS

Follow these steps to divide decimal numbers by whole numbers:

Step 1: Put a decimal point in the answer directly above the decimal point of the question.

Step 2: Carry out normal division ignoring the decimal point.

Example 9 Self Tutor

Find: **a** $4.64 \div 4$ **b** $5.28 \div 8$

a

$$\begin{array}{r|l} & 1\,.\,1\;\;6 \\ \hline 4 & 4\,.\,6\;{}^{2}4 \end{array}$$

So, $4.64 \div 4 = 1.16$

b

$$\begin{array}{r|l} & 0\,.\,6\;\;6 \\ \hline 8 & 5\,.\,2\;{}^{4}8 \end{array}$$

So, $5.28 \div 8 = 0.66$

EXERCISE 11E

1 Find:

a $3.2 \div 4$ **b** $7.5 \div 5$ **c** $1.26 \div 3$ **d** $3.57 \div 7$

e $24.16 \div 8$ **f** $2.46 \div 6$ **g** $0.72 \div 9$ **h** $81.6 \div 4$

Example 10 Self Tutor

A 6.4 m length of timber is cut into four equal lengths.
How long is each piece?

$$\begin{array}{r|l} & 1\,.\,6 \\ \hline 4 & 6\,.\,{}^{2}4 \end{array}$$

Each piece is 1.6 m long.

2 **a** How much money would each person get if €76.50 is divided equally among 9 people?

b A 10.75 kg tub of icecream is divided equally among 5 people. How much icecream does each person receive?

c A 3.5 m length of timber is cut into five equal pieces. How long is each piece?

d How many 7 kg bags of potatoes can be filled from a bag of potatoes weighing 88.2 kg?

e If £96.48 is divided equally among six people, how much does each person receive?

Example 11 Self Tutor

Find: **a** $6.3 \div 5$ **b** $3.5 \div 4$

a

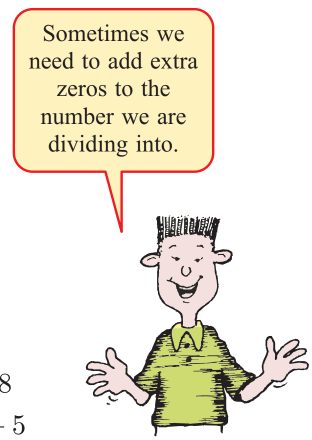
$$\begin{array}{r|l} & 1\,.\;\;2\;\;6 \\ \hline 5 & 6\,.\;{}^{1}3\;{}^{3}0 \end{array}$$

So, $6.3 \div 5 = 1.26$

b

$$\begin{array}{r|l} & 0\,.\,8\;\;7\;\;5 \\ \hline 4 & 3\,.\,5\;{}^{3}0\;{}^{2}0 \end{array}$$

So, $3.5 \div 4 = 0.875$



3 Find:

a $5.3 \div 2$ **b** $6.1 \div 5$ **c** $3.4 \div 4$ **d** $3.4 \div 8$

e $6.5 \div 2$ **f** $5.9 \div 4$ **g** $2.41 \div 2$ **h** $6.32 \div 5$

Example 12 Self Tutor

Find: $4.1 \div 3$

$$\begin{array}{r} 1\,.\,3\;\;6\;\;6\;\;6\;\;6.... \\ 3\,\overline{)\,4\,.\,{}^{1}1\;{}^{2}0\;{}^{2}0\;{}^{2}0\;{}^{2}0....} \end{array}$$

So, $4.1 \div 3 = 1.366\,66....$

4 Find:

- **a** $3.1 \div 3$
- **b** $3.5 \div 3$
- **c** $4.1 \div 9$
- **d** $2.47 \div 3$
- **e** $8.15 \div 3$
- **f** $13.6 \div 9$
- **g** $2.6 \div 7$
- **h** $6.15 \div 7$

F TERMINATING AND RECURRING DECIMALS

In the previous exercise we saw that when we divide a decimal by a whole number, the result may either end or **terminate**, or else the division will continue forever repeating or **recurring**.

In this section we look at fractions with whole number numerator and denominator.

Every fraction of this type can be written as either a **terminating** or a **recurring** decimal.

TERMINATING DECIMALS

Terminating decimals result when the rational number has a denominator which has no prime factors other than 2 or 5.

For example:
$\frac{3}{4} = 0.75$ and the only prime factor of 4 is 2
$\frac{14}{25} = 0.56$ and the only prime factor of 25 is 5
$\frac{13}{40} = 0.325$ as the prime factors of 40 are 2 and 5.

Example 13 Self Tutor

Write the following in decimal form without carrying out a division:

a $\frac{4}{5}$ **b** $\frac{9}{25}$ **c** $\frac{7}{8}$

a $\frac{4}{5} = \frac{4\times 2}{5\times 2} = \frac{8}{10} = 0.8$

b $\frac{9}{25} = \frac{9\times 4}{25\times 4} = \frac{36}{100} = 0.36$

c $\frac{7}{8} = \frac{7\times 125}{8\times 125} = \frac{875}{1000} = 0.875$

Example 14 — Self Tutor

Use division to write the following fractions as decimals: **a** $\frac{2}{5}$ **b** $\frac{5}{8}$

a
$$5\,\overline{)\,2.0}\quad \text{quotient } 0.4$$
So, $\frac{2}{5} = 0.4$

b
$$8\,\overline{)\,5.0^{2}0^{4}0}\quad \text{quotient } 0.6\ 2\ 5$$
So, $\frac{5}{8} = 0.625$

EXERCISE 11F.1

1 Write as decimals using the method of **Example 13**:

a $\frac{7}{10}$ **b** $\frac{1}{2}$ **c** $\frac{2}{5}$ **d** $\frac{3}{10}$
e $\frac{4}{5}$ **f** $\frac{1}{4}$ **g** $\frac{4}{25}$ **h** $\frac{3}{4}$
i $\frac{1}{8}$ **j** $\frac{5}{8}$ **k** $\frac{7}{20}$ **l** $\frac{6}{25}$

DEMO

2 Use division to write as a decimal:

a $\frac{3}{5}$ **b** $\frac{9}{5}$ **c** $\frac{3}{8}$ **d** $\frac{9}{8}$
e $\frac{11}{4}$ **f** $\frac{29}{5}$ **g** $\frac{39}{8}$ **h** $\frac{43}{8}$

RECURRING DECIMALS

Recurring decimals repeat the same sequence of numbers without stopping.

The fractions $\frac{1}{3}$ and $\frac{2}{3}$ provide the simplest examples of recurring decimals.

By division, $\frac{1}{3} = 0.333\,333\,33.....$ which we write as $0.\overline{3}$ and read as "point 3 recurring"

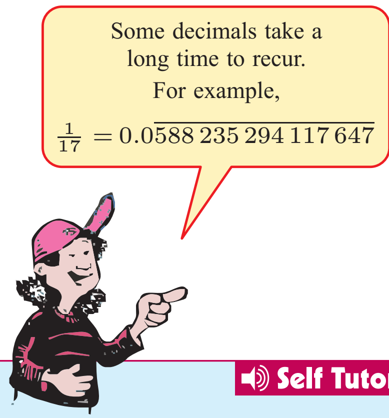
and $\frac{2}{3} = 0.666\,666\,66.....$ which we write as $0.\overline{6}$ and read as "point 6 recurring".

Recurring decimals result when the denominator of a rational number has one or more prime factors other than 2 or 5.

For example, $\frac{3}{14} = 0.214\,285\,714\,285\,714\,285\,7....$

We indicate a recurring decimal by writing the full sequence once with a line over the repeated section.

For example, $\frac{1}{3} = 0.\overline{3}$ and $\frac{3}{14} = 0.2\overline{14\,285\,7}$.



Example 15 — Self Tutor

Write as recurring decimals: **a** $\frac{4}{9}$ **b** $\frac{7}{11}$

a $\frac{4}{9}$
$= 0.4444\ ....$
$= 0.\overline{4}$

$$9\,\overline{)\,4.0^{4}0^{4}0^{4}0....}\quad \text{quotient } 0.4\ 4\ 4\ 4....$$

b $\frac{7}{11}$
$= 0.636\,363\ ....$
$= 0.\overline{63}$

$$11\,\overline{)\,7.0^{4}0^{7}0^{4}0....}\quad \text{quotient } 0.6\ 3\ 6\ 3....$$

EXERCISE 11F.2

1 Convert the following fractions to decimals. Use a bar to show the recurring digits.

a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{6}$ **d** $\frac{1}{7}$ **e** $\frac{2}{7}$

f $\frac{1}{12}$ **g** $\frac{2}{9}$ **h** $\frac{5}{6}$ **i** $\frac{3}{11}$ **j** $\frac{7}{12}$

2 **a** Copy and complete the following pattern:

Fraction:	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$
Decimal:	$0.\overline{1}$	$0.\overline{2}$							

b Comment on the value of $0.\overline{9}$.

3 Write as decimals:

a $\frac{23}{32}$ **b** $\frac{11}{16}$ **c** $\frac{17}{80}$ **d** $\frac{11}{25}$ **e** $1\frac{3}{16}$

f $\frac{3}{14}$ **g** $\frac{2}{15}$ **h** $\frac{9}{11}$ **i** $2\frac{7}{30}$ **j** $\frac{97}{50}$

k $\frac{6}{13}$ **l** $\frac{49}{160}$ **m** $3\frac{5}{12}$ **n** $\frac{31}{123}$ **o** $\frac{23}{45}$

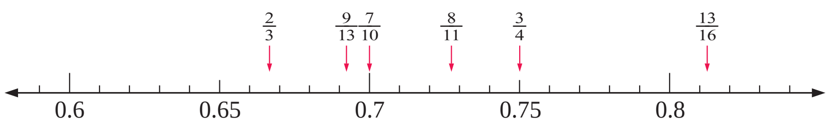
G USING A CALCULATOR

Suppose you were asked to write $\frac{2}{3}$, $\frac{3}{4}$, $\frac{13}{16}$, $\frac{7}{10}$, $\frac{8}{11}$ and $\frac{9}{13}$ in ascending order.

Converting all of these fractions to a common denominator would be long and tedious.

It would be better to convert each fraction to a decimal and use the decimals to write the fractions in order.

$$\frac{2}{3} \approx 0.666, \quad \frac{3}{4} = 0.750, \quad \frac{13}{16} \approx 0.813, \quad \frac{7}{10} = 0.700, \quad \frac{8}{11} \approx 0.727, \quad \frac{9}{13} \approx 0.692$$



So, the ascending order is: $\frac{2}{3}$, $\frac{9}{13}$, $\frac{7}{10}$, $\frac{8}{11}$, $\frac{3}{4}$, $\frac{13}{16}$

EXERCISE 11G

1 Write in ascending order using a calculator:

a $\frac{3}{10}$, $\frac{7}{22}$, $\frac{7}{20}$, $\frac{5}{17}$, $\frac{1}{3}$ **b** $\frac{4}{7}$, $\frac{3}{8}$, $\frac{5}{9}$, $\frac{5}{12}$, $\frac{7}{16}$

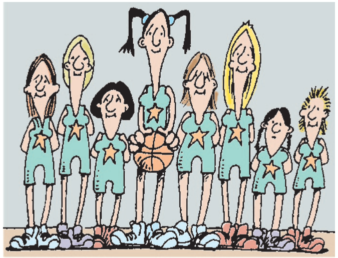
c $\frac{8}{9}$, $\frac{7}{8}$, $\frac{9}{11}$, $\frac{10}{13}$, $\frac{11}{12}$ **d** $\frac{11}{20}$, $\frac{12}{23}$, $\frac{10}{19}$, $\frac{6}{11}$, $\frac{8}{15}$

2 Write in descending order using a calculator:

a $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{11}$, $\frac{11}{17}$, $\frac{15}{23}$ **b** $\frac{8}{21}$, $\frac{3}{8}$, $\frac{5}{13}$, $\frac{6}{17}$, $\frac{4}{11}$

c $\frac{7}{20}$, $\frac{1}{3}$, $\frac{5}{16}$, $\frac{8}{23}$, $\frac{9}{25}$ **d** $\frac{14}{17}$, $\frac{16}{19}$, $\frac{17}{20}$, $\frac{20}{23}$, $\frac{3}{4}$

3 The distance around the boundary of a square is 12.66 metres. Find the length of each side of the square.

4 

The heights of the girls in the Primary School Basketball team were measured in metres and the results were:
1.56, 1.43, 1.51, 1.36, 1.32, 1.45, 1.39, 1.38

a Find the sum of the girls' heights.

b Divide the sum in **a** by the number of girls to find their *average* height.

5 A piece of wood is 6.4 m long and must be cut into short lengths of 0.36 m.

a How many full lengths can be cut?

b What length is left over?

6 How many 2.4 metre lengths of piping are needed to make a drain 360 metres long?

7 21 DVDs cost $389.55. How much does one DVD cost?

KEY WORDS USED IN THIS CHAPTER

- ascending order
- decimal number
- descending order
- index
- power
- recurring decimal
- terminating decimal

LINKS
click here

BODY MASS INDEX

Areas of interaction:
Health and social education

REVIEW SET 11A

1 Evaluate: **a** $3.018 + 20.9 + 4.836$ **b** $423.54 - 276.49$
c 4.2×1.2 **d** 0.96×0.08

2 Add fourteen point nine eight one, three point six five nine, one point zero nine eight, and twenty two point five.

3 Solve the following problems:

a Determine the total cost of 14 show bags at $7.85 each.

b Share €5885.25 equally amongst 5 people. How much does each person get?

c How much change from $100.00 would you receive if items costing $27.55, $18.30, $22.05 and $3.75 were bought?

4 In 3 seasons a vineyard produces: 638.17, 582.35 and 717.36 tonnes of grapes respectively. What was the total harvest of grapes for the 3 years?

5 Answer the **Opening Problem** on page **198**.

6 Find: **a** 6.2×10 **b** 2.158×100 **c** $5.6 \div 10$ **d** $4.2 \div 100$

7 Given that $26 \times 53 = 1378$, evaluate:

a 2.6×5.3 **b** 2.6×0.053

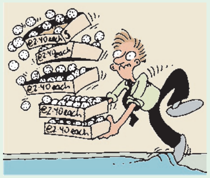
8 Convert the following into decimal numbers using division:

a $\frac{1}{7}$ **b** $\frac{5}{9}$ **c** $1\frac{1}{6}$

9 Find $\square$ if:

a $203 \div \square = 2.03$ **b** $2.03 \times \square = 2030$ **c** $0.203 \div \square = 0.002\,03$

10 A golf shop buys 10 000 golf balls from a manufacturer at €1.15 each. It sells them all for €2.40 each.



a What is the total payment that the manufacturer receives?

b How much total profit does the golf shop make?

REVIEW SET 11B

1 Evaluate: **a** $31.426 - 29.527$ **b** 28.6×0.09 **c** $4.4 + 4.04 + 0.444$ **d** $4.62 \div 3$

2 By how much is fifty six point two five one greater than eighteen point nine three seven?

3 The first horse in a 1000 metre sprint finished in 56.98 seconds. The second and third horses were 0.07 seconds and 0.23 seconds behind the winner. What were the times of the:

a second horse **b** third horse?

4 Three people with weights 56.270 kg, 94.025 kg and 68.498 kg step into a lift. How much weight is the lift carrying?

5 Find:

a 0.63×100 **b** 5.98×10^3 **c** $76 \div 1000$ **d** $0.942 \div 10^2$

6 Write the divisor to complete the equation:

a $409.6 \div \square = 4.096$ **b** $3.512 \div \square = 0.003\,512$

7 Given that $58 \times 47 = 2726$, evaluate:

a 5.8×47 **b** 5.8×0.47 **c** 5.8×4.7

8 **a** Find the difference between 246 and 239.84.

b Find: **i** 4.8×0.2 **ii** 0.03×0.5

c A square has sides of length 3.7 m. Find the total distance around its boundary.

d How much does each person get if \$82.40 is divided equally between four people?

9 A marathon runner stops for a drink $\frac{1}{3}$ of the way on his 42.195 km race. How far has he: **a** run **b** still got to run?

10 Convert the following fractions to decimals:

a $\frac{14}{25}$ **b** $\frac{8}{11}$ **c** $4\frac{19}{40}$

Chapter 12

Measurement

Contents:

In our everyday life we **measure** many things.

Measurement gives an indication of the **size** of a quantity.

The more common types of measurement are:

Measurement	*Example*
Distance or length	How far we have travelled.
Mass or weight	How heavy we are.
Time	How long a tennis match will last.
Temperature	How hot it is going to be tomorrow.
Area	The size of the block of land I need to buy.
Volume	How much concrete I need for the driveway.
Speed	How fast I travel if I get there in 2 hours.

However, in everyday life, people measure a whole range of different things and different characteristics. For instance there are measures for energy, sound, diamonds, power, elasticity, gravity, colour, smell, typing rate, and pollen count.

Some things like art, beauty, taste, desire, ambition, success, attitude and intelligence are much harder to measure.

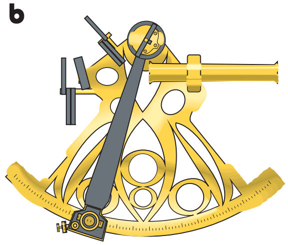
ACTIVITY 1 MEASURES AND MEASURING INSTRUMENTS

What to do:

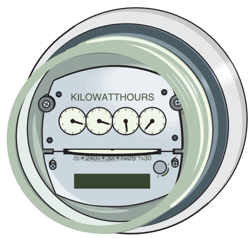
1 What do these instruments measure? Match the instrument to its name.

a

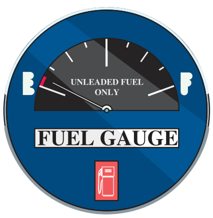
b



c



d



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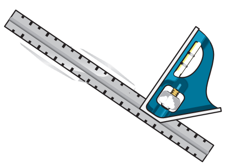

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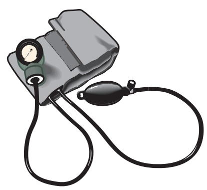

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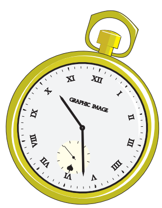


pocket watch
electricity meter
builders square
sphygmomanometer
thermometer
micrometer
fuel gauge
sextant

ACTIVITY 2 MEASURES AND WHO USES THEM

What to do:

1 Find out what is measured by the following instruments and what caused their development.

- Geiger counter
- Altimeter
- Dynamometer
- Tachometer
- Seismometer
- Hydrometer
- Theodolite
- Callipers
- Calorimeter
- Barometer

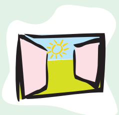
2 Find what sort of measuring tools would be used by:

- architects
- scientists
- farmers
- weather forecasters
- doctors
- pilots
- computer engineers
- mechanics
- builders
- surveyors
- sports officials
- teachers

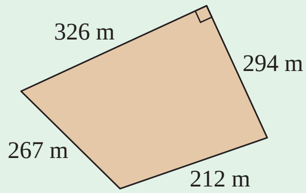
3 How would you measure the following?

- angles
- typing speed
- electricity
- location
- tides
- reading speed

OPENING PROBLEM



A farmer has a field with the dimensions shown. The farmer needs to replace the fence around the field as the current wire fence is very rusty. The existing posts will remain, but 4 strands of wire need to be attached to the posts.

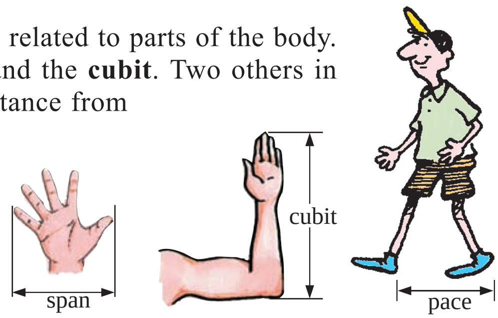


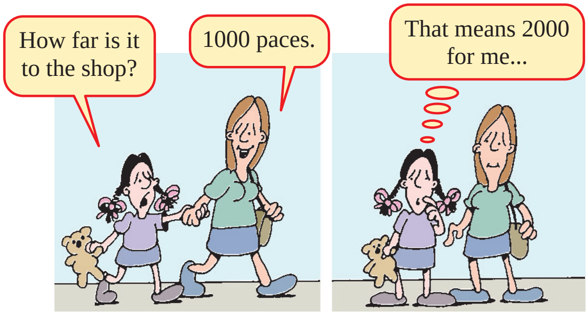
- How far is it around the field?
- What length of wire is needed given that an extra 15 m is needed for tying the wire to posts?
- What will be the total cost of the wire if its price is \$0.12 per metre?

A UNITS OF MEASUREMENT

The earliest units of measurement were lengths related to parts of the body. Two of these are illustrated below: the **span** and the **cubit**. Two others in common use were the **yard**, which was the distance from your nose to your fingertip, and the **pace**, which was the length of your stride. There were 1000 paces in a Roman mile.

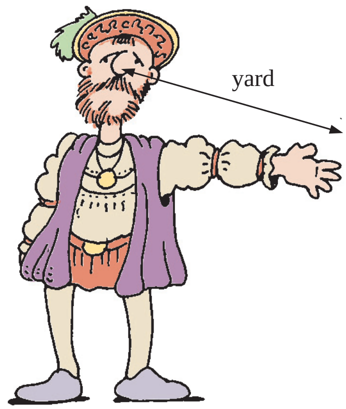
All of these measurements were inaccurate because people are different sizes.





INTERNATIONAL SYSTEM OF UNITS

In order to make these lengths more standard, King Henry VIII of England said that a yard would be the distance from *his* nose to his fingertips. This led to the British Imperial System of units which uses inches, feet, yards and miles for length, and ounces, pounds and tons for mass.



This system is still used in a few countries but the Metric System, developed in France in 1789, is now used more commonly throughout the world. The advantage of this system is that it uses powers of ten for different sizes. The basic unit for length is the **metre** (m) and for mass it is the **kilogram** (kg). Other smaller and larger units are named by using prefixes. This system of units is now known as Le Système International d'Unités or **SI** system.

LENGTH UNITS

1 millimetre (mm) $= \frac{1}{1000}$ m $= 0.001$ m

1 centimetre (cm) $= \frac{1}{100}$ m $= 0.01$ m

1 kilometre (km) $= 1000$ m

MASS UNITS

1 milligram (mg) $= \frac{1}{1000}$ g $= 0.001$ g

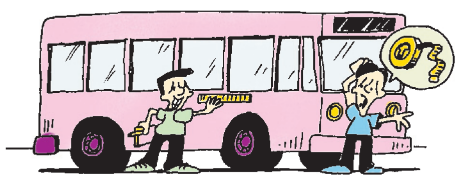
1 gram (g) $= \frac{1}{1000}$ kg $= 0.001$ kg

1 tonne (t) $= 1000$ kg

EXERCISE 12A

1 State what units you would use to measure the following:

- **a** the mass of a person
- **b** the distance between two towns
- **c** the length of a sporting field
- **d** the mass of a tablet



- **e** the length of a bus
- **f** the mass of a car
- **g** the width of this book
- **h** the mass of a truck

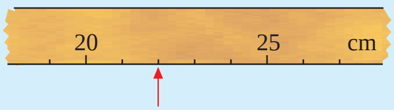
B READING SCALES

There are many instruments which are used for measuring. They usually have a **scale** marked on them. We are all familiar with a **ruler** for measuring lengths. Rulers have a scale marked in both millimetres and centimetres.

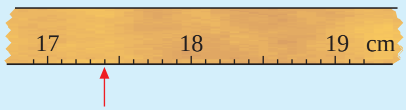
Example 1 Self Tutor

Read the following ruler measurements:

a



b

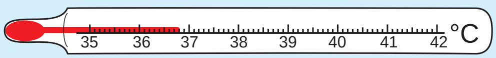


a There are 5 divisions between 20 and 25 cm, so each division is 1 cm.
The measurement is 22 cm.

b There are 10 divisions between 17 and 18 cm, so each division is $\frac{1}{10}$ of a cm.
The measurement is 17.4 cm.

Example 2 Self Tutor

Read the temperature on the following centigrade thermometer:

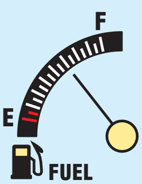


There are 10 divisions between 36^{o}C and 37^{o}C, so each division is 0.1^{o}C.
The temperature is 36.8^{o}C.

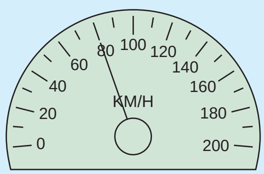
Example 3 Self Tutor

Read as accurately as possible, the measurement on:

a the fuel gauge



b the speedometer



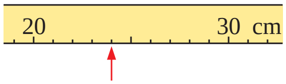
a There are eight main divisions from empty to full.
So, the fuel tank is $\frac{5}{8}$ full.

b The speed is 80 km per hour.

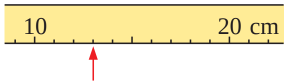
EXERCISE 12B

1 Read the following ruler measurements:

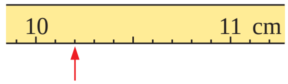
a



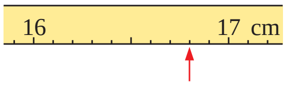
b



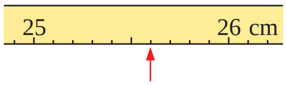
c



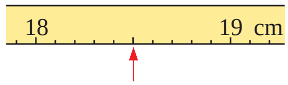
d



e

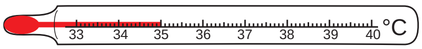


f

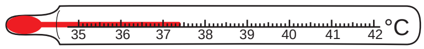


2 Read the temperatures (in ^{o}C) for the following thermometers:

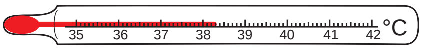
a



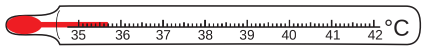
b



c

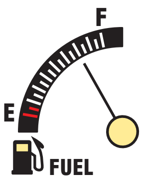


d

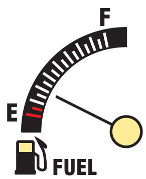


3 Read the following fuel gauges:

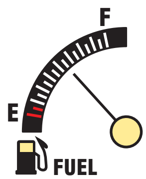
a



b

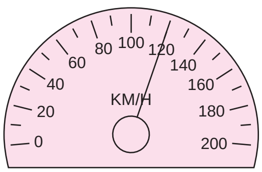


c

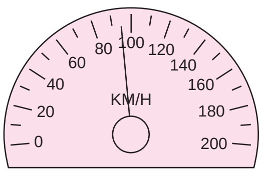


4 Read as accurately as possible, the speeds on the following:

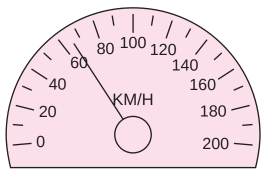
a



b

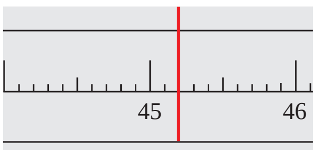


c

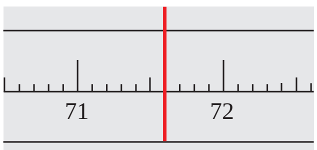


5 Find the weights, in kilograms, shown by the following bathroom scales:

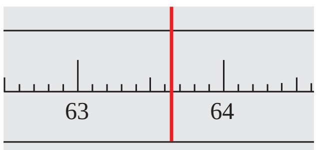
a



b

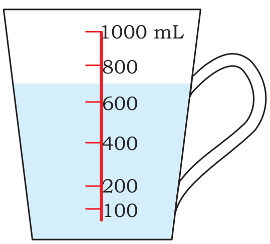


c

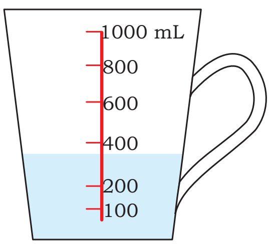


6 Find the quantity of fluid (in mL) in the following jugs:

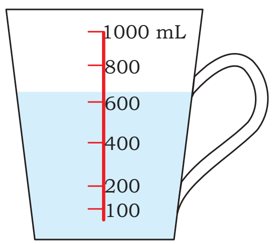
a



b



c

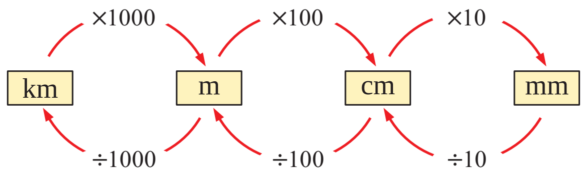


C LENGTH CONVERSIONS

When we convert from one unit to a **smaller** unit, there will be more smaller units and so we must **multiply**.

When we convert from one unit to a **larger** unit, there will be less larger units and so we must **divide**.

CONVERSION DIAGRAM



Example 4 — Self Tutor

Write the following in metres:

a 640 cm **b** 3.8 km **c** 7560 mm

a We are converting from a smaller unit to a larger one, so we divide.

640 cm
$= (640 \div 100)$ m
$= 6.4$ m

b We are converting from a larger unit to a smaller one, so we multiply.

3.8 km
$= (3.8 \times 1000)$ m
$= 3800$ m

c We are converting from a smaller unit to a larger one, so we divide.

7560 mm
$= (7560 \div 1000)$ m
$= 7.56$ m

EXERCISE 12C

1 Convert these metres into centimetres:

a 4 **b** 34 **c** 2.5 **d** 15.6 **e** 2.45 **f** 0.46

2 Convert these metres into millimetres:

a 3 **b** 45 **c** 3.6 **d** 16.2 **e** 5.46 **f** 0.09

3 Convert these centimetres into millimetres:

a 5 **b** 23 **c** 2.7 **d** 12.5 **e** 5.78 **f** 0.25

4 Convert these centimetres into metres:

a 200 **b** 3000 **c** 35 **d** 950.5 **e** 28 492 **f** 0.4

5 Convert these millimetres into centimetres:

a 20 **b** 400 **c** 450 **d** 45.6 **e** 7500 **f** 0.3

6 Convert these kilometres into metres:

a 3 **b** 75 **c** 6.5 **d** 2000 **e** 78.2 **f** 0.2

7 Convert these metres into kilometres:

a 2000 **b** 35 000 **c** 234.5 **d** 34 567 **e** 3900 **f** 2.4

8 Write the following in metres:

a 920 cm **b** 643 cm **c** 4753 cm **d** 5000 mm
e 9743 mm **f** 13 500 mm **g** 6.2 km **h** 13.5 km

9 Write the following in centimetres:

a 720 m **b** 13.8 m **c** 6.3 m **d** 134 mm
e 85 mm **f** 1328 mm **g** 5.2 km **h** 0.43 km

10 Write the following in millimetres:

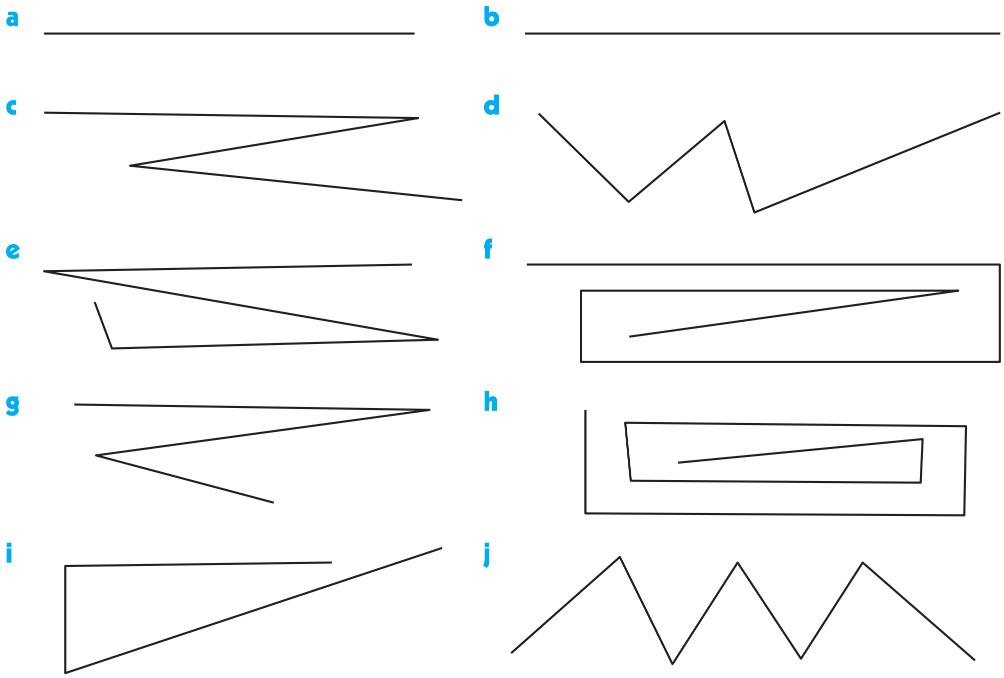
a 7 m **b** 3.4 cm **c** 78 cm **d** 0.46 m

11 Write the following in kilometres:

a 4562 m **b** 17 458 m **c** 653 000 cm **d** 16 400 cm

12 For each of the following lines:

- **i** estimate the length by looking at it carefully
- **ii** measure the length to the nearest mm using a ruler
- **iii** find the error in your estimation.



13 Convert all lengths to metres and then add:

a 3 km + 110 m + 32 cm **b** 72 km + 43 m + 47 cm + 16 mm
c 153 m + 217 cm + 48 mm **d** 15 km + 348 m + 63 cm + 97 mm
e 23 m + 47 cm + 338 mm **f** 23 km + 76 m + 318 cm + 726 mm

14 Write the following in the same units and hence write them in ascending order:

- **a** 37 mm, 4 cm
- **b** 750 cm, 8 m, 7800 mm
- **c** 1250 m, 1.3 km
- **d** 0.005 km, 485 cm, 5.2 m
- **e** 3500 mm, 347 cm, 3.6 m
- **f** 0.134 km, 128 m, 13 000 cm
- **g** 4.82 m, 512 cm, 4900 mm
- **h** 7.2 m, 7150 cm, 71 800 mm

D PERIMETER

The **perimeter** of a closed figure is a measurement of the distance around the boundary of the figure.

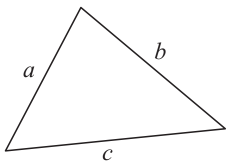
The word **perimeter** is also used to describe the boundary of a closed figure.

If the figure is closed and has straight sides, it is a **polygon** and its perimeter is found by adding the lengths of the sides.

In figures, sides having the same markings show equal lengths.

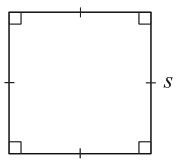
The rules for very common simple shapes are as follows:

Triangle



$P = a + b + c$

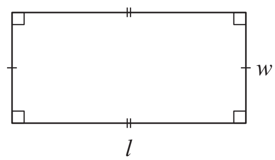
Square



$P = 4 \times s$

$P = 4 \times$ **side length**

Rectangle



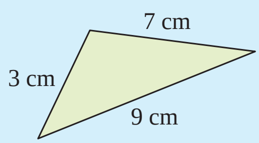
$P = (l + w) \times 2$

$P =$ (**length** + **width**) $\times$ **2**

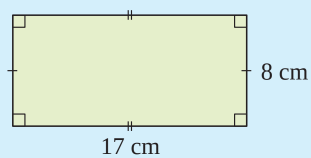
Example 5 Self Tutor

Find the perimeter of:

a



b

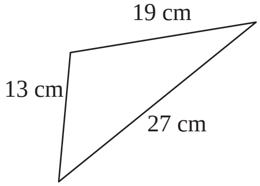


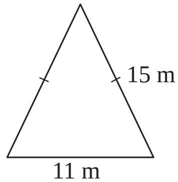
a Perimeter $= 3 + 7 + 9$ cm
$= 19$ cm

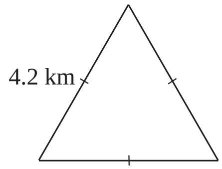
b $P = (17 + 8) \times 2$ cm
$= 25 \times 2$ cm
$= 50$ cm

EXERCISE 12D

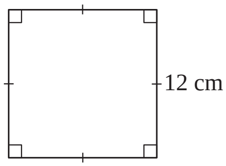
1 Find the perimeter of each of the following triangles:

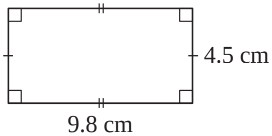
a


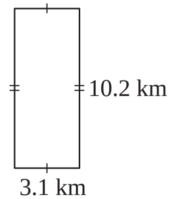
b


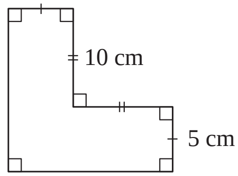
c


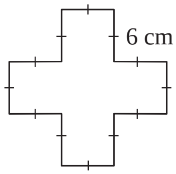
2 Find the perimeter of:

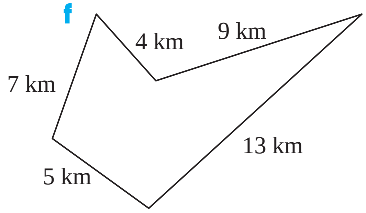
a


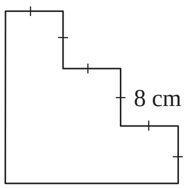
b


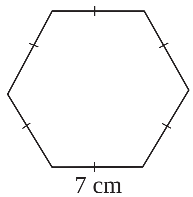
c


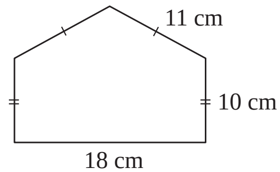
d


e


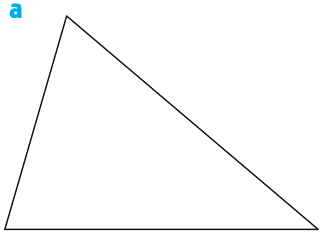
f


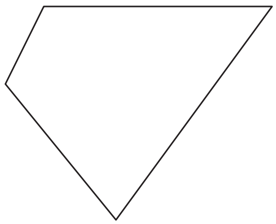
g


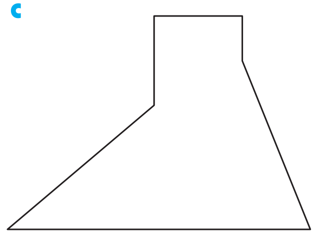
h


i


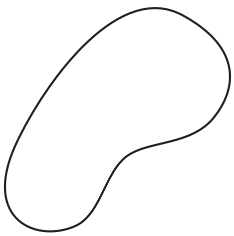
3 Estimate the perimeter of each figure, then check your estimate with a ruler.

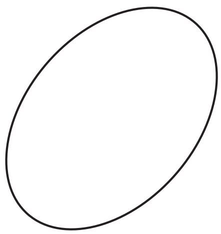
a


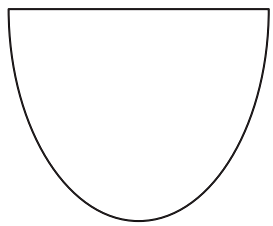
b


c


4 Use a piece of string to find the perimeter of the following:

a


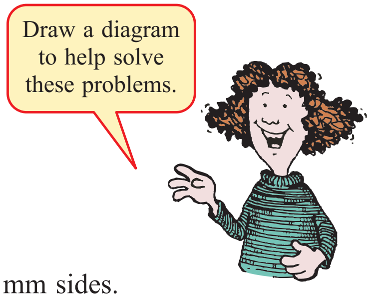
b


c


5 A rectangular paddock 120 m by 260 m is to be fenced. Find the length of the fence.

6 How far will a runner travel if he runs 5 times around a triangular block with sides 320 m, 480 m and 610 m?

7 Find the cost of fencing a square block of land with side length 75 m if the fence costs $14.50 per metre.



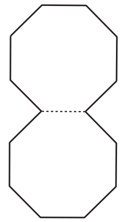
8 **a** Find the perimeter of an equilateral triangle with 35.5 mm sides.

b The perimeter of a regular pentagon is 1.35 metres. Find the length of each side.

c One half of the perimeter of a regular hexagon is 57 metres. What is the length of one of its sides?

d Find the length of the sides of a rhombus which has a perimeter of 72 metres.

e The perimeter of 2 identical regular octagons joined exactly along one side is 98 cm. What is their combined perimeter when they are separated?

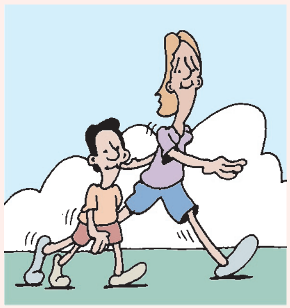


9 A rectangle has a length of 18 cm and a perimeter of 66 cm. What is the rectangle's width?

ACTIVITY 3 STEP ESTIMATION

Some of us are short and others are tall. This means that when we walk, our step lengths vary from one person to another.

Knowing your step length can enable you to estimate, with reasonable accuracy, some quite long distances.



Seani set up two flags which she measured to be 100 m apart. Using her usual walking step, she took $128\frac{1}{2}$ steps to walk between them.

100 m $\div$ 128.5 is about 0.78 m, so Seani's *average* usual step length is about 0.78 m.

When Seani walked around the school's boundary, she took 2186 steps.

Since $2186 \times 0.78 = 1705$, she said her best estimate of the school's perimeter is 1705 metres.

What to do:

1 Use a long tape measure to help place two flags exactly 100 m apart.

2 Walk with your usual steps from one flag to the other. Count the steps you take.

3 Using Seani's method, calculate your usual step length to 2 decimal places.

4 Choose *three* suitable distances around the school to estimate. Use Seani's method to estimate them.

5 Compare your estimates with other students. You could organise a competition to find the best distance estimator in your class.

E SCALE DIAGRAMS

A **scale diagram** is a drawing or plan either smaller or larger than the original, but with all sizes in the correct proportion.

Scale diagrams are used by architects, real estate agents, surveyors, and by many other professionals. House plans are a great example of the use of scale diagrams.

On each scale diagram we have a **scale**. This shows the connection between the lengths on the diagram compared with those for the real object.

A scale which says 1 : 200 or 1 represents 200 indicates that lengths on the scale diagram are 200 times larger in reality.

So, if a wall is represented by a 1 cm line on the scale diagram, it is 200 cm or 2 m in reality.

A wall which is 8 m long in reality would be $8 \text{ m} \div 200$ on the scale diagram.

This is $800 \text{ cm} \div 200 = 4 \text{ cm}$.

Example 6 Self Tutor

On a scale diagram, the scale is '1 represents 20'. Find:

a the actual length if the scale length is 3.4 cm

b the scale length if the actual length is 2.4 m.

a Actual length
$= 3.4 \text{ cm} \times 20$
$= 68 \text{ cm}$

b Scale length
$= 2.4 \text{ m} \div 20$
$= 240 \text{ cm} \div 20$
$= 12 \text{ cm}$

EXERCISE 12E

1 The scale on a diagram is 1 represents 5000.

a Find the actual length if the scale length is:
i 4 cm **ii** 5.8 cm **iii** 2.4 cm **iv** 12.6 cm

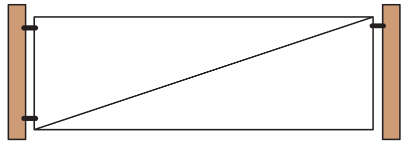
b Find the scale length if the actual length is:
i 500 m **ii** 175 m **iii** 20 m **iv** 108 m

2 The scale on a diagram is 1 represents 200.

a Find the actual length if the scale length is:
i 3 cm **ii** 4.5 cm **iii** 8.2 cm **iv** 0.8 cm

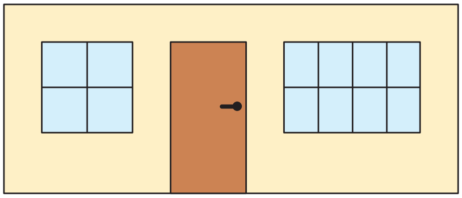
b Find the scale length if the actual length is:
i 200 m **ii** 18 m **iii** 5.6 m **iv** 12.2 m

3 The drawing of a gate alongside has a scale of 1 represents 100.
Find the actual:



- **a** width of the gate
- **b** height of the gate
- **c** length of the diagonal support.

4

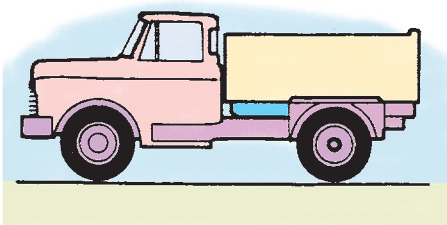


If the plan of a house wall alongside has been drawn with a scale of 1 represents 200, find the actual:

- **a** length of the wall
- **b** height of the wall
- **c** measurements of the door
- **d** measurements of the windows.

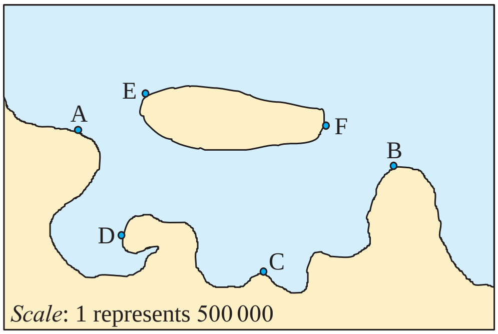
5 The drawing of the truck has the scale 1 represents 100. Find:

- **a** the actual length of the truck
- **b** the maximum height of the truck.



6 Using the scale shown on the map, find:

- **a** the actual distance shown by 1 cm
- **b** the map distance required for an actual distance of 200 km
- **c** the actual distance from
  - **i** A to B
  - **ii** D to E
  - **iii** C to F.



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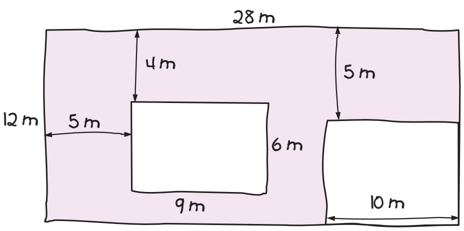
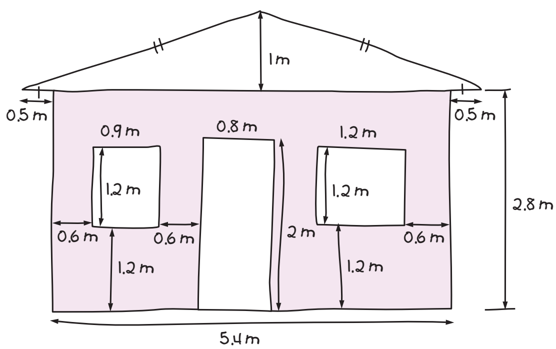


Using the measurements on the given rough sketch and a scale of 1 cm represents 2 m, draw an accurate scale diagram.

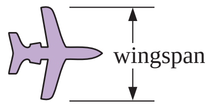


8 The front view of a house is shown in the given rough sketch. Using a scale of 1 represents 100, draw an accurate scale diagram of this view.

9 Find the scale if:

a an aeroplane has wingspan 50 m and on the diagram it is 50 cm

b a truck is 15 m long and its diagram has length 12 cm.



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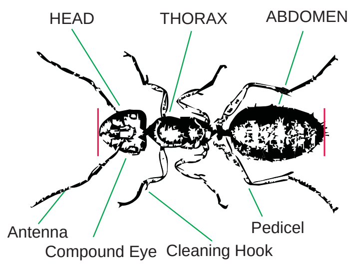


Real length = 5 mm

Alongside is a scale diagram of an ant. The actual body length of the ant between the red lines is 5 mm.

a Measure the length of the ant's body in the diagram in millimetres.

b Explain why the scale is 6 represents 1.

c Using the scale in **b**, find the actual length of the:

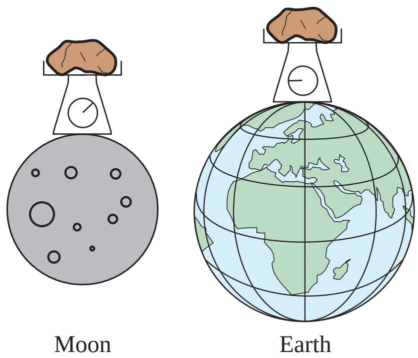
i antenna **ii** abdomen

iii thorax **iv** head.

F MASS

The **mass** of an object is the amount of matter it contains.

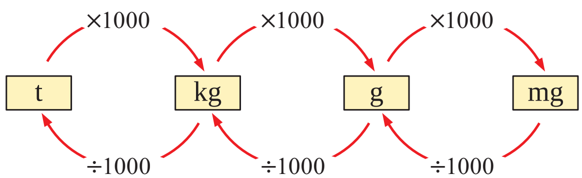
In everyday use the terms **mass** and **weight** are interchanged. In fact, they have different meaning. The mass of an object is constant; it is the same no matter where the object is. In contrast, the weight of an object is the force upon it due to gravity. For this reason, an object will have less weight on the moon than on the earth although the mass remains the same.

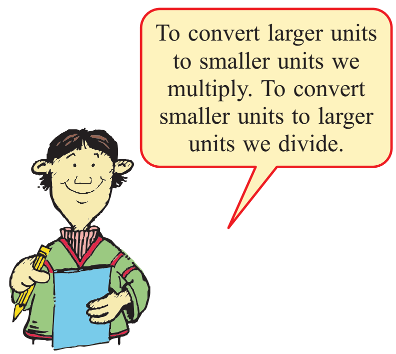


The **kilogram** (kg) is the base unit of mass in the metric system. Other units of mass which are commonly used are the milligram (mg), gram (g), and tonne (t).

1 g = 1000 mg   1 kg = 1000 g   1 t = 1000 kg

CONVERSION DIAGRAM





Example 7 — Self Tutor

Convert the following to kilograms:

a 350 g **b** 8.5 t **c** 7 500 000 mg

a
350 g
$= (350 \div 1000)\text{ kg}$
$= 0.35\text{ kg}$

b
8.5 t
$= (8.5 \times 1000)\text{ kg}$
$= 8500\text{ kg}$

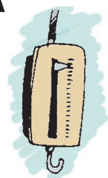
c
$7\,500\,000\text{ mg}$
$= (7\,500\,000 \div 1000)\text{ g}$
$= 7500\text{ g}$
$= (7500 \div 1000)\text{ kg}$
$= 7.5\text{ kg}$

EXERCISE 12F

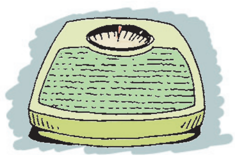
1 Give the units you would use to measure the mass of:

a a person **b** a ship **c** a tablet
d a book **e** an orange **f** a lounge suite
g a raindrop **h** a boulder **i** your school lunch
j a baseball bat **k** a refrigerator **l** a dinner plate
m a school ruler **n** a slab of concrete **o** a bulldozer
p a leaf **q** a calculator **r** a computer
s an ant **t** a horse

2

A


B


C


D


Which of the above devices should be used to measure the items in question **1**?

3 Convert these grams into milligrams:

a 2 **b** 34 **c** 350 **d** 4.5 **e** 0.3

4 Convert these tonnes into kilograms:

a 4 **b** 25 **c** 3.6 **d** 294 **e** 0.4

5 Convert these kilograms into grams:

a 6 **b** 34 **c** 2.5 **d** 256 **e** 0.6

6 Convert these milligrams into grams:

a 3000 **b** 2500 **c** 45 000 **d** 67.5 **e** 9.5

7 Convert these kilograms into tonnes:

a 4000 **b** 95 000 **c** 4534 **d** 45.6 **e** 0.8

8 Write the following in grams:

a 8 kg **b** 3.2 kg **c** 14.2 kg **d** 380 mg
e 4250 mg **f** 75 420 mg **g** 6.8 t **h** 0.56 t

9 Convert the following into kilograms:

a 13 870 g **b** 3.4 t **c** 786 g **d** 3496 mg

10 Solve the following problems:

a Find the total mass, in kilograms, of 200 blocks of chocolate each 120 grams.

b If a nail has mass 25 g, find the number of nails in a 5 kg packet.

c Find the mass in tonnes of 15 000 bricks if each brick has a mass of 2.2 kg.

G PROBLEM SOLVING

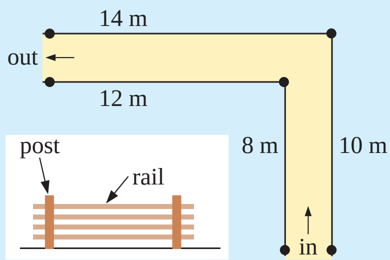
Example 8 **Self Tutor**

A cattle run is to be built in the shape shown.
The run has 4 wooden rails and a post every 2 m.

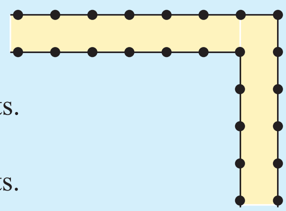
a Find the total length of wooden rails required.

b Find the number of posts required.

c If the rails cost £6.80 per metre and each post costs £12, find the total cost of building the run.



a Total length of rails $= 4 \times (14 + 10 + 12 + 8)$ m
$= 176$ m



b For the longer fence, $(14 + 10) \div 2 = 12$.
But a post is needed at both ends so we need 13 posts.
For the shorter fence, $(12 + 8) \div 2 = 10$,
and a post is needed at both ends so we need 11 posts.
So, $13 + 11 = 24$ posts are needed.

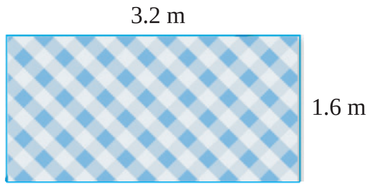
c The total cost $=$ cost of rails $+$ cost of posts
$= 176 \times £6.80 + 24 \times £12$
$= £1484.80$

When trying to solve a problem given to you in words, use the following series of steps:

Step 1: Draw a reasonably large diagram of the described situation.
Step 2: Mark clearly all dimensions and other key features on your diagram.
Step 3: Think about what the question is asking and the units you will have to work in.
Step 4: Set out your answer in a clear and logical fashion.
Step 5: Write your final answer in a sentence.

EXERCISE 12G

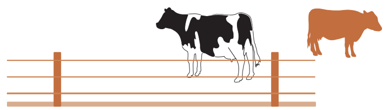
1 Martine has a rectangular table cloth with the dimensions shown. She wants to sew lace trimming along its border.



- **a** Find the length of the lace required.
- **b** If the lace costs €4.65 per metre, find the total cost of the lace Martine needs.

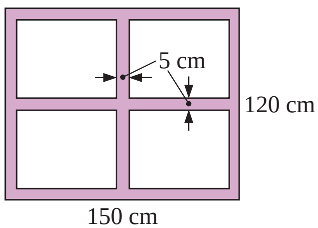
2 A tree trunk weighs 3.2 tonnes and can be cut into 80 planks. What is the mass of each plank?

3 A farmer fences a 250 m by 400 m rectangular paddock with a 3 strand wire fence.



- **a** Find the total length of wire needed.
- **b** Find the cost of the wire if wire costs $2.40 per metre.

4



A carpenter has to make a window frame with the dimensions shown. What is the total length of timber he requires?

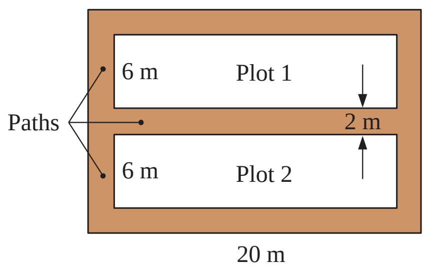
5 A supermarket buys cartons of canned tuna. Each carton contains 24 cans and each can weighs 325 g. Find the mass of a full carton in kilograms.

6 A bale of lucerne hay weighs approximately 14 kg.

- **a** Find the approximate mass carried by a truck loaded with 66 bales of lucerne.
- **b** Is it carrying more or less than a tonne? Show your working.

7

- **a** Henry edges his garden with railway sleepers. If his garden has two plots as shown, find the total length of sleepers required.
- **b** If each sleeper is 2 m long and weighs 40 kg, find:
  - **i** the total number of sleepers needed
  - **ii** the total mass of sleepers.



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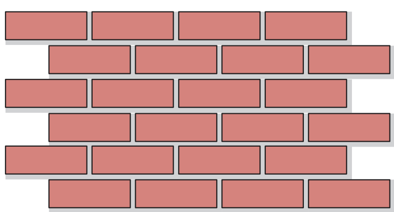
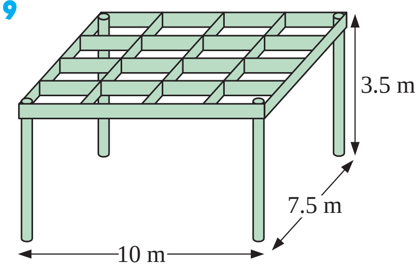


- **a** A house-proud couple wish to build a brick fence along the 30 m front of their block of land. If they want 12 rows of bricks and each brick is 20 cm long, find the number of bricks required.
- **b** If each brick weighs 2.5 kg, find the total mass in tonnes of the bricks needed.

9 

A builder needs to construct a pergola with the dimensions shown. The support posts cost $15 per metre and the timber for the top costs $4.50 per metre.

a Find the total length of timber for the top and hence the cost of this timber.

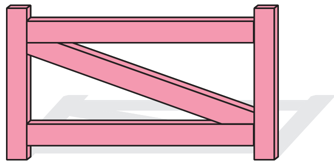
b Find the cost of the posts.

c Find the total cost of building the frame for the pergola if nails and other extras cost $27.

10 A fish tank weighs 25 kg when empty and 253 kg when full of water. Given that 1 litre of water has a mass of 1 kilogram, how many litres of water have been added to the tank?

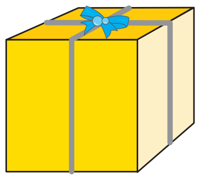
11 **a** Using the scale diagram alongside, find the total length of timber required to make the gate frame.

b If the timber costs $4.50 per metre, find the total cost of the timber.



Scale: 1 represents 60

12 Allowing 20 cm for the bow, what length of ribbon is needed to tie around the cube shaped box shown?



KEY WORDS USED IN THIS CHAPTER

- actual length
- mass
- perimeter
- scale length
- kilogram
- measure
- rectangle
- square
- length
- metre
- scale diagram
- triangle

LINKS click here

CALCULATING YOUR CARBON FOOTPRINT

Areas of interaction:
Environments, Community and service

REVIEW SET 12A

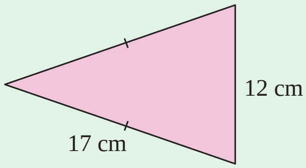
1 Convert:

a 356 cm to mm **b** 3200 g to kg **c** 450 m to km

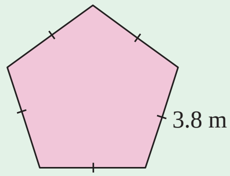
d 83 000 kg to t **e** 7.63 m to mm **f** 630 cm to m

2 Find the perimeter of:

a

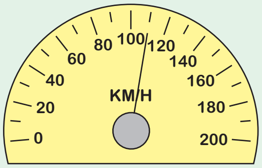


b

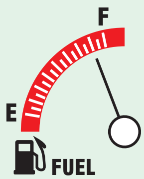


3 Read the following scales:

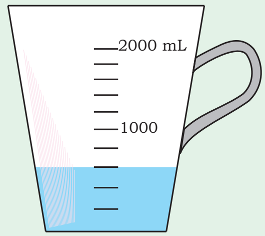
a



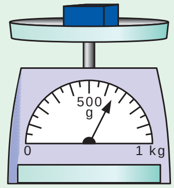
b



c



d



4 A scale diagram has the scale 1 represents 500 000.

a Find the actual length if the scale length is:

i 3.8 cm **ii** 6.4 cm **iii** 12.2 cm

b Find the scale length if the actual length is:

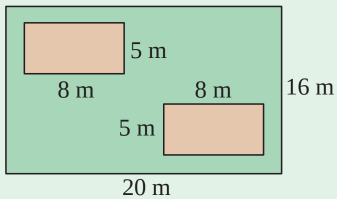
i 50 km **ii** 22 km **iii** 130 km

5 Kym competes in the 200 metre, 400 metre, 800 metre, 1500 metre, and 5000 metre running events on sports day. How many kilometres does she run in total?

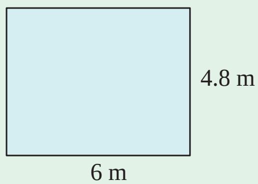
6 Find the total mass in kg of 1500 oranges if the average mass of an orange is 180 g.

7 If a truck can carry 1400 kg of soil, how many truckloads will be needed to remove 42 tonnes of soil?

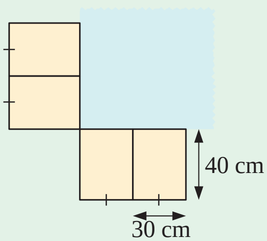
8 Find the total length of edging required to surround the lawn and two garden beds shown.



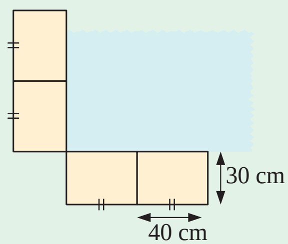
9 Rectangular tiles are 40 cm by 30 cm. They are used to form one row around a 6 m by 4.8 m swimming pool. Find the number of tiles needed if they are placed on the orientation:



a



b



REVIEW SET 12B

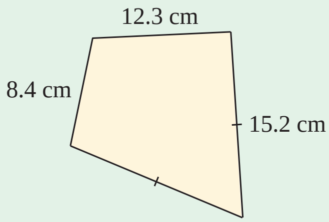
1 Convert:

a 3480 g to kg **b** 8623 mm to m **c** 4.6 g to mg

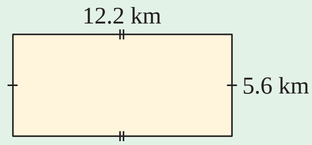
d 5.4 m to cm **e** 13.2 t to kg **f** 13.3 km to m

2 Find the perimeter of:

a

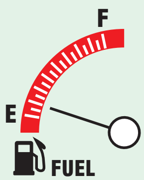


b

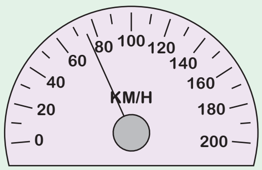


3 Read the following scales:

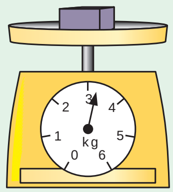
a



b



c



d

4 How many 1.8 kg bricks can be carried by a truck which has a load limit of 3.6 tonnes?

Hint: The load limit is the maximum mass the truck is allowed to carry.

5 A scale diagram has the scale 1 represents 2 500 000.

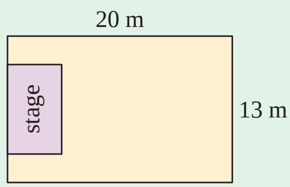
a Find the actual length if the scale length is: **i** 4.8 cm **ii** 0.7 cm

b Find the scale length if the actual length is: **i** 120 km **ii** 98 km

6 A rectangular concert hall with the dimensions shown has a frieze running along its wall just below the ceiling.

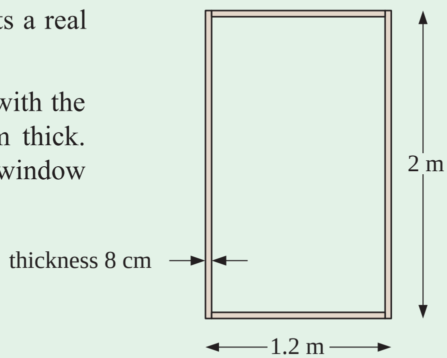
a Find the total length of frieze.

b If the frieze costs €38.80 per metre, what is its total cost?



7 A 2.3 cm line on a scale diagram represents a real length of 460 m. What is the scale?

8 A window frame is made from pine wood with the dimensions as shown. The timber is 8 cm thick. What is the total length of timber in the window frame?



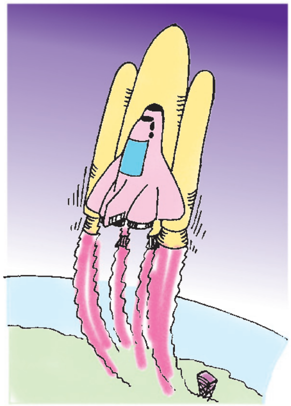
Chapter 13

Directed numbers

Contents:

The set of **whole numbers** 0, 1, 2, 3, 4, 5, ... are useful for solving many mathematical problems. However, there are certain situations where these numbers are not sufficient.

You are probably familiar with the **countdown** for a rocket: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, BLAST OFF!



What comes after zero if we keep counting backwards?

It may seem that we have 'run out' of numbers when we reach zero, but there are many situations where we need to be able to keep counting and where an answer of less than zero has a sensible meaning.

Which of these ideas can you explain, either in words or with a diagram?

- 10 metres below sea level
- 5 degrees below freezing
- 3 floors below ground level
- owing €30
- a loss of £4500

OPENING PROBLEMS

Problem 1:

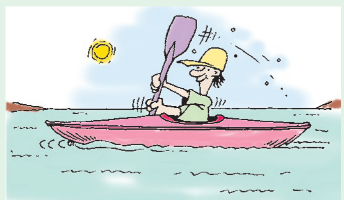
I had \$15 in my cheque account and had to write a cheque for \$20. How much do I have in my cheque account now? How much do I need to deposit to have a zero balance?

Problem 2:

A group of student bushwalkers experienced a maximum daily temperature of 21°C. At night, the temperature dropped to 3°C below zero. What change in temperature did they experience?

Problem 3:

A kayaker on Lake Eyre in South Australia is 16 m below sea level. A climber standing on the top of Mt Everest in Nepal is 8848 m above sea level. How much higher is the climber than the kayaker?



A OPPOSITES

The **Opening Problems** both involve **opposites**.

These are:
- *having* money in a bank account and *owing* money to a bank account
- temperature *above* zero and temperature *below* zero
- height *above* sea level and height *below* sea level.

DISCUSSION

Prepare a list of *ten* opposites which involve numbers.

Instead of using words to distinguish between opposites, we can use **positive** and **negative** numbers.

NEGATIVE NUMBERS

Negative numbers are written with a **negative sign (−)** before the number.

For instance:

- '10 metres below sea level' would be written as −10
- 'owing €30' would be written as −30
- '3 floors below ground level' would be written as −3.

In each case a measurement is being taken from a reference position of zero such as sea level or ground level.

POSITIVE NUMBERS

Positive numbers are the opposite of negative numbers.

They can be written with a **positive sign (+)** before the number, but we normally see them with no sign at all and we *assume* the number is positive.

For instance:

- '10 metres above sea level' would be written as +10 or just 10
- having €30 would be written as +30
- 3 floors above ground level would be written as +3.

Again, the measurement is being taken from a zero reference position.

Consider again the **Opening Problems**:

- Owing the bank \$5 would be represented as −5, whereas having a deposit of \$5 would be represented as +5 or just 5.
- A temperature of 21°C above zero would be 21, but 3°C below zero would be −3.
- A height of 16 m below sea level would be −16, whereas 8848 m above sea level would be 8848.

Some common uses of positive and negative signs are listed in the given table:

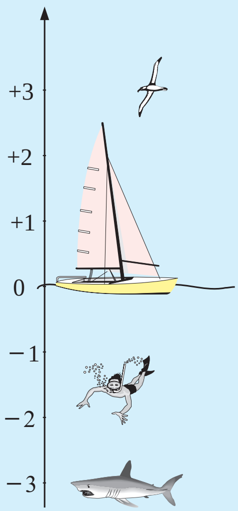
Positive (+)	*Negative (−)*	*Positive (+)*	*Negative (−)*
above	below	fast	slow
increase	decrease	win	lose
profit	loss	north	south
right	left	east	west

Example 1 **Self Tutor**

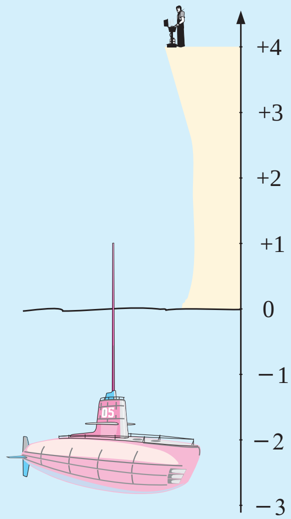
Write the positive or negative number for the position of each object.

The reference position is the water level.

a



b



a Positions *above* the water level are marked off with *positive numbers*, so the bird is at $+3$.
The boat is level with the water, so it is at 0.
Positions *below* the water level are marked off with *negative numbers*. The diver is at -1.5 and the shark is at -3.

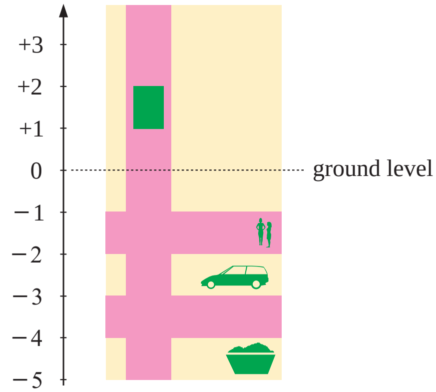
b The clifftop is at $+4$, the periscope is at $+1$, the water is at 0, and the submarine is at -2.

EXERCISE 13A

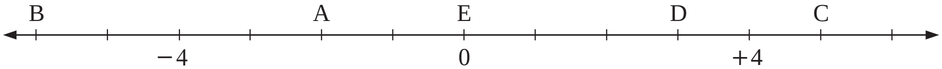
1 Copy and complete the following table:

	Statement	*Directed number*	*Opposite to statement*	*Directed number*
a	20 m above sea level	$+20$	20 m below sea level	-20
b	45 km south of the city			
c	a loss of 2 kg in weight			
d	a clock is 2 min fast			
e	she arrives 5 min early			
f	a profit of $4000			
g	2 floors above ground level			
h	10^oC below zero			
i	an increase of €400			
j	winning by 34 points			

2 Write positive or negative numbers for the position of the lift, the car, the parking attendant, and the rubbish skip. Use the bottom of each object to make your measurements.



3 If right is positive and left is negative, write numbers for the positions of A, B, C, D and E using zero as the reference position.



4 Write these temperatures as positive or negative numbers. Zero degrees is the reference point.

a 11^o above zero **b** 6^o below zero **c** 8^o below zero
d 29^o above zero **e** 14^o below zero

5 Write these gains or losses as positive or negative numbers:

a \$30 loss **b** €200 gain **c** \$431 loss
d £751 loss **e** RM 809 gain **f** ¥39 000 gain

6 If north is the positive direction, write these positions as positive or negative numbers:

a 7 metres north **b** 15 metres south **c** 115 metres south
d 362 metres north **e** 19.6 metres south

7 If the ground floor or street level is regarded as zero, write a directed number for the following positions:

a 6 floors above ground level **b** 3 floors below ground level
c 29 floors above ground level **d** 7 floors below ground level
e 4 floors below ground level

8 If right is positive, write a number for the position from zero which is:

a 7 units left **b** 5 units right **c** 12 units left
d 9 units right **e** 23 units left

9 State the combined effect of the following:

a a withdrawal of \$7 followed by a deposit of \$10
b a £7 withdrawal followed by a £6 withdrawal
c a rise in temperature of 13^oC followed by a fall of 8^oC
d a fall of 12^oC followed by a rise of 7^oC
e a 4 km trip east followed by a 3 km trip west
f a 7 km trip south followed by a 7 km trip north

g going up 5 floors in a lift and then coming down 6 floors

h a loss in mass of 4 kg followed by a gain in mass of 2 kg.

10 A baby boy weighed 3409 grams at birth. The record of his weight for the first five days showed the following:

Day 1:	28 g loss	Day 2:	15 g loss	Day 3:	13 g loss
Day 4:	17 g gain	Day 5:	29 g gain		

a Write each day's gain or loss as a positive or negative number.

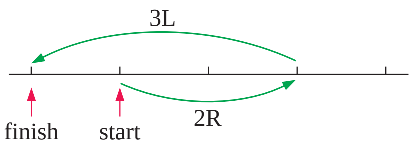
b What was the baby's weight at the end of the five days?

11 Luigi's boat is anchored in a harbour 7 metres from the jetty. As the tide rises and falls, it drifts 3 m away from the jetty, then 5 m towards it, then 6 m away from it. What is the boat's position now?

12 Helene has €155 in the bank. How much will be in her account after the following transactions?

Week 1:	Deposit €18	Week 2:	Withdraw €17	Week 3:	Withdraw €38
Week 4:	Withdraw €23	Week 5:	Deposit €29		

13 Suppose $2R$ means a trip to the right 2 units and $3L$ means a trip to the left 3 units.
$2R + 3L = 1L$ is the combined trip.
Find the combined trips for the following:



a $2R + 4L$ **b** $5L + 1R$ **c** $3R + 2R$ **d** $5L + 4L$

14 Find the combined trips for the following:

a $4R + 2L$ **b** $1R + 3R + 5L$ **c** $7L + 8L + 4R$

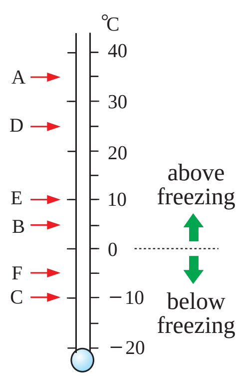
15 If $4\uparrow$ means go upwards 4 units and $2\downarrow$ means go downwards 2 units, then $4\uparrow + 2\downarrow = 2\uparrow$ is the combined effect. Find the combined effect of:

a $2\uparrow + 5\downarrow$ **b** $3\downarrow + 4\uparrow$ **c** $7\downarrow + 6\downarrow$

16 Find the combined effect of the following:

a $1\uparrow + 2\uparrow + 6\downarrow$ **b** $9\uparrow + 3\downarrow + 6\downarrow$ **c** $3\downarrow + 4\downarrow + 5\uparrow$

17 The temperatures of cities A, B, C, D, E and F were recorded at 12 noon on a certain day last year.



a What was the temperature of each of the cities?

b How many ^{o}C is city D warmer than city:

i E **ii** B **iii** F **iv** C?

c How many ^{o}C is city C cooler than city:

i A **ii** E **iii** F **iv** B?

d What is the difference in temperature between:

i A and B **ii** D and E

iii E and C **iv** F and C

v B and F **vi** D and F?

B DIRECTED NUMBERS AND THE NUMBER LINE

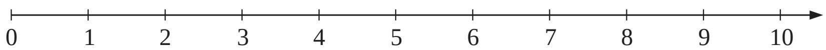
In order to describe situations in which opposites occur, mathematicians introduced **directed numbers**.

DIRECTED NUMBERS

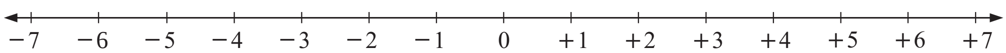
All negative numbers, zero, and positive numbers form the set of all **directed numbers**.

Directed numbers have both size and direction, and they can be illustrated on a **number line**.

We have used number lines before to place numbers in order. We placed zero on the left and marked numbers off in equal intervals to the right.



Suppose we make a mirror image of the numbers to the right of zero so the number line stretches in both directions. The numbers to the **right of zero** are the positive numbers, and the numbers to the **left of zero** are the negative numbers.



Pairs of numbers like -7 and 7 are exactly the same distance from 0 but on opposite sides, so they are called **opposites**.

Example 2 — Self Tutor

What is the opposite of **a** $+4$ **b** -9?

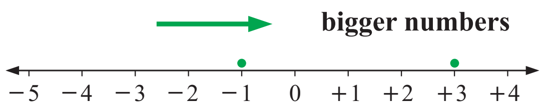
a The opposite of $+4$ is -4 because they are the same distance from zero but on opposite sides.

b The opposite of -9 is $+9$.

COMPARING AND ORDERING NUMBERS

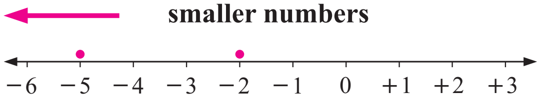
Using the position of numbers on a number line makes it easy to compare their size and arrange them in order.

As you move along the number line from *left* to *right*, the numbers increase in size. The number furthest to the right is the largest.



$+3$ is *bigger* than -1 because it is further to the right on the number line.

As you move along the number line from *right* to *left*, the numbers decrease in size. The number furthest to the left is the smallest.



−5 is *smaller* than −2 because it is further to the left on the number line.

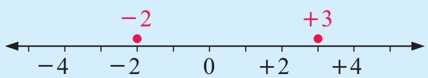
Remember that the symbol $>$ stands for '**is greater than**'
and the symbol $<$ stands for '**is less than**'.

So, we could write these two statements as $+3 > -1$ and $-5 < -2$.

Example 3 **Self Tutor**

a Show +3 and −2 on a number line and write a sentence comparing their size.
b Write the statement $-7 > -4$ in words, then state whether it is true or false.

a



Since +3 is further to the right, we can say that +3 is greater than −2. We could also say −2 is less than +3.

b The statement reads 'negative 7 is greater than negative 4'.
This is false because −7 is to the **left** of −4, and so it is smaller than −4.

Summary:

- Positive numbers are to the right of zero. Negative numbers are to the left of zero.
- 5 and −5 are opposites as they are both 5 units from zero but in opposite directions.
- 0 is the only number which is neither positive nor negative.
- 4 is to the right of −1 and $4 > -1$. −2 is to the right of −5 and $-2 > -5$.
- The further to the **right** a number is on the number line, the **greater** its value.
- The further to the **left** a number is on the number line, the **smaller** its value.

EXERCISE 13B

Draw a number line to help you with these questions:

1 Write the opposite of these numbers:

a +8 **b** −5 **c** 0 **d** 11 **e** −2
f +6.4 **g** $-3\frac{1}{2}$ **h** 56 **i** −23 **j** −23.6

2 Use a number line to:

a increase 2 by 3 **b** increase −1 by 3 **c** decrease 5 by 2
d decrease −1 by 3 **e** increase −4 by 3 **f** increase −2 by 1
g decrease 3 by 6 **h** decrease −2 by 2 **i** increase −3 by 5

3 Which is larger?

a +5 or +10 **b** +6 or −3 **c** −4 or +4
d +7 or −1 **e** −6 or −2 **f** −5 or −12

4 Which is smaller?

a 15 or 12 **b** 8 or -2 **c** -3 or 3

d -7 or -9 **e** -2 or 2 **f** -6 or -6.5

5 Write *true* or *false* for the following:

a $6 < -3$ **b** $13 > -5$ **c** $0 > -4$

d $7 < -2$ **e** $11 > -5$ **f** $-8 > -1$

g $-7 > -3$ **h** $-17 < 1$ **i** $-5 > -12$

6 Add $<$ or $>$ in the square to make each statement true:

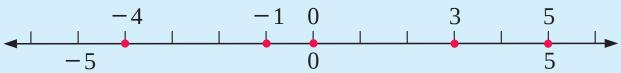
a $4 \square -1$ **b** $-4 \square -11$ **c** $8 \square -8$

d $-1 \square -11$ **e** $-6 \square -8$ **f** $-9 \square -13$

g $0 \square -8$ **h** $-6 \square 0$ **i** $-7 \square -5.5$

Example 4 Self Tutor

On a number line locate the values of: $\{5, 3, 0, -1, -4\}$



7 Draw number lines to show the following sets of numbers. Use a different number line for each set.

a $\{-2, 0, 3\}$ **b** $\{4, 3, 2, 0, -1, -5\}$

c $\{-5, 3, -2, 0, 4, 1\}$ **d** $\{6, -3, 4, -1, 0, -6\}$

8 **a** Arrange in *ascending* order: $\{-3, 0, -4, -1, 4\}$

b Arrange in *descending* order: $\{-2, 2, 5, 0, -1\}$

9 Four friends have the following bank balances: Monica −\$592, Joey \$311, Rachel \$852 and Ross −\$312. Place them in order of richest to poorest.

10 The temperatures of five cities were: Sydney 12°C, New York −3°C, Mexico City 15°C, Moscow −7°C and London 0°C. Place them in order of coldest to hottest.

11 Arrange these numbers from smallest to largest:

a $-5, 8, -2$ **b** $4, -3, -4, 0$

c $2.5, -1.2, 4, -3.1$ **d** $-9.5, -8.9, -10, -9.7$

e $3\frac{1}{2}, -2\frac{1}{4}, 1, -1\frac{1}{5}$ **f** $-\frac{1}{8}, -\frac{7}{8}, \frac{5}{8}, -\frac{3}{8}, -\frac{5}{8}$

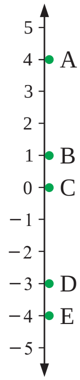
12 **a** Which number is furthest from 7?

i 3 or 15 **ii** 10 or -1 **iii** -20 or 28

b Which number is furthest from -3?

i 5 or -8 **ii** -10 or 6 **iii** 32 or -28

13 This number line is vertical. As you go up the numbers increase, and as you go down the numbers decrease. Write the directed number for each of the points marked on the number line, and write true or false for the following statements:



- **a** B is higher than D
- **b** A < E
- **c** D is lower than A
- **d** B < C
- **e** C > E
- **f** C < B
- **g** B and D are opposites
- **h** A and E are opposites.

14 What number is halfway between the following?

- **a** 0 and 12
- **b** 0 and 20
- **c** 6 and 10
- **d** 1 and 11
- **e** 0 and −4
- **f** −2 and 2
- **g** −6 and −2
- **h** −4 and 2

HISTORICAL NOTE

Fibonacci, an Italian mathematician from Pisa, was one of the first scholars of the 13th century to recognise the use of negative numbers.

While he was solving a financial problem he obtained a negative answer. Instead of rejecting this solution as many had before him, he analysed it and realised its significance. He wrote 'this problem is insoluble unless it is conceded that the first man had a debt.'

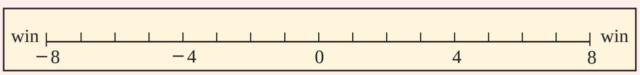
Debts are thus applications of negative numbers.

ACTIVITY 1 DIRECTED NUMBER GAME FOR 2 PLAYERS

You will need:

PRINTABLE NUMBER LINE

- 2 different coloured dice showing the numbers 1 to 6. Choose one die to represent the positive numbers 1 to 6 and the other to represent the negative numbers −1 to −6.
- 2 counters
- a number line



Object of the game: To move your counter over one end of the number line, i.e., > 8 or < −8.

How to play: Start the game with both counters on zero. Take it in turns to throw both dice and move your own counter according to the numbers thrown. Keep going until one player goes over one end. That person wins!

C USING A NUMBER LINE TO ADD AND SUBTRACT

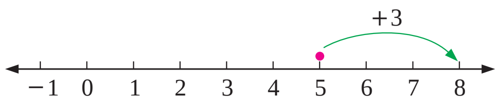
A number line is useful for adding and subtracting directed numbers.

ADDING A POSITIVE

When we **add** a positive number to another, we start at the first number mentioned and then move to the **right** the amount added.

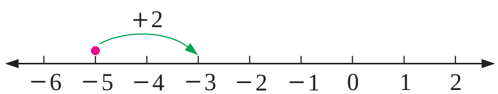
For $5 + 3$, we
"start at 5 and go 3 units to the right"

We end up at 8, so $5 + 3 = 8$.



Similarly, for $-5 + 2$ we
"start at -5 and go 2 units to the right"

We end up at -3, so $-5 + 2 = -3$.

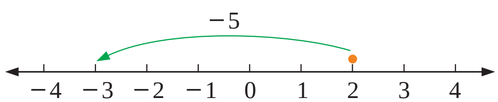


SUBTRACTING A POSITIVE

When we **subtract** a positive number from another, we start at the first number mentioned and move the required number of units to the **left**. So, subtraction is the opposite of addition.

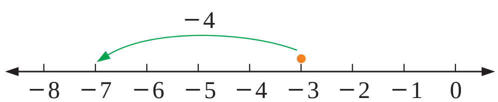
For $2 - 5$, we
"start at 2 and move 5 units to the left"

We end up at -3, so $2 - 5 = -3$.



Similarly, for $-3 - 4$ we
"start at -3 and move 4 units to the left"

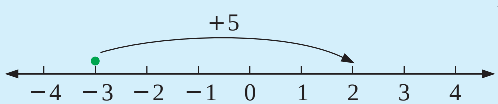
We end up at -7, so $-3 - 4 = -7$.



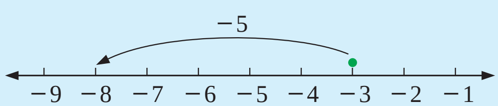
Example 5 **Self Tutor**

Find, using a number line: **a** $-3 + 5$ **b** $-3 - 5$

a $-3 + 5 = 2$



b $-3 - 5 = -8$



EXERCISE 13C

1 Calculate the following **additions** by moving to the **right** along a number line:

a $5+3$	**b** $-5+3$	**c** $4+2$	**d** $-4+2$
e $-7+7$	**f** $-6+6$	**g** $-8+8$	**h** $-3+3$
i $0+4$	**j** $0+2$	**k** $-9+10$	**l** $-7+13$
m $-5+8$	**n** $-4+10$	**o** $-1+9$	**p** $-2+7$

2 Calculate the following **subtractions** by moving to the **left** along a number line:

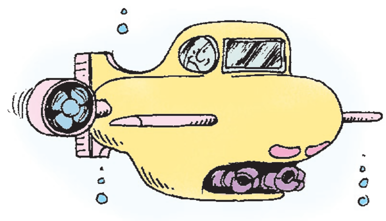
a $7-4$	**b** $-7-4$	**c** $5-8$	**d** $-5-8$
e $0-6$	**f** $0-3$	**g** $3-7$	**h** $-3-7$
i $6-9$	**j** $9-6$	**k** $8-7$	**l** $7-8$
m $-4-0$	**n** $-5-2$	**o** $3-5$	**p** $-1-5$

3 Calculate the following by moving along a number line:

a $8-3$	**b** $-6+8$	**c** $-5+3$	**d** $-4+4$
e $-1-3$	**f** $0-5$	**g** $-9+9$	**h** $-4+1$
i $5-5$	**j** $-3-2$	**k** $3-7$	**l** $-2+6$
m $0+4$	**n** $-6+6$	**o** $0-3$	**p** $-8-1$

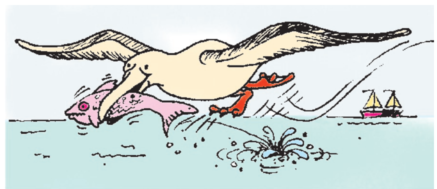
4 Use a number line to solve the following problems:

a A mini-submarine is 25 m below sea level and rises 18 m. What is its new depth?



b The temperature overnight was -8^oC but it has risen 15^oC by noon. What is the temperature at noon?

c A bird gliding 12 m above sea level sees a fish and dives 14 m vertically down to catch it. At what depth was the fish?

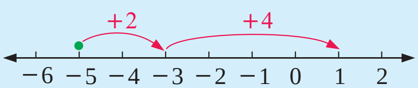


d The temperature at dusk was 9^oC and it fell by 14^oC during the night. What was the lowest temperature reached?

Example 6 Self Tutor

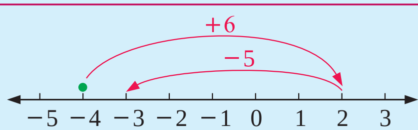
Find using a number line: **a** $-5+2+4$ **b** $-4+6-5$ **c** $6-4-7$

a



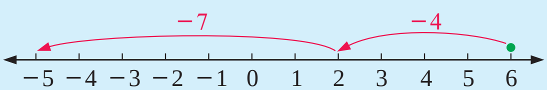
$-5+2+4=1$

b



$-4+6-5=-3$

c



$6-4-7=-5$

5 Find using a number line:

- **a** $3-1-4$
- **b** $-2-1-3$
- **c** $-3+1+4$
- **d** $3-5-1$
- **e** $3+2-7$
- **f** $5-4-3$
- **g** $-8-2+2$
- **h** $-1-2-4$
- **i** $4-9+2$

6 Chao has \$10 in his wallet. Dong owes him \$18, but Chao owes Guang \$5. If all debts are paid, how much will Chao have in his wallet?

7 Find using a number line:

- **a** $7+6-2-8$
- **b** $-7+11+1-5$
- **c** $5-6+2-8$
- **d** $-2-3+8-5$
- **e** $4-10+3-2$
- **f** $-5-2+9-3$

ACTIVITY 2 LUCKY DIP

Lucky Dip is a game for two people.

PRINTABLE GRIDS

How to play:

1. Each player draws up a grid like those alongside. This sheet represents their Lucky Dip box containing 16 small bags. Each player marks any 3 squares with the numbers 2, 4, and 5.
   2 represents a bag containing \$2, 4 represents \$4, and 5 represents \$5. All the other bags are empty.

Joanne

A	B 5	C	D
E	F	G	H
I	J	K 2	L
M	N	O	P 4

Troy

A 4	B	C	D
E	F 2	G	H 5
I	J	K	L
M	N	O	P

2. Players take turns to pick a bag from their opponent's Lucky Dip box, by calling the letter of a square.
3. If a player does not get a bag containing money then he or she loses \$2. If a bag containing money is selected then that amount is gained.
4. The player with the most money after 8 selections each is the winner, unless one player has gained all of the opponent's money in less than 8 selections. Each player must have the same number of selections.

Sample game:

Joanne's Lucky Dip box is on the left and Troy's is on the right.

The game ends after 7 selections with Joanne winning.

Set up your own Lucky Dip sheet and play against a partner!

	Selection		Scorecard	
Round	*Troy*	*Joanne*	*Troy*	*Joanne*
1	G	C	-2	-2
2	P(\$4)	J	2	-4
3	L	K	0	-6
4	A	A(\$4)	-2	-2
5	N	H(\$5)	-4	3
6	M	N	-6	1
7	B(\$5)	F(\$2)	-1	3

D ADDING AND SUBTRACTING NEGATIVES

ADDING A NEGATIVE NUMBER

We know that $4+3=7$, but what is the value of $4+-3$?

Consider the following true statements:

$$4+3=7$$
$$4+2=6$$
$$4+1=5$$
$$4+0=4$$

As the number being added to 4 decreases by 1, the final answer also decreases by 1.

Continuing this pattern gives:

$4+-1=3$	compared with	$4-1=3$
$4+-2=2$		$4-2=2$
$4+-3=1$		$4-3=1$
$4+-4=0$		$4-4=0$

So, **adding** a **negative** number is the same as **subtracting** a **positive** number.

For example, $2+-6$ is the same as $2-6$.

SUBTRACTING A NEGATIVE NUMBER

We know that $4-3=1$, but what is the value of $4--3$?

Consider the following true statements:

$$4-3=1$$
$$4-2=2$$
$$4-1=3$$
$$4-0=4$$

Notice that as the number being subtracted decreases by 1, the answer increases by 1.

Continuing this pattern gives:

$4--1=5$	compared with	$4+1=5$
$4--2=6$		$4+2=6$
$4--3=7$		$4+3=7$
$4--4=8$		$4+4=8$

So, **subtracting** a **negative** number is the same as **adding** a **positive** number.

For example, $3--5$ is the same as $3+5$.

Example 7 Self Tutor

Simplify and then evaluate:

a $2+-5$ **b** $2--5$ **c** $-2+-5$ **d** $-2--5$

a $2+-5$
$=2-5$
$=-3$

b $2--5$
$=2+5$
$=7$

c $-2+-5$
$=-2-5$
$=-7$

d $-2--5$
$=-2+5$
$=3$

EXERCISE 13D

1 Simplify and then use a number line to evaluate:

a $7+-3$	**b** $7--3$	**c** $-7+-3$	**d** $-3+-7$
e $3+-7$	**f** $3--7$	**g** $-3--7$	**h** $-7--3$
i $5-11$	**j** $11-5$	**k** $5--11$	**l** $11--5$
m $-5+-11$	**n** $-5--11$	**o** $-11+-5$	**p** $-11--5$
q $-6+-1$	**r** $-2+-4$	**s** $6+-2$	**t** $-5+-3$
u $2+-6$	**v** $-6+-4$	**w** $-6+-13$	**x** $-15+-5$

2 A steward working in a hotel starts his day on the ground floor. To fulfil his duties he goes up 7 floors, up 3 floors, down 5 floors, down 6 floors, up 4 floors, down 3 floors, and then up 8 floors. Which floor does he finish on?

3 Simplify if possible, and hence evaluate:

a $-4+-3$	**b** $5--5$	**c** $-6-2$	**d** $4--8$
e $8+-1$	**f** $7--11$	**g** $-2+-1$	**h** $6-9$
i $-6-9$	**j** $-4--2$	**k** $16+-25$	**l** $-16--25$
m $31+-45$	**n** $56--12$	**o** $39+-15$	**p** $-21--16$

Example 8 Self Tutor

Simplify and find: **a** $4+-6--3$ **b** $-2--5+-7$

a
$$\begin{aligned}&4+-6--3\\&=4-6+3 \quad \{\text{simplifying}\}\\&=-2+3\\&=1\end{aligned}$$

b
$$\begin{aligned}&-2--5+-7\\&=-2+5-7 \quad \{\text{simplifying}\}\\&=3-7\\&=-4\end{aligned}$$

4 Simplify and find:

a $2--1+7$	**b** $11+-3--8$	**c** $-3--2+-3$
d $-5+-2--3$	**e** $6-11--5$	**f** $9-13+-8$
g $-1+4--6$	**h** $2+-3--5$	**i** $5--2+-3$
j $6-6--1$	**k** $-7--2+3$	**l** $-5--2+-6-11$

5 St Moritz recorded the following maximum temperatures for a week:
Mon 5^oC, Tues -3^oC, Wed -7^oC,
Thurs 2^oC, Fri 1^oC, Sat -4^oC,
Sun -1^oC.
What was the *average* daily maximum temperature for the week?

To find the *average*, add the 7 temperatures and then divide this sum by 7.

INVESTIGATION MAGIC SQUARES

4	3	8
9	5	1
2	7	6

A **magic square** is one filled with **consecutive whole numbers** so that each row, column, and diagonal has the same sum.

For example, this magic square contains the numbers 1 to 9 and has the **magic sum** of 15 along every row, column, and diagonal.

What to do:

1 Copy and complete the following magic squares:

a

4		8
	7	
		10

b

		7	12
15		9	6
	5		
8	11	2	

2 Magic squares may also contain negative numbers.

2	-5	0
-3	-1	1
-2	3	-4

a Is the square alongside a magic square? If so, what is the magic sum?

b Make a new magic square by adding 2 to each number in the magic square given. State the new magic sum.

c Make a new magic square by subtracting 3 from each number in the magic square given. State the new magic sum.

d Compare the magic sums **a**, **b** and **c**. If you started with the square in **a** and added 3 to each number, can you predict the new magic sum?

3 Copy the following magic squares and try to complete them:

a

-4		0
	-1	
		2

b

3			-9
-8			
-7		-4	5
6		-1	-6

E MULTIPLYING DIRECTED NUMBERS

We have already seen how to correctly add and subtract negative numbers. In this section we look for rules for their multiplication.

For example, we know that $4 \times 3 = 12$, but how do we calculate:

- 4×-3
- -4×3
- -4×-3?

Consider the following true statements:

$$4 \times 3 = 12$$
$$4 \times 2 = 8$$
$$4 \times 1 = 4$$
$$4 \times 0 = 0$$

As the number that 4 is multiplied by decreases by 1, the answer decreases by 4.

If we continue this pattern we get:

$$4 \times -1 = -4$$
$$4 \times -2 = -8$$
$$4 \times -3 = -12$$

From these examples it seems that a **positive** times a **negative** gives a **negative**.

Likewise, from the pattern:

$$2 \times 4 = 8$$
$$1 \times 4 = 4$$
$$0 \times 4 = 0$$
$$-1 \times 4 = -4$$
$$-2 \times 4 = -8$$
$$-3 \times 4 = -12$$

it seems that a **negative** times a **positive** gives a **negative**.

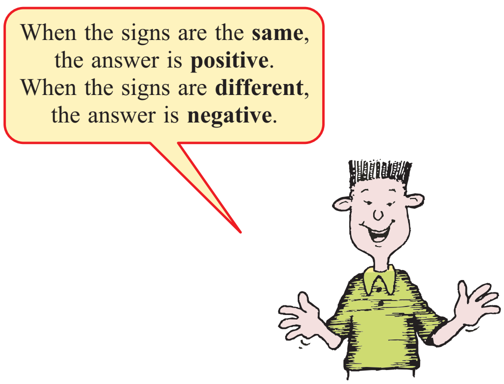
Also, from the pattern:

$$-3 \times 3 = -9$$
$$-3 \times 2 = -6$$
$$-3 \times 1 = -3$$
$$-3 \times 0 = 0$$
$$-3 \times -1 = 3$$
$$-3 \times -2 = 6$$
$$-3 \times -3 = 9$$

it seems that a **negative** times a **negative** gives a **positive**.

RULES FOR MULTIPLICATION

- **(positive) × (positive) = (positive)**
- **(positive) × (negative) = (negative)**
- **(negative) × (positive) = (negative)**
- **(negative) × (negative) = (positive)**



Example 9 — Self Tutor

Simplify:

a 2×5 **b** 2×-5 **c** -2×5 **d** -2×-5

a $2 \times 5 = 10$ **b** $2 \times -5 = -10$ **c** $-2 \times 5 = -10$ **d** $-2 \times -5 = 10$

EXERCISE 13E

1 Simplify:

a 2×3 **b** 2×-3 **c** -2×3 **d** -2×-3
e 8×-2 **f** 8×2 **g** -8×2 **h** -8×-2
i 7×11 **j** -7×-11 **k** 7×-11 **l** -7×11
m 0×3 **n** -2×0 **o** -3×-6 **p** -5×-5

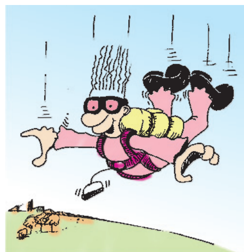
2 Determine the missing number in each of the following:

a $-2 \times \square = -16$ **b** $-2 \times \square = 16$ **c** $5 \times \square = 10$
d $-5 \times \square = 10$ **e** $\square \times 4 = -12$ **f** $\square \times -4 = 12$
g $-4 \times \square = 20$ **h** $-4 \times \square = -20$ **i** $3 \times \square = -15$
j $-3 \times \square = -15$ **k** $\square \times -6 = 18$ **l** $\square \times -6 = -18$

3 Use a negative sign to help solve the following questions:

a A gambler loses \$8 per race for seven successive races. How much does he lose in total?

b A skydiver falls 200 metres per second for 30 seconds. How many metres does he fall?



Example 10 **Self Tutor**

Simplify: **a** $-2 \times 5 \times -3$ **b** $(-3)^2$ **c** $(-2)^3$

a $\underbrace{-2 \times 5}_{} \times -3$
$= -10 \times -3$
$= 30$

b $(-3)^2$
$= -3 \times -3$
$= 9$

c $(-2)^3$
$= \underbrace{-2 \times -2}_{} \times -2$
$= 4 \times -2$
$= -8$

4 Simplify:

a $3 \times -2 \times 5$ **b** $-2 \times -1 \times -3$ **c** $-1 \times 3 \times -4$
d $(-7)^2$ **e** $(-1)^3$ **f** $4 \times -1 \times -5$
g $5 \times -2 \times -4$ **h** $-7 \times -2 \times 2$ **i** $(-2)^3$
j -2×5^2 **k** $-2 \times (-3)^2$ **l** $(-2)^2 \times -6$

5 Do $(-2)^2$ and -2^2 have the same value?

6 Calculate:

a $(-1)^2$ **b** $(-1)^3$ **c** $(-1)^4$
d $(-1)^5$ **e** $(-1)^6$ **f** $(-1)^7$

What do you notice?

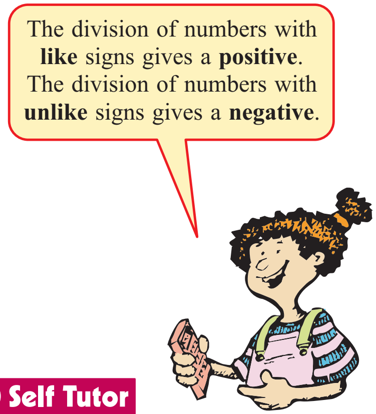
F DIVIDING DIRECTED NUMBERS

In this section we look for rules for the division of negative numbers.

We know that $12 \div 4 = 3$, but how do we calculate:
- $12 \div -4$
- $-12 \div 4$
- $-12 \div -4$?

The rules for **division** are identical to those for multiplication. This is not surprising because multiplication and division are **inverse operations**.

For example, $\div$ by 2 is the same as $\times$ by $\frac{1}{2}$.



RULES FOR DIVISION

(positive) $\div$ (positive) = (positive)
(positive) $\div$ (negative) = (negative)
(negative) $\div$ (positive) = (negative)
(negative) $\div$ (negative) = (positive)

Example 11 — Self Tutor

Calculate:

a $-6 \div 2$ **b** $8 \div -4$ **c** $\dfrac{-14}{-2}$

a $-6 \div 2 = -3$ **b** $8 \div -4 = -2$ **c** $\dfrac{-14}{-2} = 7$

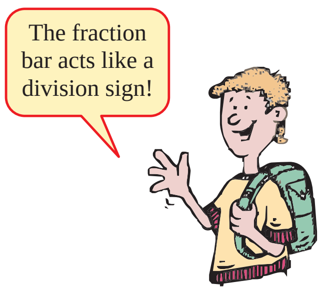
EXERCISE 13F

1 Calculate:

a $14 \div 7$ **b** $14 \div -7$ **c** $-14 \div 7$ **d** $-14 \div -7$
e $30 \div 5$ **f** $-30 \div -5$ **g** $-30 \div 5$ **h** $30 \div -5$
i $8 \div 8$ **j** $8 \div -8$ **k** $-8 \div 8$ **l** $-8 \div -8$
m $24 \div 4$ **n** $24 \div -4$ **o** $-24 \div -4$ **p** $-24 \div 4$

2 Calculate:

a $\dfrac{12}{3}$ **b** $\dfrac{-12}{3}$ **c** $\dfrac{12}{-3}$ **d** $\dfrac{-12}{-3}$
e $\dfrac{22}{2}$ **f** $\dfrac{22}{-2}$ **g** $\dfrac{-22}{2}$ **h** $\dfrac{-22}{-2}$
i $\dfrac{18}{9}$ **j** $\dfrac{18}{-9}$ **k** $\dfrac{-18}{-9}$ **l** $\dfrac{-18}{9}$



3 Find the missing number in each of the following:

a $24 \div \square = -4$ **b** $24 \div \square = 4$ **c** $-18 \div \square = 9$
d $-18 \div \square = -9$ **e** $-27 \div \square = -3$ **f** $-27 \div \square = 3$
g $\square \div -5 = 7$ **h** $\square \div -5 = -7$ **i** $\square \div -2 = -8$
j $\square \div -2 = 8$ **k** $\square \div 3 = -5$ **l** $\square \div -3 = 5$
m $\square \div -4 = -4$ **n** $\square \div -4 = 4$ **o** $7 \div \square = -7$
p $-7 \div \square = 7$ **q** $\square \div \square = 1$ **r** $\square \div \square = -1$

4 Use a negative sign to help solve the following questions:

a A company owned equally by four people has a debt of \$320 000. What is each person's share of the debt?

b One night in Siberia the temperature drops 18^{o}C in six hours. What is the average temperature change per hour?

The *average* temperature change is the total temperature change divided by the number of hours.

G COMBINED OPERATIONS

The order of operations rules also apply to negative numbers.

- **B**rackets are evaluated first.
- **E**xponents are calculated next.
- **D**ivisions and **M**ultiplications are done next, in the order that they appear.
- **A**ddition and **S**ubtractions are then done, in the order that they appear.

Example 12 **Self Tutor**

Use the correct order of operations rules to calculate:

a $5 + -8 \times 3$ **b** $-5 - 15 \div -5$

a $5 + -8 \times 3$
$= 5 + -24$ {multiplication first}
$= 5 - 24$ {simplify}
$= -19$

b $-5 - 15 \div -5$
$= -5 - -3$ {division first}
$= -5 + 3$ {simplify}
$= -2$

Remember to use BEDMAS!

EXERCISE 13G

1 Find, using the order of operations rules:

- **a** $3 + -7 \times 2$
- **b** $-2 - 3 \times -4$
- **c** $-4 - 18 \div 3$
- **d** $(5 - 10) \times (3 - 5)$
- **e** $-10 + 2 \times -4$
- **f** $3 \times -4 + -5 \times -2$
- **g** $(8 - 12) \times 3 - 7$
- **h** $8 - 12 \times (3 - 7)$
- **i** $8 - 12 \times 3 - 7$
- **j** $7 - 2 \times -3 + 4 \times -5$

2 Abidin's company makes a €70 000 loss per month for four months and then a €40 000 profit for each of the next eight months. What was the year's result?

3 Debbie's Dresses show the following sales record over a six week period:

Week 1: $1214 profit   Week 2: $867 profit   Week 3: $126 loss
Week 4: $992 profit   Week 5: $543 loss   Week 6: $2150 profit.

- **a** What is Debbie's overall profit or loss during this period?
- **b** What is Debbie's average weekly earnings during this period?

4 The temperature of a bottle of liquid is 18°C. The bottle is placed in a freezer that cools the liquid at 5°C per hour. What is its temperature after 4 hours?

5 To explore for gold, a mining company uses a drilling rig to take core samples from below the ground. Gold samples were found at the levels shown in the table:

- **a** Which sample is closest to ground level?
- **b** Which sample is the deepest?
- **c** What is the difference in depth between sample B and D?
- **d** The cost of drilling is £600 per m. What is the cost of taking sample A?

Sample	*Level*
A	−113 m
B	−42 m
C	−119 m
D	−78 m

Example 13 Self Tutor

Calculate:

a $\dfrac{5 \times -12}{7 - 3}$ **b** $\dfrac{-36}{-3 \times -4}$

a $\dfrac{5 \times -12}{7 - 3}$
$= \dfrac{-60}{4}$
$= -15$

b $\dfrac{-36}{-3 \times -4}$
$= \dfrac{-36}{12}$
$= -3$

For more complicated fractions, work out the numerator and the denominator first, and then divide.

6 Calculate:

- **a** $\dfrac{3 \times -2}{6}$
- **b** $\dfrac{-4 \times -2}{-8}$
- **c** $\dfrac{3 \times -4}{6}$
- **d** $\dfrac{12}{-2 \times -3}$
- **e** $\dfrac{3 \times -5}{7 - 2}$
- **f** $\dfrac{-3 \times -4}{5 - 1}$
- **g** $\dfrac{-3 \times -6}{5 - 7}$
- **h** $\dfrac{3 \times -6}{-2 \times 3}$

H USING YOUR CALCULATOR

Scientific calculators have a [(−)] or [+/−] key to specify a negative number.

On most calculators we press this key *before* the number, for example, [(−)] 2.

On some older calculators, however, we press it *after* the number, for example, 2 [+/−] .

You will need to check what keys your calculator has and the sequence in which they need to be pressed.

Example 14 — Self Tutor

Find the following answers using your calculator:

a $-14 + -71$ **b** $22 - -45$ **c** $-8 \times -4 \div (7 - 11)$

Answers

a Press [(−)] 14 [+] [(−)] 71 [ENTER] — -85

or Press 14 [+/−] [+] 71 [+/−] [=]

b Press 22 [−] [(−)] 45 [ENTER] — 67

c Press [(−)] 8 [×] [(−)] 4 [÷] [(] 7 [−] 11 [)] [ENTER] — -8

EXERCISE 13H

1 Use your calculator to evaluate:

a $-29 + 51 - 36$ **b** $-20 - -37 + 53$ **c** $-41 - 35 + 28$

d $-29 - -71 - 25$ **e** 17×-25 **f** $-2100 \div 30$

g $-30 \div -5 \times -4$ **h** $-10 \times 24 \div 15 - -8$ **i** $-450 \times 4 \div -18$

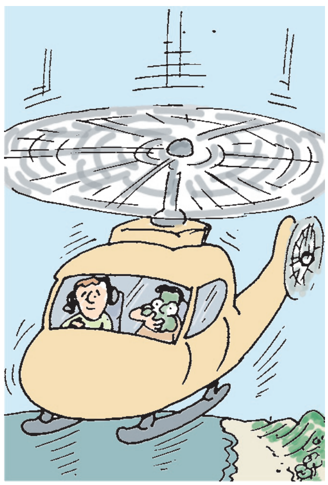
2 Solve the following problems using your calculator:

a In windy conditions a helicopter rises 30 m, falls 45 m, rises 20 m, falls 10 m, rises 15 m, then falls 12 m. How far is it now above or below its original position?

b Lumina has €673 in the bank and she makes the following transactions: a withdrawal of €517, a deposit of €263, a deposit of €143, and a withdrawal of $317. What is her new bank balance?

c Mr Jones owes £150 to each of 17 creditors. How much does he owe altogether?

d Abdul wanted to buy a nice car, so he saved RM 240 per week for 5 years. How much extra money does he need to borrow to buy a car valued at RM 86 000?



KEY WORDS USED IN THIS CHAPTER

- addition
- inverse operation
- number line
- subtraction
- directed number
- multiplication
- opposite
- division
- negative number
- positive number

REVIEW SET 13A

1 **a** Evaluate $3-5$ using a number line.

b Insert an inequality symbol to make the following statement true: $7 \square -12$

c Evaluate $(-1)^3$.

d Arrange in ascending order: $\{1, -3, 0, -2, -5, 4, 3\}$

e State the combined effect of borrowing 8 books and returning 4.

f Use a number line to decrease 1 by 4.

g Simplify -6×2.

h What is the result of subtracting -7 from 15?

i Simplify $\dfrac{-12}{-4}$.

2 **a** Find $(-3)^3$.

b Simplify $4 \times (-1)^2$.

c Evaluate $6 - 11 - 2$.

d Copy and complete: negative $\times$ negative = ...

e Decrease 4 by 6.

f Find $17 - 22$.

g By what must I divide -8 to obtain 2?

h Insert $>$ or $<$ between -6 and 0 to make a true statement.

i Simplify $\dfrac{(-2)^2}{-4}$.

3 **a** Xuen's business has \$7500 in the bank. She must pay each of her 7 employees a wage of \$327 per week for 3 weeks. How much money will be remaining in the bank?

b Which is the greater distance:
moving from 63 metres below sea level to 33 metres above sea level, or
moving from 289 metres below sea level to 365 metres below sea level?

REVIEW SET 13B

1 **a** State the opposite of going up 4 flights of stairs.

b Simplify $-7-2$ using a number line.

c Write down 3 consecutive directed numbers, the smallest being -4.

d Find $5-8$ using a number line.

e Use a number line to increase -9 by 11.

f Simplify $-(-2)^2$.

g Arrange in ascending order: $\{-6, 5, 0, -2, -4\}$

2 Calculate:

a $-5+12$ **b** $-3-8$ **c** $15+-6-4$

d $\dfrac{3\times-6}{4-2}$ **e** $\dfrac{-4\times-8}{16}$ **f** $\dfrac{43-7}{-8--2}$

3 **a** What is the combined effect of depositing €83 and withdrawing €57?

b If ground level is marked as 0 and the top of a 2 metre high fence is assigned the integer 2, what integer would be assigned to:

i a shrub 1 metre high **ii** a 10 m tall tree

iii the bottom of a well 15 m below ground level?

c A shopkeeper bought a refrigerator for £575 and increased the price by £90 to make a profit. He then gave a discount of £55. What was the final selling price of the refrigerator?

d A person weighing 127 kg wishes to reduce his weight to 64 kg in 9 months. What average weight reduction is needed each month to achieve his aim?

ACTIVITY DRAWING A CIRCLE USING A PIECE OF STRING

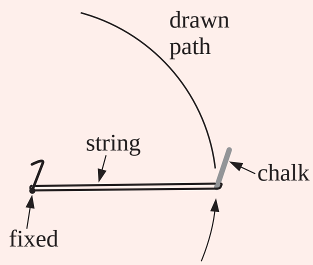
Before you begin this activity, ask your teacher where there is a suitable area of concrete which you can draw on with chalk.

What to do:

1 Take a piece of string about 1 metre long and tie its ends together.

2 Place a peg at one end of the loop and a piece of chalk at the other end, as shown.

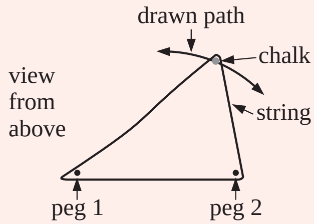
3 Hold the peg still on the ground. Keep the string taut while moving the chalk. The chalk will trace a circle with the peg at its centre.



Extension:

4 Place your string loop over two pegs. Ask a friend to help you keep the pegs still. Keep the string taut while moving the chalk.
What shape is traced out?

5 Experiment with the same piece of string, putting the pegs closer together, then putting the pegs further apart. What effect does this have on the shape formed?

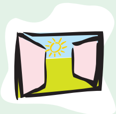


Chapter 14

Percentage

Contents:

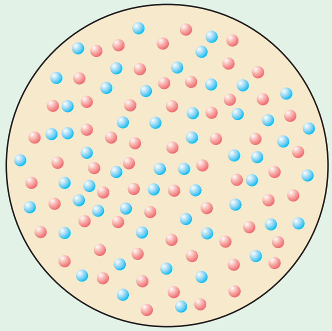
OPENING PROBLEMS



Problem 1:

Mahari has a collection of blue and red beads. She wants to string them together to form a bracelet. Can you write the number of red beads compared with the total number of beads as:

- a fraction
- a decimal
- a percentage?



Problem 2:

- If a store advertises a 25% off sale, what percentage of the normal cost of an item would you have to pay?
- Can you write the amount you would have to pay as a fraction of the usual amount?

A PERCENTAGES

From the chapter on fractions, you might remember the difficulty of comparing some fractions.

Fractions with the same denominators like $\frac{1}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ and $\frac{8}{5}$ were easy to compare but fractions with different denominators like $\frac{1}{4}$, $\frac{3}{10}$, $\frac{7}{25}$ or $\frac{37}{20}$ needed to first be converted to fractions with the same denominator.

Percentages are special kinds of fractions because their denominator is always 100.

Rather than write the fraction $\frac{12}{100}$, we would write 12%.

$100\% = \frac{100}{100} = 1$, so 100% represents the whole amount.

The word percent comes from the Latin meaning *out of every hundred*.

Percentages are comparisons of a portion with the whole amount, which we call 100%.

For example, $12\% = \frac{12}{100}$, and means '12 out of every 100'.

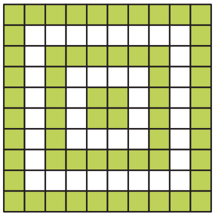
EXERCISE 14A

1 In each of the following patterns there are 100 tiles. For each pattern:

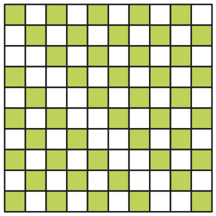
  i write the number of coloured tiles as a fraction of the total, leaving your answer with the denominator 100

  ii write a percentage which shows the proportion of squares shaded.

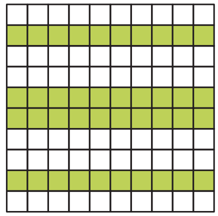
a



b

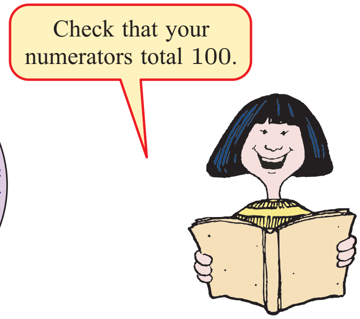


c



2 In this circle there are 100 symbols. For each of the different symbols present:

- **a** count how many there are
- **b** write the proportion as a fraction of 100
- **c** write the proportion as a percentage.

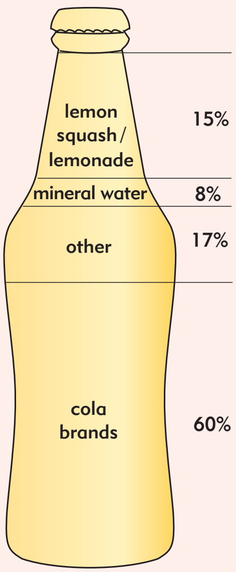


3 For the numbers from 1 to 100 inclusive, write as a percentage the proportion which:

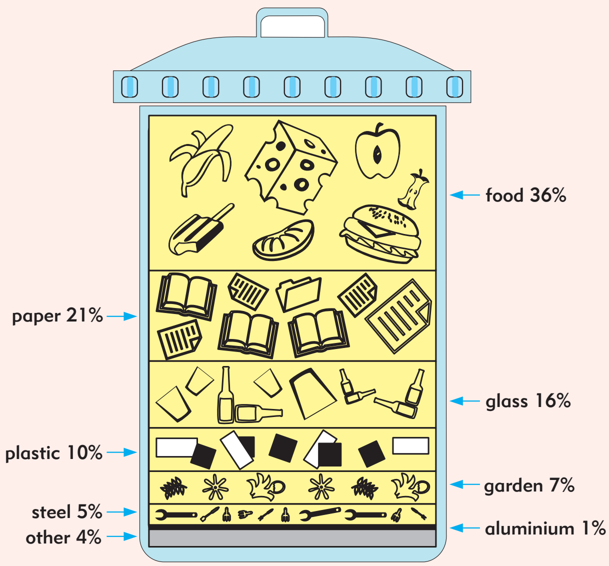
- **a** are odd
- **b** are exactly divisible by 5
- **c** are multiples of 4
- **d** can be divided by 10 exactly
- **e** contain the digit 1
- **f** have only 1 digit
- **g** are prime numbers
- **h** are composite numbers.

ACTIVITY 1 CATCHING ATTENTION WITH PERCENTAGE

Here are some examples of eye-catching graphs which use percentages to create an impact.



Sales of all carbonated softdrinks



Contents of a garbage can

What to do:

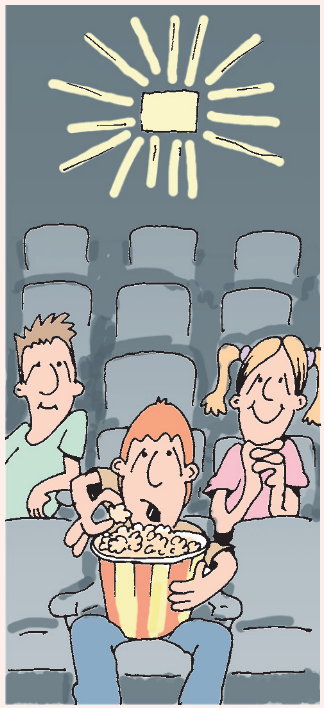
Think of some eye-catching ways you could present different types of information in percentage form. Remember when you represent a percentage, you need to give a symbol or statement which explains what the whole quantity is.

ACTIVITY 2 EVERYDAY USE OF PERCENTAGE

What to do:

1 Read the following everyday examples of the use of percentages:

- In my street 25% of the homes have roses growing in the front garden.
- Sixty five percent of students at my school voted for a greater variety of fresh fruit in the school canteen.
- Twenty seven percent of primary school age children do not eat fruit and vegetables.
- Our netball goal shooter Alice had a 68% accuracy rate for the whole season.
- Sarah improved by 10% in her times table tests.
- Our country's unemployment rate dropped to 8.1%.
- Last year over 52% of 5-14 year old children living in Switzerland played sport outside school hours.
- House prices near the beach increased by 15% in the last year.
- Nearly 27% of the population visited a museum in 2008.
- The number of children attending the local cinema during the school holidays has dropped 12% on last year's attendance.
- The humidity at 9 am was 46% and at 3 pm it was 88%.
- After the weekend rainfalls the reserviour was at 75% capacity.



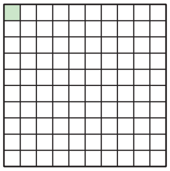
2 For each of the above examples, suggest how and why these percentages may have been worked out.

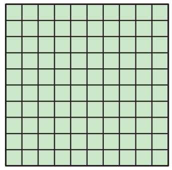
3 What is a census? How is a census conducted? Why is a census conducted? What types of questions may be asked? Why are percentages important here?

4 What census do schools conduct, and why?

B CONVERTING FRACTIONS TO PERCENTAGES

If an object is divided into 100 equal parts then each part is 1 percent and is written as 1%.

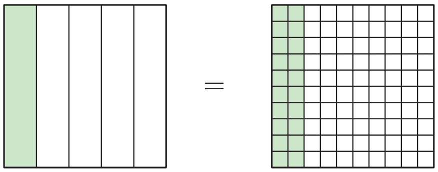




Thus $\frac{1}{100} = 1\%$ and $\frac{100}{100} = 100\%$

Most common fractions and decimal fractions can be changed into percentage form by first converting into an equal fraction with a denominator of 100.

For example:



The shaded part of both squares is the same.

In the first square $\frac{1}{5}$ is shaded.

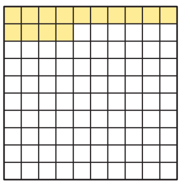
In the second square $\frac{20}{100}$ is shaded.

So, $\frac{1}{5} = \frac{20}{100} = 20\%$.

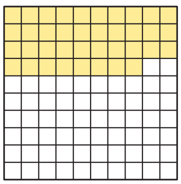
EXERCISE 14B

1 What percentage is represented by the following shaded diagrams?

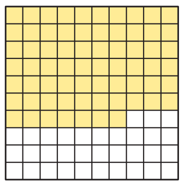
a



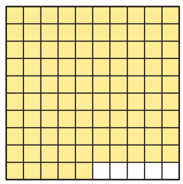
b



c

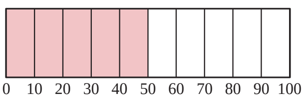


d

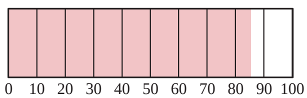


2 Estimate the percentage shaded:

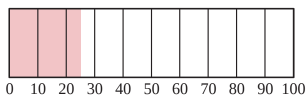
a



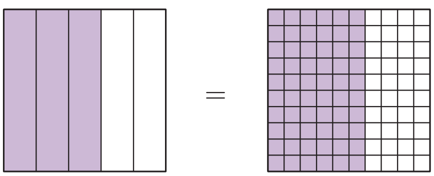
b



c



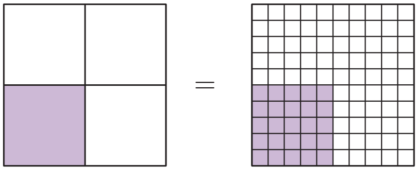
3 **a**



Copy and complete:

$\frac{\square}{\square} = \frac{\square}{100} = \square\%$

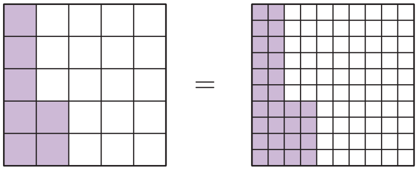
b



Copy and complete:

$\frac{\square}{4} = \frac{\square}{\square} = \square\%$

c



Copy and complete:

$\frac{\square}{\square} = \frac{\square}{\square} = \square\%$

Example 1 — Self Tutor

Write as percentages:

a $\frac{19}{100}$ **b** $\frac{76.8}{100}$ **c** $\frac{557}{1000}$

a $\frac{19}{100}$
$= 19\%$

b $\frac{76.8}{100}$
$= 76.8\%$

c $\frac{557}{1000}$
$= \frac{557 \div 10}{1000 \div 10}$
$= \frac{55.7}{100}$
$= 55.7\%$

4 Write the following fractions as percentages:

a $\frac{31}{100}$ **b** $\frac{3}{100}$ **c** $\frac{37}{100}$ **d** $\frac{54}{100}$
e $\frac{79}{100}$ **f** $\frac{50}{100}$ **g** $\frac{100}{100}$ **h** $\frac{85}{100}$
i $\frac{6.6}{100}$ **j** $\frac{34.5}{100}$ **k** $\frac{75}{1000}$ **l** $\frac{356}{1000}$

Example 2 — Self Tutor

Write as percentages:

a $\frac{2}{5}$ **b** $\frac{13}{25}$

a $\frac{2}{5}$
$= \frac{2 \times 20}{5 \times 20}$
$= \frac{40}{100}$
$= 40\%$

b $\frac{13}{25}$
$= \frac{13 \times 4}{25 \times 4}$
$= \frac{52}{100}$
$= 52\%$

5 Write the following as fractions with denominator 100, and then convert to percentages:

a $\frac{7}{10}$ **b** $\frac{1}{10}$ **c** $\frac{9}{10}$ **d** $\frac{1}{2}$ **e** $\frac{1}{4}$
f $\frac{3}{4}$ **g** $\frac{3}{5}$ **h** $\frac{4}{5}$ **i** $\frac{7}{20}$ **j** $\frac{11}{20}$
k $\frac{7}{25}$ **l** $\frac{19}{25}$ **m** $\frac{23}{50}$ **n** $\frac{47}{50}$ **o** 1

6 Write these statements in full:

a Fourteen percent means fourteen out of every ....

b If 53% of the students in a school are girls, 53% means the fraction $\frac{\cdots\cdots}{\cdots\cdots}$.

7 Refer to the illustration given and then complete the table which follows:

	Students	*Number*	*Fraction*	*Fraction with denom.* 100	*Percentage*
a	wearing shorts				
b	with a ball				
c	wearing skirts				
d	wearing shorts and with a ball				
e	wearing track pants, baseball cap and green top				
f	wearing shorts or track pants				
g	every student in the picture				

Example 3 **Self Tutor**

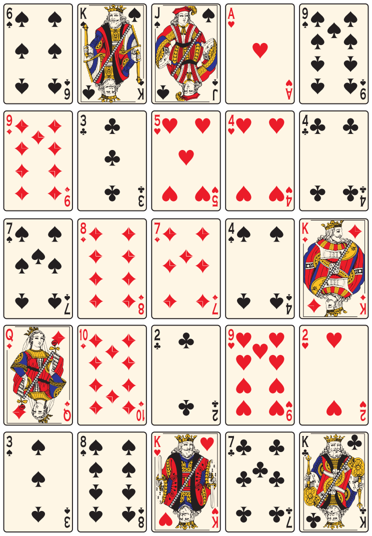
In a class of 25 students, 6 have black hair.
What percentage of the class have black hair?

$$\text{The fraction with black hair} = \frac{6}{25} = \frac{6\times 4}{25\times 4} = \frac{24}{100}$$

So, 24% of the class has black hair.

8 In a class of 25 students, 13 have blue eyes. What percentage of the class have blue eyes?

9 There are 35 basketball players in the Tigers club. 14 of them are boys. What percentage are girls?

10 

A pack of 52 playing cards has been shuffled. You can view the whole pack by clicking on the icon. Suppose the 25 cards shown are dealt from the pack.

a What percentage of the cards shown are:
- **i** hearts
- **ii** black
- **iii** picture cards
- **iv** spades?

b If an ace is 1 and picture cards are higher than 10, what percentage of the cards shown are:
- **i** 10 or higher
- **ii** 5 or lower
- **iii** higher than 5 and less than 10?

c In the full pack of cards, what percentage are:
- **i** red
- **ii** diamonds
- **iii** either spades or clubs?

C CONVERTING PERCENTAGES TO FRACTIONS

Percentages are easily converted into fractions. We first write the percentage as a fraction with a denominator of 100, and then express the fraction in its lowest terms.

Example 4 **Self Tutor**

Express as fractions in lowest terms:

a 70% **b** 85%

a 70% $= \frac{70}{100} = \frac{70 \div 10}{100 \div 10} = \frac{7}{10}$

b 85% $= \frac{85}{100} = \frac{85 \div 5}{100 \div 5} = \frac{17}{20}$

Convert to a fraction with denominator 100, then write in simplest form.

EXERCISE 14C

1 Write as a fraction in lowest terms:

a 43%	**b** 37%	**c** 50%	**d** 30%
e 90%	**f** 20%	**g** 40%	**h** 25%
i 75%	**j** 95%	**k** 100%	**l** 3%
m 5%	**n** 44%	**o** 37%	**p** 80%
q 99%	**r** 21%	**s** 32%	**t** 15%
u 200%	**v** 350%	**w** 125%	**x** 800%

Example 5 — Self Tutor

Express 2.5% as a fraction in lowest terms.

$$\begin{aligned} & 2.5\% \\ = & \frac{2.5}{100} \\ = & \frac{2.5 \times 10}{100 \times 10} \quad \{\text{to remove the decimal}\} \\ = & \frac{25}{1000} \\ = & \frac{25 \div 25}{1000 \div 25} \\ = & \frac{1}{40} \end{aligned}$$

2 Write as a fraction in lowest terms:

- **a** 12.5%
- **b** 7.5%
- **c** 0.5%
- **d** 17.3%
- **e** 97.5%
- **f** 0.2%
- **g** 0.05%
- **h** 0.02%

D CONVERTING DECIMALS TO PERCENTAGES

To write a decimal number as a percentage we **multiply it by 100%**.

Since $100\% = \frac{100}{100} = 1$, multiplying by 100% is the same as multiplying by 1. We therefore do not change the value of the number.

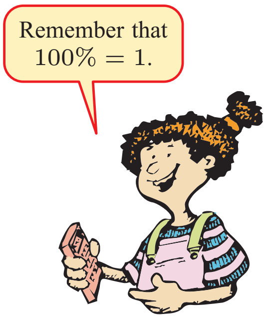
Example 6 — Self Tutor

Convert to a percentage by multiplying by 100%:

a 0.27 **b** 0.055

a 0.27
$= 0.27 \times 100\%$
$= 27\%$

b 0.055
$= 0.055 \times 100\%$
$= 5.5\%$



Another way of converting a **fraction to a percentage** is to first convert it to a decimal.

Example 7 — Self Tutor

Change to percentages by multiplying by 100%:

a $\frac{4}{5}$ **b** $\frac{3}{4}$

a $\frac{4}{5}$
$= 0.8$
$= 0.8 \times 100\%$
$= 80\%$

b $\frac{3}{4}$
$= 0.75$
$= 0.75 \times 100\%$
$= 75\%$

EXERCISE 14D

1 Convert into percentage form by multiplying by 100%:

a 0.37 **b** 0.89 **c** 0.15 **d** 0.49
e 0.73 **f** 0.05 **g** 1.02 **h** 1.17

2 Convert into percentage form by multiplying by 100%:

a 0.2 **b** 0.7 **c** 0.9 **d** 0.4
e 0.074 **f** 0.739 **g** 0.0067 **h** 0.0018

3 Convert to a percentage by first writing as a decimal:

a $\frac{1}{10}$ **b** $\frac{8}{10}$ **c** $\frac{4}{10}$ **d** $\frac{3}{5}$
e $\frac{2}{5}$ **f** $\frac{1}{2}$ **g** $\frac{3}{20}$ **h** $\frac{1}{4}$
i $\frac{19}{20}$ **j** $\frac{3}{50}$ **k** $\frac{39}{50}$ **l** $\frac{17}{25}$
m $\frac{3}{8}$ **n** 1 **o** $\frac{11}{100}$ **p** $\frac{7}{8}$
q $\frac{1}{3}$ **r** $\frac{2}{3}$

4 Copy and complete these patterns:

a 1 is 100%	**b** $\frac{1}{5} = 20\%$	**c** $\frac{1}{3}$ is $33\frac{1}{3}\%$	**d** $\frac{1}{4}$ is ......
$\frac{1}{2}$ is 50%	$\frac{2}{5} =$	$\frac{2}{3}$ is	$\frac{2}{4} = \frac{1}{2}$ is ....
$\frac{1}{4}$ is......	$\frac{3}{5} =$	$\frac{3}{3}$ is	$\frac{3}{4} =$ ......
$\frac{1}{8}$ is......	$\frac{4}{5} =$		$\frac{4}{4} =$ ......
$\frac{1}{16}$ is......	$\frac{5}{5} =$		

E CONVERTING PERCENTAGES TO DECIMALS

To write a percentage as a decimal number, we **divide by 100%**.

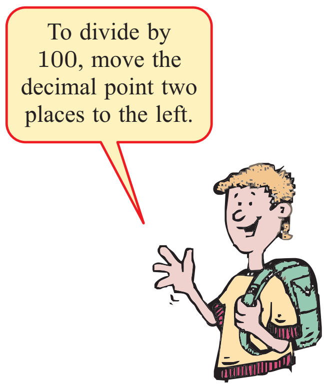
To achieve this we can first write the percentage as a common fraction with denominator 100.

Example 8 Self Tutor

Write as a decimal:
a 21% **b** $12\frac{1}{2}\%$

a 21%
$= \frac{21}{100}$
$= \frac{21.}{100}$
$= 0.21$

b $12\frac{1}{2}\%$
$= 12.5\%$
$= \frac{12.5}{100}$
$= \frac{12.5}{100}$
$= 0.125$



It is worthwhile remembering the conversions in the following table:

Percentage	*Common Fraction*	*Decimal Fraction*
100%	1	1.0
75%	$\frac{3}{4}$	0.75
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
10%	$\frac{1}{10}$	0.1

Percentage	*Common Fraction*	*Decimal Fraction*
5%	$\frac{1}{20}$	0.05
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0.\overline{3}$
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0.\overline{6}$
$12\frac{1}{2}\%$	$\frac{1}{8}$	0.125
$6\frac{1}{4}\%$	$\frac{1}{16}$	0.0625
$\frac{1}{2}\%$	$\frac{1}{200}$	0.005

EXERCISE 14E

1 Write as a decimal:

a 50% **b** 30% **c** 25% **d** 60%
e 85% **f** 5% **g** 45% **h** 42%
i 15% **j** 100% **k** 67% **l** 125%

2 Write as a decimal:

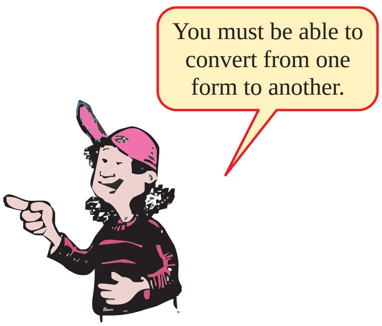
a 7.5% **b** 18.3% **c** 17.2% **d** 106.7%
e 0.15% **f** 8.63% **g** $37\frac{1}{2}\%$ **h** $6\frac{1}{2}\%$
i $\frac{1}{2}\%$ **j** $1\frac{1}{2}\%$ **k** $\frac{3}{4}\%$ **l** $4\frac{1}{4}\%$

3 Copy and complete the table below:

	Percent	*Fraction*	*Decimal*
a	20%		0.2
b	40%	$\frac{2}{5}$	
c			0.5
d		$\frac{3}{4}$	
e			0.85
f		$\frac{2}{25}$	

	Percent	*Fraction*	*Decimal*
g			0.35
h	12.5%		
i		$\frac{5}{8}$	
j	100%		
k		$\frac{3}{20}$	
l			0.375

4 **a** Write 45% as a fraction and as a decimal. The fraction must be in simplest form.
b Write $\frac{7}{25}$ as a decimal and as a percentage.
c Write $\frac{1}{5}\%$ as a decimal number and as a fraction.
d Write 250% as a decimal and as a fraction.



F PLOTTING NUMBERS ON A NUMBER LINE

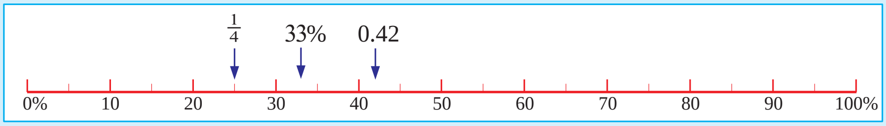
Plotting numbers on a number line can be difficult, especially when the numbers are given as a mixture of fractions, decimals, and percentages. However, we can make the comparison easier by converting all fractions and decimals to percentages.

Example 9 **Self Tutor**

Convert $\frac{1}{4}$, 0.42, and 33% to percentages and plot them on a number line.

- $\frac{1}{4} \times 100\% = 25\%$
- $0.42 \times 100\% = 42\%$
- 33% is already a percentage

We use the percentages to arrange the numbers in order from lowest to highest.



EXERCISE 14F

1 Convert each set of numbers to percentages and plot them on a number line:

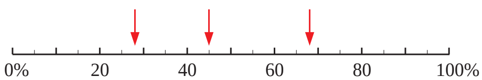
a $\frac{3}{5}$, 70%, 0.65 **b** 55%, $\frac{9}{20}$, 0.83 **c** 0.93, 79%, $\frac{17}{20}$

d 0.85, $\frac{3}{4}$, 92% **e** $\frac{27}{50}$, 67%, 0.59 **f** 47%, 0.74, $\frac{18}{30}$

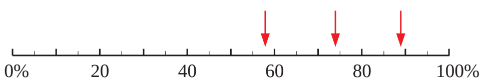
g $\frac{3}{4}$, 0.65, 42% **h** 0.39, 58%, $\frac{7}{20}$, $\frac{2}{5}$ **i** $\frac{5}{8}$, 73%, $\frac{13}{20}$, 0.47

2 Write each of the following number line positions as fractions with denominator 100, as decimals, and also as percentages:

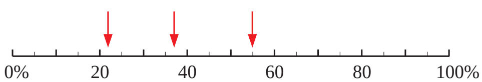
a

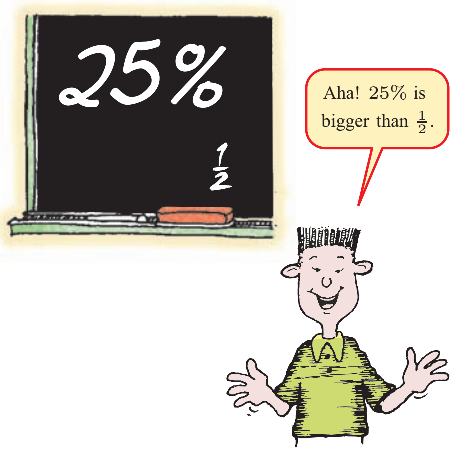


b



c





3 Comment on the cartoon opposite.

G SHADED REGIONS OF FIGURES

When we shade regions of figures to illustrate percentages, it is important that the region is the correct size.

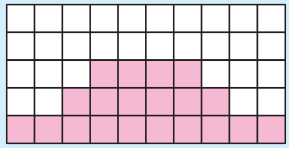
In some cases we may divide the figure into a number of equal parts, and then shade the appropriate number of them.

Example 10 **Self Tutor**

For the given figure:

a what fraction of the figure is unshaded

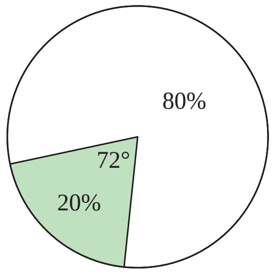
b what percentage of the figure is unshaded?



a There are 50 squares in total.
30 squares are unshaded.
So, $\frac{30}{50} = \frac{3}{5}$ is unshaded.

b $\frac{3}{5} \times 100\% = 60\%$
So, 60% is unshaded.

When we divide up a circle, we need to remember there are 360^o in a full turn.

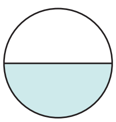
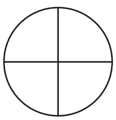
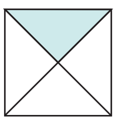


Suppose we wish to shade 20% of a circle.

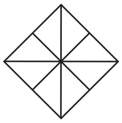
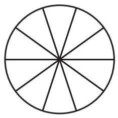
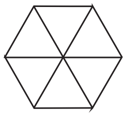
If 100% is 360^o then 1% is 3.6^o and so 20% is 72^o.

EXERCISE 14G

1 Copy and complete the following table, filling in the shading where necessary:

	Figure	*Fraction shaded*	*Percentage shaded*	*Percentage unshaded*
a				
b		$\frac{3}{4}$		
c				

PRINTABLE WORKSHEET

	Figure	Fraction shaded	Percentage shaded	Percentage unshaded
d			25%	
e				30%
f		$\frac{1}{6}$		

2 Construct a square with 10 cm sides. Divide it into 100 squares with 1 cm sides.

- **a** How many squares must you shade to leave 65% unshaded?
- **b** In lowest terms, what fraction of the largest square is then unshaded?

3 Construct a rectangle 10 cm by 5 cm. Divide it into 1 cm squares. Shade 7 squares blue, 9 squares red, and 20 squares yellow. What percentage of the rectangle is:

a red **b** blue **c** unshaded **d** either red or blue?

4 Use a compass to draw a circle.
Colour 50% of your circle red, 25% blue, 10% orange, 5% green, 5% purple, 5% yellow.
What fraction of the whole circle is now:

a blue **b** red **c** orange
d orange or blue **e** red or purple or yellow **f** coloured?

5 Divide a circle into 5 equal sectors of 20%. Colour $\frac{1}{5}$ of the circle red, 40% yellow, $\frac{1}{5}$ blue, and 20% green. If you drew 4 such identical circles:

- **a** what percentage of *all* the circles would be **i** blue **ii** red
- **b** what fraction of the 4 circles would be yellow?

6 Click on the icon for a worksheet which gives more practice questions like those in question **1**.

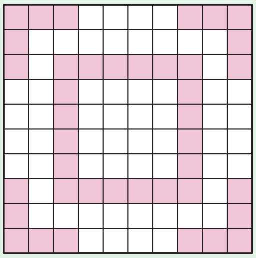
PRINTABLE WORKSHEET

KEY WORDS USED IN THIS CHAPTER

- decimal
- denominator
- fraction
- lowest terms
- number line
- numerator
- percentage

REVIEW SET 14A

1 In the following pattern there are 100 tiles.

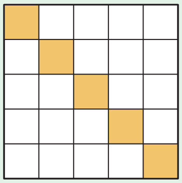


a Write the number of coloured tiles as a fraction of 100.

b Write the number of coloured tiles as a percentage of the total number of tiles.

2 Find the percentage of the numbers from 1 to 100 inclusive, which are exactly divisible by 12.

3 What percentage of the diagram is unshaded?



4 **a** Write $\frac{3}{4}$ as a percentage.

b Write 66% as a fraction in lowest terms.

c Convert 0.125 into a percentage.

d Write 82% as a decimal.

5 **a** Write $\frac{7}{25}$ with a denominator of 100.

b Change $\frac{1}{3}$ to a percentage.

c Convert 2.15 into a percentage.

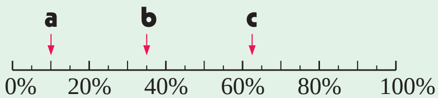
6 In a group of 40 children, 14 are allergic to peanuts. What percentage of the group are not allergic to peanuts?

7 Write as a decimal:

a 81% **b** 10.8% **c** $8\frac{1}{2}\%$

8 Write $\frac{3}{4}$, 0.78 and 72% as percentages and then plot them on a number line.

9 Write the number line positions indicated as fractions with denominator 100, as decimals, and using % notation:



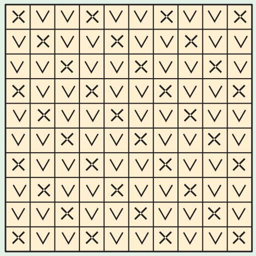
10 Construct a square with 5 cm sides. Divide it into 1 cm squares.

a How many squares must you shade in order to leave 20% unshaded?

b In lowest terms, what fraction of the original square is then unshaded?

REVIEW SET 14B

1 In the following pattern there are 100 symbols.

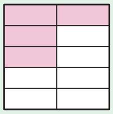
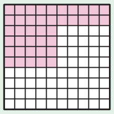


a Count the number of each type of symbol.

b Write the number of each type as a fraction of the total.

c Write the number of each type as a percentage of the total.

2 Find the percentage of numbers from 1 to 100 inclusive, which contain the digit 4.

3 If  = , copy and complete: $\frac{\square}{10} = \frac{\square}{\square} = \square\%$

4 **a** Write 0.47 as a percentage.

b Write 40% as a fraction in lowest terms.

c Write $\frac{2}{3}$ as a percentage.

d Write $12\frac{1}{2}\%$ as a decimal.

5 Write the following fractions as percentages:

a $\frac{27}{100}$ **b** $\frac{3.5}{100}$ **c** $\frac{18}{25}$ **d** $\frac{13}{20}$

6 45 students decided to attend the annual quiz night. 27 of them won at least one prize during the night. What percentage of the students won at least one prize?

7 Write as a fraction in lowest terms:

a 16% **b** 250% **c** 8.5% **d** 0.01%

8 **a** Convert 0.45 to a percentage.

b Write 5.79% as a decimal.

9 Convert to percentages and plot on a number line:

a $\frac{2}{5}$, 0.75, and 56% **b** $\frac{1}{8}$, 52%, and 0.8

10 Use a compass to draw a circle. Divide the circle into 8 equal sectors of 12.5%. Colour $\frac{1}{8}$ of the circle blue, 25% red, $\frac{3}{8}$ white and $\frac{1}{4}$ green. If you drew 4 such identical circles:

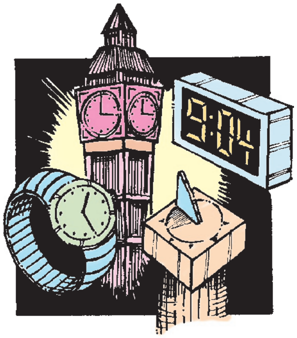
a what percentage of all 4 circles would be white

b what fraction of all 4 circles would be red?

Chapter 15

Time and temperature

Contents:

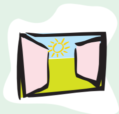


For most of us, time seems to control our lives.

Questions like the following all involve time:

How long until the bus leaves?
What time do you have to be at school?
When did the Second World War finish?
For how long did Elizabeth I reign in England?
What time does the ice hockey start?
How long will it take us to travel to Tokyo?

OPENING PROBLEM



Simran is sitting in London Heathrow airport. His flight to his home in Mumbai departs at 10:15 pm. It is 7163 km from London to Mumbai, and the flight is scheduled to take 9 hours.

Things to think about:

- What is the *average speed* of the scheduled flight?
- What is the *time difference* between London and Mumbai?
- When the flight takes off in London, what is the time in Mumbai?
- What is the *local time* in Mumbai when the plane is scheduled to land?

ACTIVITY 1 — TIME OUT

Work in small groups to discuss the following topics.

What to do:

1 List ways that the following would be aware of time changes:

a animals in the wild
b domestic animals
c human infants
d farmers
e sailors at sea 300 years ago

2 Outline some of the problems people may have had in measuring time using:

a candles
b water clocks
c shadow sticks
d sundials
e sand-glasses
f pendulum clocks
g mechanical clocks

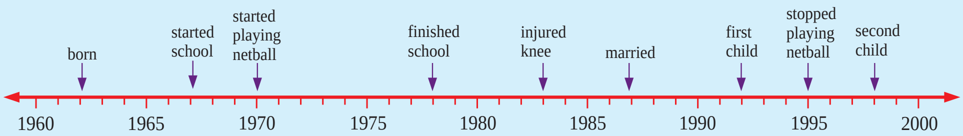
3 List 20 occupations where time or timing is very important, for example musicians and restaurant chefs. For each of the occupations listed, write 2 consequences for wrong timing.

A TIME LINES

Time lines are simple graphs which display times or dates, and key events that correspond to these times.

Example 1 — Self Tutor

The following time line shows some of the important dates in Sarah's life:



Use the time line to determine:

a when Sarah

i was born **ii** was married **iii** finished school **iv** injured her knee

b for how long Sarah played netball

c the age difference between Sarah's two children.

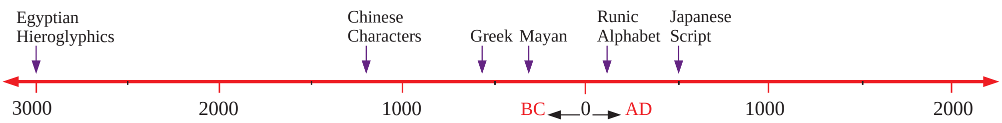
a **i** 1962 **ii** 1987 **iii** 1978 **iv** 1983

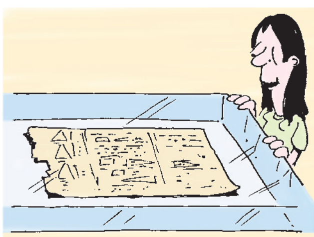
b She started in 1970 and finished in 1995, so she played for 25 years.

c Her first child was born in 1992 and the second was born in 1998, so there is 6 years' difference in age.

EXERCISE 15A

1 The following time line shows when various methods of writing first appeared:



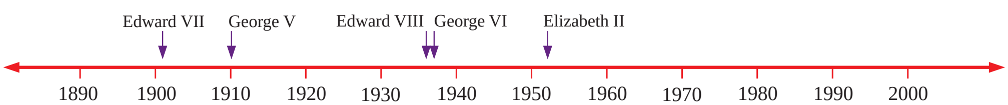


a Estimate when the Runic Alphabet first appeared.

b How long was there between the appearance of Egyptian Hieroglyphics and Chinese Characters?

c Which form of writing appeared most recently?

2 The following time line shows the monarchs of England during the 20th century:

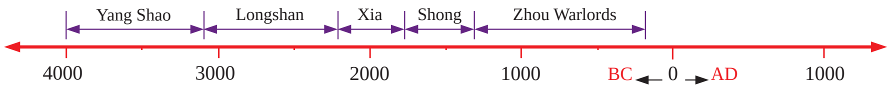


a Which monarch reigned longest in the 20th century?

b How long did the reign of George VI last?

c How much longer was the reign of Edward VII than Edward VIII?

3 The following time line shows the period of various Chinese civilizations:



Use the time line and a ruler to:

a find the Chinese civilization which lasted longest

b determine the period for which the Xia dynasty lasted

c find how much longer the Longshan civilization lasted compared with the Shong civilization.

ACTIVITY 2 DEVELOPING TIME LINES

The purpose of this activity is to draw at least one time line based on your research of relevant information.

What to do:

1 Decide on a suitable topic to research. Here are some suggestions, but you should research a topic which particularly interests you:

- Presidents of the USA
- Wars of the 20th century
- Significant dates in your family's history, such as birthdays, marriages, and special holidays.

2 Choose a suitable scale for your time line and complete it with the relevant information.

3 Write down 4 questions based on your time line, and ask another student to answer them.

ACTIVITY 3 ESTIMATING A MINUTE

In small groups or as a whole class, arrange a competition to find who is best at estimating one minute.

One student or the teacher has a watch and others try to estimate a minute without seeing a watch or clock.

B UNITS OF TIME

We are all familiar with the concept of time and the measurement of time in years, months, weeks, days, hours, minutes, and seconds.

RELATIONSHIP BETWEEN TIME UNITS

1 year	1 week = 7 days
= 12 months	1 day = 24 hours
= 52 weeks (approximately)	1 hour = 60 minutes
= 365 days (or 366 in a leap year)	1 minute = 60 seconds

The number of days in the month varies:

January	31	May	31	September	30
February	28 (29 in a leap year)	June	30	October	31
March	31	July	31	November	30
April	30	August	31	December	31

A **leap year** occurs if the year is divisible by 4 but not by 100, except if the year is divisible by 400. For example:
1996 was a leap year
2000 was a leap year
2100 will not be a leap year.

Long times

1 decade = 10 years
1 century = 100 years
1 millennium = 1000 years

Short times

1 millisecond = $\frac{1}{1000}$ of a second
1 microsecond = $\frac{1}{1\,000\,000}$ of a second
1 nanosecond = $\frac{1}{1\,000\,000\,000}$ of a second

Example 2 — Self Tutor

Convert 3 days, 9 hours and 42 minutes to minutes.

3 days
$= 3 \times 24$ hours
$= 3 \times 24 \times 60$ min
$= 4320$ min

9 hours
$= 9 \times 60$ min
$= 540$ min

3 days	4320
9 hours	540
+ 42 mins	42
Total	4902 min

EXERCISE 15B

1 Convert to minutes:

- **a** 7 hours 24 min
- **b** 3 days 5 hours 43 min
- **c** 12 days 15 hours 36 min
- **d** 2 weeks 3 days 8 hours 17 min

2 Convert to seconds:

- **a** 40 min 38 s
- **b** 3 h 35 min 27 s
- **c** 14 h 12 min 43 s
- **d** 22 h 52 min 11 s

3 Find the number of minutes in:

- **a** one day
- **b** one week
- **c** one 365 day year

4 Find the number of seconds in:

- **a** one day
- **b** one fortnight
- **c** one 365 day year

5 Consider a four year period which includes a leap year. Find the number of:

- **a** days
- **b** hours
- **c** minutes

in this period.

6 Find the following:

- **a** 3 h 7 min + 5 h 23 min
- **b** 5 h 17 min + 3 h 25 min + 4 h 35 min
- **c** 7 h 53 min − 3 h 36 min
- **d** 11 h 43 min + 2 h 24 min + 5 h 16 min
- **e** 17 h 42 min − 12 h 53 min
- **f** 10 h 32 min + 5 h 47 min − 7 h 57 min

Example 3 — Self Tutor

Convert 1635 hours to days and hours.

Using a calculator, $\frac{1635}{24} = 68.125$

So, there are 68 days and 0.125 of a day remaining.

$0.125 \times 24 = 3$ hours, so the answer is 68 days, 3 hours.

Click on the icon to load a game for improving your time estimation skills.

7 Convert to days and hours:

- **a** 124 h
- **b** 552 h
- **c** 873 h
- **d** 2167 h

8 Convert:

- **a** 67 680 min to days
- **b** 31 717 min to days, hours and minutes

RESEARCH — MEASURING TIME IN DIFFERENT CULTURES

What to do:

1. Research the development of the current Western calendar.
2. Find out what the Gregorian calendar is.
3. Compare the calendars of Christian, Muslim, Jewish and Chinese people.
4. List the days of great importance to these people and write them on a single calendar.

C DIFFERENCES IN TIME

Calculating the difference between two times or dates is very important. Time difference calculations are frequently made by travellers, credit card owners, and businesses.

Examples of date notation:

31/12/08	Australia
31.12.08	Germany
12.31.08	USA
2008-12-31	ISO standard

Dates are often represented using three numbers, though the order in which they are written differs from place to place. In this course we write dates in the form (day)/(month)/(year). So, the date 13/6/08 is the shorthand way of writing "the thirteenth day of the sixth month (June) of the year 2008".

Example 4 — Self Tutor

Karaline will turn 17 on 23/4/10. What does this mean?

Karaline's 17th birthday is on the 23rd of April 2010.

Example 5 — Self Tutor

How many days is it from April 24th to July 17th?

April has 30 days, so there are $30 - 24 = 6$ days remaining in April.

	April	6
	May	31
	June	30
+	July	17
	Total	84 days

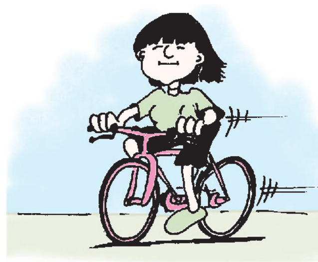
EXERCISE 15C

1 Write out in sentence form the meaning of:

- **a** Wei joined the club on 17/12/07
- **b** Jon arrived on 13/3/06
- **c** Piri is departing for Malaysia on 30/7/10
- **d** Sam will turn 21 on 28/5/14

2 Find the number of days from:

- **a** March 11th to April 7th
- **b** May 11th to June 23rd
- **c** July 12th to November 6th
- **d** September 19th to January 8th
- **e** January 7th to March 16th in a non-leap year
- **f** February 6th to August 3rd in a leap year
- **g** 6/7/12 to 2/11/12
- **h** 7/2/13 to 17/5/13



3 Lou Wong is saving money to buy a bicycle costing $279. Today is the 23rd of March and the shop will hold the bicycle until May 7th at this price.

- **a** How many days does Lou have to save for the bicycle?
- **b** How much needs to be saved each day to reach the $279 target?

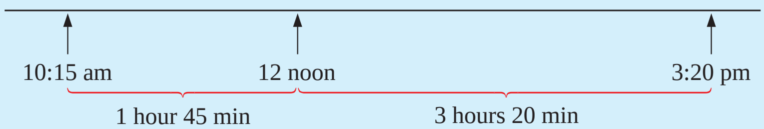
4 Sean can save €18 a day. Today is May 9th, and on November 20th he wishes to travel to Fiji on a package deal costing €5449. If he does not reach the target of €5449 he will have to borrow the remainder from a bank.

- **a** How many days does he have available for saving?
- **b** What is the total he will save?
- **c** Will he need to borrow money? If so, how much?

5 On 17th July 2005 Sung Kim proudly announced that he had been in New Zealand for 1000 days. On what day did he arrive in New Zealand?

Example 6 **Self Tutor**

It is now 10:15 am and the plane departs at 3:20 pm. How long is it before departure?



Time before departure = time before noon + time afternoon
= 1 hour 45 min + 3 hours 20 min
= 4 hours 65 min
= 5 hours 5 min

	h	min
	1	45
+	3	20
	4_1	65_5
	5	5

Example 7 **Self Tutor**

What is the time 3 hours 40 minutes before 9:15 am?

The time is 9:15 − 3 hours 40 minutes
= 9 hours + 15 min − 3 hours − 40 min
= 6 hours + 15 min − 40 min
= 5 hours + 60 min + 15 min − 40 min
= 5 hours + 35 mins

	h	min
	~~9~~ 8	~~15~~ 75
−	3	40
	5	35

So, the time is 5:35 am.

6 What is the time:

a 4 hours after 3:00 am	**b** 5 hours after 8:00 pm
c 34 minutes after 6:15 am	**d** 45 minutes after 7:21 pm
e 2 hours 13 min after 8:19 pm	**f** 3 hours 27 min after 12:42 pm
g 2 hours 55 min before 2 pm	**h** 5 hours 18 minutes before noon
i 1 hour 47 mins before 1:30 pm	**j** 3 hours 16 minutes before 2 am Mon?

7 What is the time difference from:

a 3:24 am to 11:43 am	**b** 7:36 pm to 10:55 pm
c 8:29 am to 3:46 pm	**d** 5:32 am to 6:24 pm
e 3:18 pm to 11:27 am next day	**f** 4:29 pm to 2:06 am next day
g 2:23 pm Sun to 5:11 pm Mon	**h** 3:42 am Tues to 7:36 pm Fri?

8 If a courier travelling between two cities takes $1\frac{1}{4}$ hrs for a one way trip, how many round trips can she do in an 8 hour working day?

9 Herbert was born in 1895. How old was he when he had his birthday in 1920?

10 It takes Jill 4 seconds to put each can into her supermarket display. If there are 120 cans to be displayed, how long will it take Jill to complete the job?

11 Mary's watch loses 3 seconds every hour. If it shows the correct time at 8 am on Wednesday, how slow will it be when the real time is 5 pm on Friday of the same week?

12 A high tide happens every 6 hours and 20 minutes. If the next high tide is at 1:25 am on Monday, list the times for the next 8 high tides after that one.

13 How many times will the second hand of a clock pass 12 in a 24 hour period starting at 11:30 pm?

D READING CLOCKS AND WATCHES

ACTIVITY 4 HOW LONG DOES IT TAKE?

You will need:

A stopwatch or digital watch with similar functions and a partner to work with.

What to do:

1. Become familiar with what the watch can do and how you should operate it.
2. Read through the list of activities.
3. Organise how and when you can do them together.

4 Prepare your own copy of the chart.
5 Estimate the times you think it will take to do each activity.
6 Take it in turns to do the activity or operate the watch.
7 Find the difference between your estimated time and the actual time it takes for each activity. Who is more accurate?

Activity	*Estimated Time*	*Actual Time*	*Difference*
Count from 1 to 200 by ones			
Accurately write down your 8 and 9 times tables			
Carefully read aloud one page from a novel			
Walk 100 metres			
Record the total commercial time in 30 min of TV			
Make a cup of coffee			

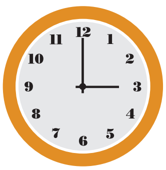
How many times each day do you look at a clock, watch or timetable? There are many occasions each day when we need to determine 'the time'. For example:

- the time that lesson 2 starts today
- the time when the next bus or train departs
- the time dinner will be served tonight.

12-HOUR CLOCKS

Traditional analogue clocks give us 12-hour time.

For example:



reads 3 o'clock and could be 3:00 am or 3:00 pm

reads 20 minutes past 8 o'clock and could be either 8:20 am or 8:20 pm.

am stands for *ante meridiem* which means 'before the middle of the day'.
pm stands for *post meridiem* which means 'after the middle of the day'.

24-HOUR CLOCKS

24-hour time is often used in transport timetables such as railway and airport schedules. It is used to avoid confusion between am and pm times.

For example, on a 12-hour clock **8:15** could mean 8:15 am or 8:15 pm.

A 24-hour clock overcomes this problem by displaying **8:15** for the morning (am) and **20:15** for the afternoon (pm).

20:15 means 20 hours 15 minutes since midnight, which is 8 hours 15 minutes since midday, that is, 8:15 pm.

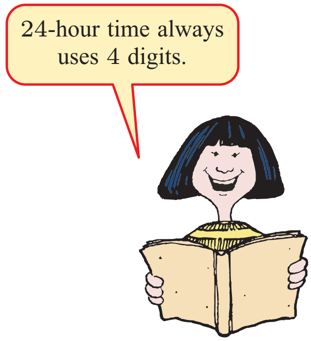
FOUR DIGIT NOTATION

7:15 am appears as 7:15 and is written as 0715 hours.

8:15 pm appears as 20:15 and is written as 2015 hours.

In the following examples we compare 12-hour time and 24-hour time.

12-*hour time*	*Digital display*	24-*hour time*
midnight	0:00	0000 hours
7:42 am	7:42	0742 hours
midday (noon)	12:00	1200 hours
11:29 pm	23:29	2329 hours



Example 8 **Self Tutor**

a Convert 7:38 am into 24-hour time.

b Convert the digital display 17:38 into 12-hour time.

a 7:38 am is 0738 hours

b 17:38 is 5:38 pm

EXERCISE 15D

1 Write as 24-hour time:

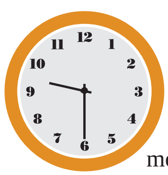
a 3:13 am **b** 11:17 am **c** midnight **d** 12:47 pm
e 5:41 pm **f** noon **g** 8:19 pm **h** 11:59 pm

2 Write the following 24-hour times as 12-hour times:

a 0300 hours **b** 0630 hours **c** 1800 hours **d** 1200 hours
e 0615 hours **f** 1545 hours **g** 2017 hours **h** 2348 hours

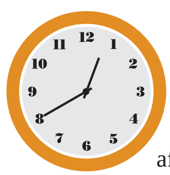
3 Write the following analogue times as 24-hour times:

a



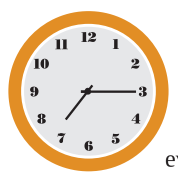
morning

b



afternoon

c



evening

4 What, if anything, is wrong with the following 24-hour times:

a 0862 hours **b** 0713 hours **c** 2541 hours?

5 The following arrivals appear on a display at Singapore Changi Airport:

a Convert each arrival time into 12-hour time.

b At what time is the Singapore Airlines flight from Rome arriving?

c If a thunderstorm delays all arrivals to the airport by 1 hour 35 minutes, what new arrival time is expected for:

i BA10 **ii** QF14

ARRIVALS		
Flight	*From*	*Arr. Time*
JAL130	Tokyo	14:50
BA10	London	15:50
SQ71	Rome	16:25
QF14	Perth	16:45
EM16	Dubai	17:15

E TIMETABLES

Timetables are tables of information which tell us when events are to occur.

The timetable alongside gives information about phases of the moon and the rising and setting of the planets of our solar system. We can observe, for example, that:

- the next full moon is on October the 6th
- Mercury rises at 5:59 am tomorrow
- Saturn sets at 8:23 pm tomorrow.

The Moon

New	First $\frac{1}{4}$	Full	Last $\frac{1}{4}$
Sep 21	Sep 29	Oct 6	Oct 12

The Sun and Planets

Tomorrow	Rise	Set
Sun	6:15 am	6:07 pm
Moon	2:29 am	1:00 pm
Mercury	5:59 am	5:20 pm
Venus	5:51 am	5:09 pm
Mars	4:43 am	3:11 pm
Jupiter	6:01 am	6:34 pm
Saturn	9:10 am	8:23 pm

EXERCISE 15E

1 This timetable shows tide times on a particular day.

Tide times				
Port Xenon	12:55 AM	0.8 m	7:21 AM	2.5 m
	1:56 PM	1.2 m	7:13 PM	1.8 m
Port Dowell	5:15 AM	1.4 m	11:45 AM	1.1 m
	1:46 PM	1.2 m	9:22 PM	0.6 m
Windcok	2:47 AM	0.9 m	9:53 AM	2.4 m
	5:12 PM	1.2 m	8:41 PM	1.3 m
Joseph's Bay	3:20 AM	0.9 m	10:22 AM	2.6 m
	5:24 PM	1.3 m	9:13 PM	1.5 m
Paradise Point	3:22 AM	1.4 m	7:57 AM	0.9 m
	12:19 PM	1.3 m	9:08 PM	0.6 m
Sunny Inlet	12:29 AM	0.5 m	9:03 AM	1.3 m
	11:29 PM	0.4 m		

a When is the tide highest in the morning at Port Xenon?

b When is the tide lowest in the afternoon at Paradise Point?

c What is the lowest tide at Joseph's Bay in the morning and at what time does it occur?

d What is the highest tide at Port Dowell in the afternoon and at what time does it occur?

2 Consider the timetable for an Adelaide tourist bus service in the summer season:

a How many bus services are available?

b What is the latest departure time?

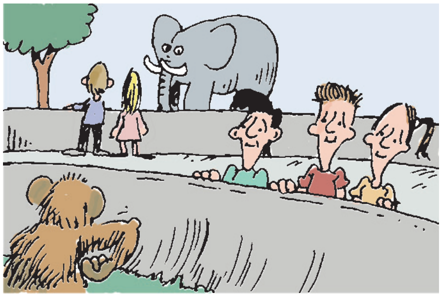
c What is the earliest arrival time back at the depot?

d How long does it take between arrivals at

i the Adelaide Zoo and the Stonyfell Winery

ii the Murray Mouth and Victor Harbor?

Departure Times	*Bus A*	*Bus B*	*Bus C*	*Bus D*	*Bus E*	*Bus F*
City depot	7:30	7:45	8:00	8:15	8:30	8:45
Adelaide Oval	7:40	7:55	8:10	8:25	8:40	8:55
Adelaide Zoo	8:20	8:35	8:50	9:05	9:20	9:35
Stonyfell Winery	10:15	10:30	10:45	11:00	11:15	11:30
Hahndorf	11:20	11:35	11:50	12:05	12:20	12:35
River Murray Mouth	1:00	1:15	1:30	1:45	2:00	2:15
Victor Harbor	1:40	1:55	2:10	2:25	2:40	2:55
Port Adelaide	3:15	3:30	3:45	4:00	4:15	4:30
The Museum	4:00	4:15	4:30	4:45	5:00	5:15
Arrive at City depot	5:00	5:15	5:30	5:45	6:00	6:15



e How long does a complete trip last?

f If you wanted to be at Victor Harbor no later than 2:00, what bus should you take?

g If a friend is meeting the bus at Port Adelaide at 3:30, what bus is it best to travel on?

3 Consider the given Carlingford to Wynyard train timetable:

a What does it mean by:

i arr **ii** dep?

b If I catch the 4:17 pm train at Rydalmere, what time will I arrive at Central?

c At what time will I have to catch the train from Dundas in order to arrive at Lidcombe by 6:00 pm?

d If I miss the 5:00 pm train from Clyde, what is the earliest time I can now arrive at Wynyard?

CARLINGFORD-WYNYARD TRAIN TIMETABLE

		p.m.	p.m.	p.m.	p.m.	p.m.	p.m.	p.m.
Carlingford		3.32	4.11	4.45	5.23	5.55	6.26	6.52
Telopea		3.34	4.13	4.47	5.25	5.57	6.28	6.54
Dundas		3.36	4.15	4.49	5.27	5.59	6.30	6.56
Rydalmere		3.38	4.17	4.51	5.29	6.01	6.32	6.58
Camellia		3.40	4.19	4.53	5.31	6.03	6.34	7.00
Rosehill UA		3.42	4.21	4.55	5.33	6.05	6.36	7.02
Clyde ………	**arr**	3.45X	4.24X	4.58X	5.36X	6.08X	6.39X	7.05
	dep	3.51	4.26	5.00	5.48	6.18	6.48	7.06
Lidcombe…..	**arr**							
	dep	3.57	4.31	5.06	5.54	6.24	6.54	7.12
Strathfield….	**arr**	4.02	4.36	5.11	5.59	6.29	6.59	7.18X
	dep	4.03	4.37	5.12	6.00	6.30	7.00	7.23
Central……..	**arr**	4.17	4.50	5.26	6.14	6.44	7.14	7.36
	dep	4.18	4.51	5.27	6.15	6.45	7.15	7.37
Townhall		4.21	4.54	5.30	6.18	6.48	7.18	7.40
Wynyard		4.24	4.57	5.33	6.20	6.50	7.20	7.42

e **i** If I come out of the cinema at 3:45 pm at Carlingford, what is the time of the first train that I can catch to Strathfield?

ii At what time will this train reach Strathfield?

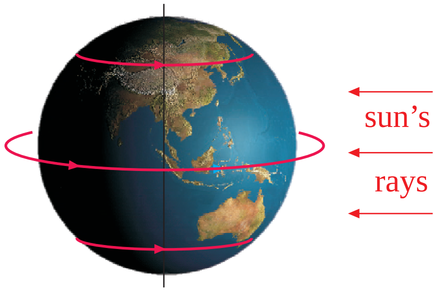
iii I have an errand at Strathfield that will take no more than half an hour. What is the shortest time that I will have to wait for the next train that I can catch to Townhall?

f Calculate the time it takes for the **i** 3:32 pm **ii** 5:23 pm **iii** 6:52 pm trains from Carlingford to reach Central. Can you suggest a reason for the differences in times?

F TIME ZONES

The Earth rotates from west to east about its axis. This rotation causes day and night.

As the sun rises in the Chinese city of Beijing, India is still in darkness. It is therefore later in the day in Beijing. By the time the sun rises in India, Beijing has already experienced $2\frac{1}{2}$ hours of daylight.



TRUE LOCAL TIME

The Earth rotates a full 360^o every day.

This is 360^o in 24 hours
or 15^o in 1 hour $\{360 \div 24 = 15\}$
or 1^o in 4 minutes. $\{\frac{1}{15}$ of 60 min $= 4$ min$\}$

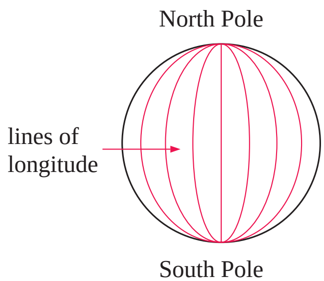
Places on the same **line of longitude** share the same **true local time**.

However, using true local time would cause many problems.

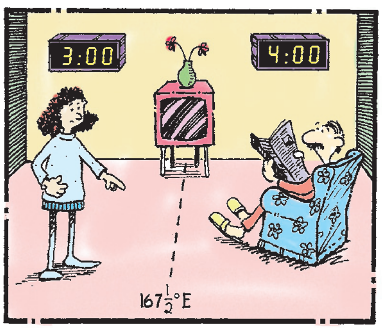
For example, any movement to the east or west would mean the time would be different, so the time on one side of a city would be different to the time on the other side.

Some states of the USA and Australia stretch across almost 15^o of longitude. This would mean time differences of up to one hour within the same state.

The solution to these problems was to create **time zones**.



STANDARD TIME ZONES

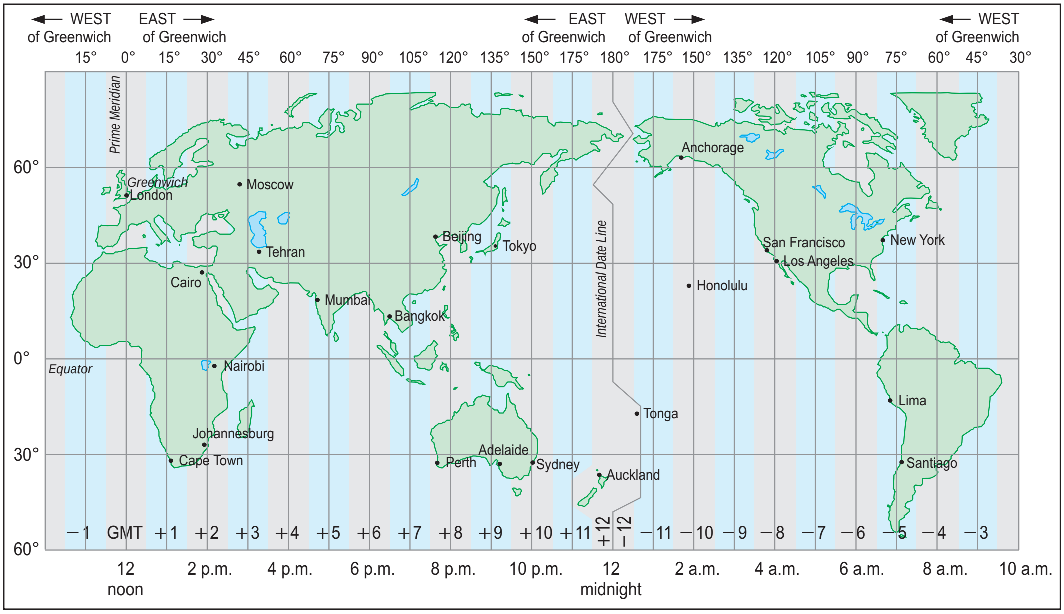


The following map shows lines that run between the North and South Poles. They are not straight like the lines of longitude but sometimes follow the borders of countries, states or regions, or natural boundaries such as rivers and mountains.

The first line of longitude, 0^o, passes through Greenwich near London. This first or **prime meridian** is the starting point for 12 time zones west of Greenwich and 12 time zones east of Greenwich.

Places which lie within a time zone share the same **standard time**. Standard Time Zones are mostly measured in 1 hour units, but there are also some $\frac{1}{2}$ hour units.

Time along the prime meridian is called **Greenwich Mean Time (GMT)**.



Places to the east of the prime meridian are **ahead** of or **later** than GMT.

Places to the west of the prime meridian are **behind** or **earlier** than GMT.

The numbers in the zones show how many hours have to be added or subtracted from Greenwich Mean Time to work out the **standard time** for that zone.

Example 9 **Self Tutor**

If it is 12 noon in Greenwich, what is the standard time in:

a Sydney **b** San Francisco?

a Sydney is in a zone marked $+10$.
So, the standard time in Sydney is 10 hours ahead of GMT.
The standard time in Sydney is 10 pm.

b San Francisco is in a zone marked -8.
So, the standard time in San Francisco is 8 hours behind GMT.
The standard time in San Francisco is 4 am.

EXERCISE 15F

Use the Standard Time Zone map to answer these questions:

1 If it is 12 noon in Greenwich, what is the standard time in:

a Moscow **b** Beijing **c** Adelaide **d** Santiago?

2 If it is 12 midnight Monday in Greenwich, what is the standard time in:

a Cairo **b** Mumbai **c** Tokyo **d** London?

3 If it is 10 pm on Tuesday in Beijing, what is the standard time in:

a New York **b** San Francisco **c** Tehran **d** Johannesburg?

4 If it is 2:45 am on Sunday in Tokyo, what is the standard time in:

a San Francisco **b** Mumbai **c** Lima **d** Auckland?

5 If it is 3 pm in Moscow on Friday, what is the standard time in:

a Nairobi **b** Beijing **c** London **d** Los Angeles?

G AVERAGE SPEED

Speed is a measure of how fast something is travelling. Most objects do not travel with exactly the same speed all the time, so we say the speed *varies*.

The **average speed** of an object in a given period of time is the constant speed at which it would need to travel in that time period so as to travel the same distance.

The average speed can be calculated using the formula:

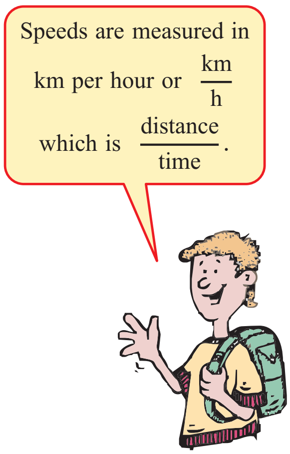
$$\textbf{average speed} = \frac{\textbf{distance travelled}}{\textbf{time taken}}$$

For example, if a car travels a distance of 180 km in two hours then its average speed is $\frac{180 \text{ km}}{2 \text{ hours}} = 90$ km per hour. On average it travels 90 km in each hour.

Example 10 **Self Tutor**

Find the average speed of a car which travels 720 km in 9 hours.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{720 \text{ km}}{9 \text{ hour}}$$
$$= 80 \text{ km per hour.}$$



FINDING DISTANCES TRAVELLED

If we travel at 60 km per hour for 3 hours, we will travel a distance of $60 \times 3 = 180$ km.

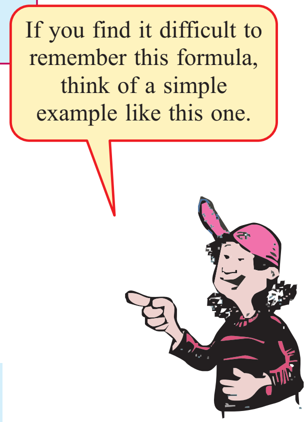
Notice that we worked out $60 \times 3 = 180$
(60 = speed, 3 = time, 180 = distance)

So, $\textbf{distance} = \textbf{average speed} \times \textbf{time taken}$

Example 11 — Self Tutor

How far would you travel in 5 hours at an average speed of 96 km per hour?

$$\begin{aligned}\text{Distance} &= \text{speed} \times \text{time}\\ &= 96 \times 5\\ &= 480 \text{ km}\end{aligned}$$



FINDING TIME TAKEN

To travel 200 km at 100 km per hour, it would take us 2 hours.

Notice that $\frac{200}{100} = 2$ (200 → distance, 2 → time, 100 → speed)

So, $$\textbf{time taken} = \frac{\textbf{distance}}{\textbf{average speed}}$$

Example 12 — Self Tutor

How long would it take to travel 300 km at 75 km per hour?

$$\begin{aligned}\text{time} &= \frac{\text{distance}}{\text{speed}}\\ &= \frac{300}{75}\\ &= 4 \text{ hours}\end{aligned}$$

EXERCISE 15G

1 Find the average speed of a vehicle which travels:

- **a** 540 km in 6 hours
- **b** 840 km in 12 hours
- **c** 664 km in 8 hours
- **d** 846 km in 9 hours.

2 If a vehicle is travelling at 90 km per hour, find how far it will travel in:

- **a** 7 hours
- **b** 5 hours
- **c** 10 hours
- **d** 3.5 hours
- **e** 11 hours 24 mins

Hint: 24 min $= \frac{24}{60}$ hours

3 How far would you travel in:

- **a** 3 hours at an average speed of 85 km per hour
- **b** 8 hours at an average speed of 110 km per hour
- **c** $4\frac{1}{2}$ hours at an average speed of 98 km per hour
- **d** 2 hours 15 mins at an average speed of 76 km per hour?

4 Find how long it will take to travel:

a 90 km at 30 km per hour
b 720 km at 120 km per hour
c 440 km at 80 km per hour
d 750 km at 90 km per hour
e 208 km at 64 km per hour.

H TEMPERATURE CONVERSIONS

In most of the world, temperatures are measured in **degrees Celsius** (^{o}C).

0^{o}C is the temperature at which pure water freezes.

DEMO

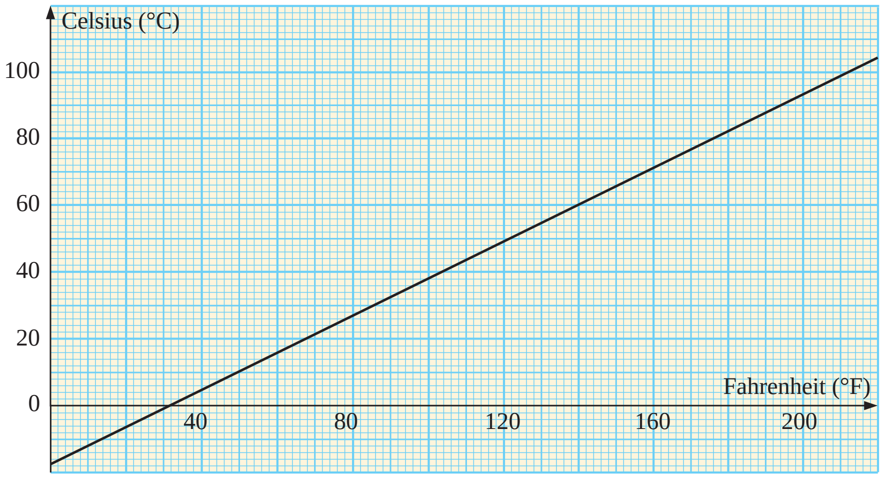
100^{o}C is the temperature at which pure water boils.

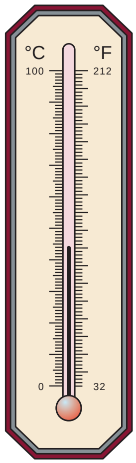
In a few countries, such as the USA, the **Fahrenheit scale** (^{o}F) is still used.

32^{o}F is the temperature at which pure water freezes.

212^{o}F is the temperature at which pure water boils.

The following graph allows us to convert from ^{o}C to ^{o}F or from ^{o}F to ^{o}C.





EXERCISE 15H

DEMO

1 Convert these ^{o}C temperatures into ^{o}F temperatures:

a 50^{o}C **b** 80^{o}C **c** 20^{o}C **d** -5^oC

2 Convert these ^{o}F temperatures into ^{o}C temperatures:

a 100^{o}F **b** 50^{o}F **c** 80^{o}F **d** 0^{o}F

3 The formula for converting ^{o}C temperatures into ^{o}F temperatures is $F = 1.8 \times C + 32$.
Use this formula to check your answers to question **1**.

4 The formula for converting ^{o}F temperatures into ^{o}C temperatures is
$C = 5 \times (F - 32) \div 9$.

a Use the formula to check your answers to question **2**.

b If you are in New York and the temperature is 90^{o}F, what is the temperature in ^{o}C?

ACTIVITY 5 TEMPERATURES ONLINE

Use the internet to compare temperatures in three different world cities.

On the same graph plot the daily maximum temperatures over a fortnightly period.

KEY WORDS USED IN THIS CHAPTER

- 12-hour time
- 24-hour time
- ante meridiem
- average speed
- degrees Celsius
- degrees Fahrenheit
- distance travelled
- Greenwich Mean Time
- longitude
- post meridiem
- prime meridian
- standard time
- time line
- time taken
- time zone
- timetable

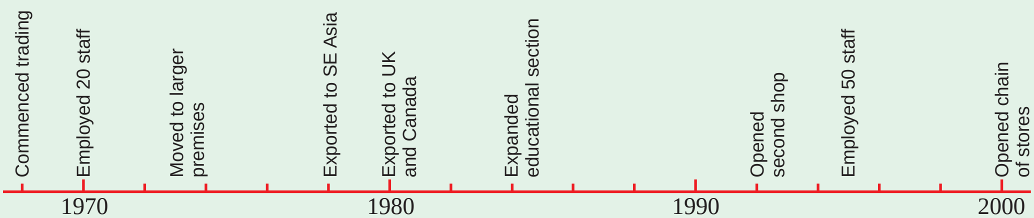
LINKS click here

HOW MANY STEPS DO YOU TAKE EACH DAY?

Areas of interaction:
Environments, Health and social education

REVIEW SET 15A

1 The following line shows some important dates in the history of The Book Company.



a Use the time line to find when The Book Company:

i started trading **ii** employed 20 staff

iii exported to SE Asia

iv expanded their educational section.

b How many years was it between when The Book Company:

i employed 20 staff and employed 50 staff

ii opened a second shop and opened a chain of stores?

2 Copy and complete:

a 7 weeks = ☐ days **b** 12 minutes = ☐ seconds

c $9\frac{1}{4}$ hours = ☐ minutes **d** 1 millennium = ☐ years

3 Find the following:

a 9 hours 38 mins + 6 hours 45 mins + 4 hours 18 mins

b 7 hours 27 min − 3 hours 49 mins

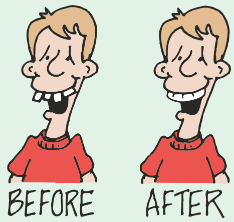
4 To reach her goal of running 500 km before the season starts, a hockey player plans to run 5 km each day. If the season starts on the 8th of September, when should she start her running?

5 Josh began saving 15 dollars a day from the 4th of April. He needs $3000 to have his teeth straightened on September 27th.

a How many days does he have to save?

b What is the total he will have saved by then?

c How much will he still owe the orthodontist?



6 Write the following in **i** 12-hour time **ii** 24-hour time:

a quarter to seven in the morning **b** quarter past midnight

c half past nine at night.

7 The following schedule of arrivals has appeared on a TV monitor at New Orleans Airport.

a Give the arrival time for the plane from Orlando in 12-hour time.

b Give the difference in time between the arrival of the two planes from:

i Houston **ii** Memphis.

c If the plane from Dallas is one hour and twenty minutes late, at what time will it arrive?

ARRIVALS		
Flight	*From*	*Arrival time*
438	Atlanta	1218
1236	Dallas	1225
524	Houston	1355
2618	Memphis	1440
1029	Houston	1530
264	Orlando	1545
4350	St Louis	1620
3256	Memphis	1655

8 Use the Standard Time Zone map on page **289** to answer the following questions:

a If it is 11 am on Saturday in Greenwich, what is the standard time in:

i Moscow **ii** Santiago?

b If it is 8 pm on Tuesday in New York, what is the standard time in:

i Cairo **ii** Mumbai?

9 How far would I ride in 3 hours if I can travel at 18 km per hour on my bicycle?

10 Find the speed of a car which travels 752 km in 8 hours.

REVIEW SET 15B

1 Convert:

a 19 hours 54 minutes to minutes **b** 475 hours to days

2 Find the number of:

a days from 7th July to 22nd October

b hours from 11 pm Monday to noon the following Thursday

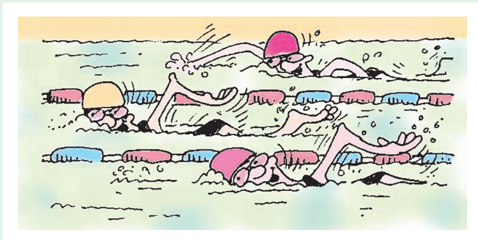
c minutes from 9:47 am to 11:08 am
d seconds from 11:59 pm to 12:04 am.

3 Find:
a 4 hour 8 min + 5 hour 35 min + 3 hour 47 min
b 2 days 9 hours 18 min + 3 days 15 hours 45 min
c 3 hours 15 min − 1 hour 57 min

4 How many days are there from:
a 15th January to 4th April in a leap year
b 1st January 2007 to 31st December 2008
c 1st January 1999 to 31st December 2008?

5 How many Tuesdays were there in 2008 if January 1st was a Tuesday?

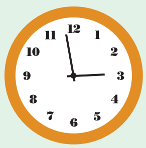
6 **a** In the 1500 m event, the first placed swimmer's time was 14 min 58.29 seconds. Second place was 2.78 seconds slower, with third place a further 4.35 seconds behind. Find the second and third placed swimmers' times.



b A marathon runner finished 2 minutes 13 seconds slower than his personal best time of 2 hours 58 minutes and 48 seconds. What was his finishing time?
c If the sun rose at 5:24 am and set at 7:43 pm, how many hours and minutes of daylight were there?

7 Write the following 24-hour times as 12-hour times:
a 0415 hours **b** 1300 hours **c** 2335 hours

8 Write this **pm** time as:
a as an analogue time in words
b in 12-hour digital time
c in 24-hour time.



9 Use the Standard Time Zone map on page **289** to answer the following questions.
a If it is 12 noon in Greenwich, what is the standard time in:
i Cape Town **ii** Anchorage?
b If it is 6:05 am on Thursday in Tokyo, what is the standard time in:
i Johannesburg **ii** Honolulu?

10 What distance is travelled by a car in 4 hours if it is travelling at 87 km per hour?

11 How long would it take me to travel 25 km if I can ride at 20 km per hour on my bicycle?

12 Convert these ^{o}F temperatures into ^{o}C temperatures:
a 140^{o}F **b** 23^{o}F

13 Use the formula $F = 1.8 \times C + 32$ to answer the following questions:

a If it is 5^oC in London, what is the temperature in ^oF?

b If it is 40^oC in Cairo, what is the temperature in ^oF?

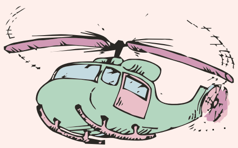
ACTIVITY RECORD TIMES

Time is used to compare and record achievements and events.

What to do:

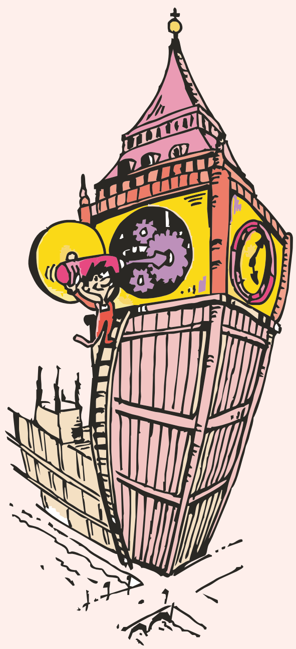
Use the conversion tables on page **279** to help you answer the following:

1 The world's oldest surviving clock was completed in England in 1386. How many years ago was that?

2 

Two helicopter pilots flew around the world in 17 days 6 hours 4 minutes and 25 seconds. Find the total flight time in seconds.

3 John Fairfax and Sylvia Cook started rowing across the Pacific Ocean from San Francisco on 26th April 1972. They reached Hayman Island in Australia on 22nd April 1973. How many days did their crossing take?



4 Yiannis Kouros started a 1000 km run on Saturday 26th November 1984 at 1 pm. At what time did he finish if it took him 136 hours and 17 minutes to complete the distance?

5 Big Ben stopped at noon on 4th April 1977. He was repaired and restarted at noon on 17th April 1977. For how many minutes did Big Ben stop?

6 Recent records show that the greatest age that a human has lived is one hundred and twenty one years.

a For how many decades did she live?

b Over how many centuries did her life span?

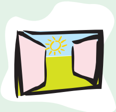
7 Find some time records in your family or class.

Chapter 16

Using percentages

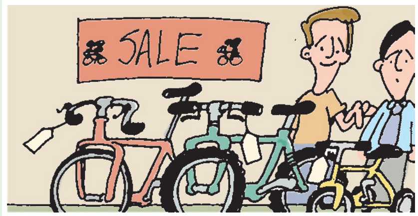
Contents:

OPENING PROBLEMS



Problem 1:
What is 15% of $2400?

Problem 2:
A bicycle normally sells for €400.
An 18% discount is given.
What will the bicycle sell for?



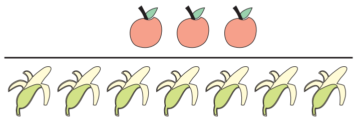
A COMPARING QUANTITIES

Percentages are often used to compare quantities, so it is useful to be able to express one quantity as a percentage of another.

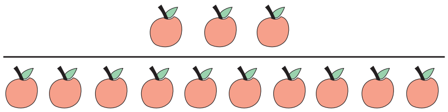
We must be careful to only compare **like with like**.

For example:

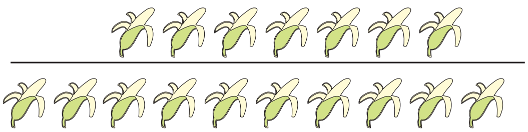
We cannot express 3 apples as a percentage of 7 bananas



but we can express 3 apples as a percentage of 10 apples



or 7 bananas as a percentage of 10 bananas



.

We must also make sure that the quantities are compared in the same units.

For example, if we are asked to express "35 cm as a percentage of 7 m" we would normally convert the larger unit to the smaller one. So we would find "35 cm as a percentage of 700 cm".

We cannot express "5 bicycles as a percentage of 45 cars", but we can express "5 bicycles as a percentage of 50 vehicles" as bicycles are a type of vehicle.

ACTIVITY CHOOSING A COMMON NAME OR SAME UNIT

When we are given different quantities to compare, we often need to choose a common name that describes them.

For example, a German Shepherd and a Rottweiler are both dogs.

What to do:

1. Choose a common name which could be used to describe each of the given items:
   - **a** coffee, tea
   - **b** hamburgers, pizza
   - **c** train, bus, tram
   - **d** fins, wetsuit, goggles
   - **e** e-mail, letters, fax, telephone
   - **f** saxophone, clarinet, trumpet
   - **g** museum, art gallery
   - **h** zloty, euro, pound, dollar
   - **i** rice, barley, wheat, oats
   - **j** locusts, termites, millipedes, mice
2. For each of the above examples above, prepare a statement and a question which uses the common name you have chosen.
   For example, "Of the people who had breakfast in the hotel dining room, 12 ordered coffee and 28 ordered tea. What percentage of the people who ordered a hot drink ordered tea?"

ONE QUANTITY AS A PERCENTAGE OF ANOTHER

Martina's water container holds 20 litres whereas Fabio's holds 50 litres.

20 litres compared with 50 litres is $\frac{20}{50} = \frac{2}{5}$ as a fraction and is $\frac{40}{100} = 40\%$.

To express one quantity as a percentage of another, we first write them as a fraction and then convert the fraction to a percentage.

Example 1 Self Tutor

Express the first quantity as a percentage of the second:

a 12 hours, 5 days **b** 800 m, 2 km

a
$$\frac{12 \text{ hours}}{5 \text{ days}} = \frac{\overset{1}{\cancel{12}} \text{ h}}{5 \times \underset{2}{\cancel{24}} \text{ h}} = \frac{1}{10} = 10\%$$

b
$$\frac{800 \text{ m}}{2 \text{ km}} = \frac{800 \text{ m}}{2000 \text{ m}} = \frac{800 \div 20}{2000 \div 20} = \frac{40}{100} = 40\%$$

EXERCISE 16A

1. Express the first quantity as a percentage of the second:
   - **a** 10 km, 50 km
   - **b** 20 cm, 100 cm
   - **c** 3 m, 4 m
   - **d** 90^o, 360^o
   - **e** 5 L, 100 L
   - **f** 45^o, 90^o
   - **g** 125 mL, 750 mL
   - **h** 6 months, 4 years
   - **i** 50 g, 1 kg
   - **j** 48 kg, 1 tonne
   - **k** 120^o, 360^o
   - **l** 5 mm, 8 cm

m 25 cm, 0.5 m
n 48 min, 2 hours
o 180 cm, 3 m
p 3 min, $\frac{1}{2}$ hour
q 5 mg, 2 g
r 6 hours, 2 days

Example 2 Self Tutor

Express a mark of 17 out of 25 as a percentage.

$$\frac{17 \text{ marks}}{25 \text{ marks}} = \frac{17}{25}$$
$$= \frac{17 \times 4}{25 \times 4}$$
$$= \frac{68}{100}$$
$$= 68\%$$

2 Express as a percentage:

a 17 marks out of 20
b 11 marks out of 25
c 29 marks out of 40
d 72 marks out of 80
e 37 marks out of 50
f 138 marks out of 200.

Example 3 Self Tutor

Out of 1250 cars sold last month, 250 were made by Ford.
Express the Ford sales as a percentage of total sales.

$$\frac{250 \text{ cars}}{1250 \text{ cars}} = \frac{250}{1250}$$
$$= \frac{250 \div 250}{1250 \div 250}$$
$$= \frac{1}{5}$$
$$= 20\%$$

So, the Ford sales were 20% of the total sales.

3 Express as a percentage:

a 427 books sold out of a total 500 printed
b 650 square metres of lawn in a 2000 square metre garden
c 27 400 spectators in a 40 000 seat stadium
d An archer scores 95 points out of a possible 125 points.

4 What percentage is:

a 42 of 60
b 34 of 40
c 440 mL of 2000 mL
d 3 minutes of one hour
e 175 g of 1 kg
f 48 seconds of 2 min
g 420 kg of 1 tonne
h 16 hours of 1 day
i 174 cm of 1 m?

B FINDING PERCENTAGES OF QUANTITIES

To find a percentage of a quantity, we could first convert the percentage to a fraction. We then find the required fraction of the given quantity.

For example, 50% of 40 is $\frac{1}{2}$ of 40, which is 20.

Example 4 **Self Tutor**

Find: **a** 10% of 7 m (in cm) **b** 35% of 4000 people

a 10% of 7 m
$= \frac{10}{\cancel{100}_1} \times \cancel{700}^{7}$ cm
$= 10 \times 7$ cm
$= 70$ cm

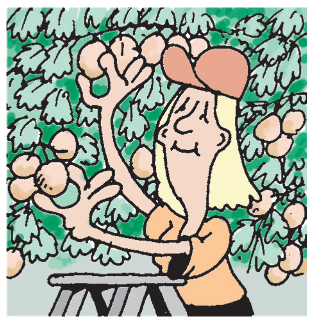
b 35% of 4000 people
$= \frac{35}{\cancel{100}_1} \times 40\cancel{00}$ people
$= 35 \times 40$ people
$= 1400$ people

EXERCISE 16B

1 Find:

- **a** 20% of 360 hectares
- **b** 25% of 4200 square metres
- **c** 5% of 9 m (in cm)
- **d** 40% of 400 tonnes
- **e** 10% of 3 hours (in min)
- **f** 8% of 80 metres (in cm)
- **g** 30% of 2 tonnes (in kg)
- **h** 4% of 12 m (in mm)
- **i** 15% of 12 hours (in min)
- **j** 75% of 250 kilolitres

2 A school with 485 students takes 20% of them on an excursion to the museum. How many students are left at school?



3 An orchardist picks 2400 kg of apricots for drying. If 85% of the weight is lost in the drying process, how many kilograms of dried apricots are produced?

4 A council collects 4500 tonnes of rubbish each year from its ratepayers. If 27% is recycled, how many tonnes is that?

5 A marathon runner improves her best time of 4 hours by 5%. What is her new best time?

6 Damian was 1.5 metres tall at the beginning of the school year. At the end of the year his height had increased by 5.6%. What was his new height?

7 Which is the larger amount?

- **a** 40% of a litre or $\frac{1}{3}$ of a litre
- **b** 20% of one metre or $\frac{1}{4}$ of a metre
- **c** 33% of 1000 or $\frac{1}{3}$ of 1000
- **d** 30% of a kg or 315 g

8 45% of an energy drink is sugar. How many grams of sugar would there be in a 450 g can of this drink?

9 Simon used 20% of a 4 L can of paint. How much paint was left?

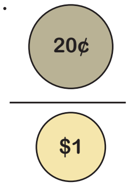
10 30% of a farmer's crop was barley, and the rest was wheat. If he planted 2400 acres in total, how many acres were planted with wheat?

C PERCENTAGES AND MONEY

All over the world, money is used for trading goods and services. Percentages are commonly used in situations involving money, especially decimal currencies.

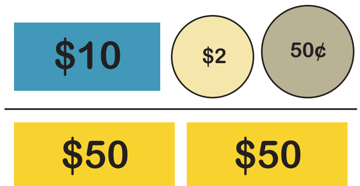
Look at the following examples:

20 cents out of 1 dollar is:



$= \frac{20 \text{ cents}}{100 \text{ cents}} = \frac{20}{100} = 20\%$

$12.5\% = \frac{12.5}{100}$ = \$12.50 out of every \$100 or



Example 5 — Self Tutor

Find: **a** 15% of \$200 **b** 8% of \$3500

a 15% of \$200
$= \frac{15}{100} \times \$200$
$= 15 \times \$2$
$= \$30$

b 8% of \$3500
$= \frac{8}{100} \times \$3500$
$= 8 \times \$35$
$= \$280$

The word 'of' indicates that we multiply.

Another way to find one quantity as a percentage of another is to first find the fraction and then multiply by 100%.

Example 6 — Self Tutor

Express £32 as a percentage of £80.

£32 as a percentage of £80
$= \frac{32}{80} \times 100\%$
$= \frac{320}{8}\%$
$= 40\%$

Multiplying by 100% is really multiplying by 1, so the actual value does not change.

EXERCISE 16C

1 Find:

- **a** 10% of \$40
- **b** 30% of €180
- **c** 70% of £210
- **d** 11% of €20
- **e** 20% of \$150
- **f** 45% of RM 9700
- **g** 83% of £720
- **h** 36% of \$450
- **i** 8% of €4850
- **j** 12% of 2950 rupees
- **k** 37% of £700
- **l** 54% of €2500
- **m** 17.5% of RMB 4000
- **n** 6.8% of £40
- **o** 10.9% of ¥50 000

2 Express:

- **a** \$5 as a percentage of \$20
- **b** €15 as a percentage of €150
- **c** £3 as a percentage of £20
- **d** ¥4000 as a percentgae of ¥80 000
- **e** €25 as a percentage of €125
- **f** 8 rubles as a percentage of 120 rubles
- **g** £1.50 as a percentage of £30
- **h** 35 cents as a percentage of \$1.40
- **i** €8 as a percentage of €240
- **j** £40 as a percentage of £600
- **k** \$334 as a percentage of \$33 400
- **l** €9.95 as a percentage of €99.50.

D PROFIT AND LOSS

PROFIT

If we sell an item for **more** than we paid for it, we say we have made a **profit**.

For example, suppose we buy a bicycle for €375 and then sell it for €425.
We have made a profit of €425 − €375 = €50.

LOSS

If we sell an item for **less** than we paid for it, we say we have made a **loss**.

For example, suppose we buy a computer game for \$155 and then sell it for \$90.
We have made a loss of \$155 − \$90 = \$65.

PROFIT AND LOSS AS A PERCENTAGE

Profit and loss are often expressed as a percentage of the cost price.

To do this we compare the profit or loss with the cost price using a fraction, and then multiply it by 100% to convert to a percentage.

$$\textbf{percentage profit} = \frac{\textbf{profit}}{\textbf{cost price}} \times 100\%$$

$$\textbf{percentage loss} = \frac{\textbf{loss}}{\textbf{cost price}} \times 100\%$$

Example 7 **Self Tutor**

An autographed baseball bat is bought for \$125 and sold for \$150.

a What is the profit made?

b Express the profit as a percentage of the cost price.

a Profit
$= \$150 - \$125$
$= \$25$

b Percentage profit
$= \dfrac{\text{profit}}{\text{cost price}} \times 100\%$
$= \dfrac{\$25}{\$125} \times 100\%$
$= \dfrac{1}{5} \times 100\%$
$= 20\%$

Example 8 **Self Tutor**

Morgan paid €200 for a printer. He sold it a year later for €120, when he decided he wanted a newer model.

a What loss was made?

b Express the loss as a percentage of the cost price.

a Loss
$= €200 - €120$
$= €80$

b Percentage loss
$= \dfrac{\text{loss}}{\text{cost price}} \times 100\%$
$= \dfrac{€80}{€200} \times 100\%$
$= 40\%$

EXERCISE 16D

1 A surfboard was bought for £400 and was sold later for £300.

a What was the loss made?

b Express the loss as a percentage of the cost price.

2 A mountain bike was purchased by a cycle shop for €500 and sold to a customer for €700.

a What was the profit made?

b Express the profit as a percentage of the cost price.

3 Donald bought a property for \$80 000 and sold it a year later for \$95 000. Find his profit:

a in dollars **b** as a percentage of his cost price.

4 Kate bought a dress for RM 250 and sold it later for RM 150. Find her loss:

a in RM **b** as a percentage of her cost price.

Example 9 — Self Tutor

Emma bought a gold ring for £300 and then sold it for a 30% profit. Find:

a her profit **b** the selling price of the ring.

a Emma's profit
$= 30\%$ of £300
$= \frac{30}{100} \times £300$
$= 30 \times £3$
$= £90$

b The selling price
$=$ cost price $+$ profit
$= £300 + £90$
$= £390$

5 Francis paid €250 for a wrist watch and sold it for a 40% profit. Find:
a his profit **b** the selling price of the wrist watch.

6 Mia paid £125 for a pair of earrings and then sold them for a 60% profit. Find:
a her profit **b** the selling price of the earrings.

7 A car was bought for ¥100 000 and sold at a 17% loss. Find:
a the loss **b** the actual selling price of the car.

8 Sergio had to sell his fishing boat at a 30% loss. If it cost him €18 000, find:
a Sergio's loss in euros **b** the selling price of the boat.

E DISCOUNT

To attract customers and make sales, many businesses reduce the prices of items from the **marked price** shown on their price tags.

The amount of money by which the marked price is reduced is called the **discount**.

Discounts are often stated as a percentage of the marked price or recommended retail price.

Example 10 — Self Tutor

The marked price of a wetsuit is $200. If a 15% discount is offered, find the actual selling price of the wetsuit.

15% of the marked price
$= 15\% \times \$200$
$= \frac{15}{100} \times \$200$
$= \$30$ discount

Marked price	$200
Less 15% discount	− $30
Selling price	$170

EXERCISE 16E

1 **a** The marked price of a DVD player is $320. If a 15% discount is offered, find the actual selling price of the DVD player.

b A camera's normal price is €460. Buying it duty free reduces the price by 25%. Find the actual selling price of the camera.

c A supermarket is offering 2% discount on the total of your shopping docket. How much will you actually pay if your docket shows $130?

d The marked price of a computer is £600. If a 12% discount is offered, what is the new selling price?

2 Find the selling price after the following discounts have been made:

a $7.00 less 20%

b $36.00 less 25%

c £140 less 8%

d RM56.00 less 12%

e €19.00 less 15%

f $650 less 30%

g $112 less 18%

h £3400 less 90%

i €35.00 LESS 6%

j €470 less 20%

k $78.50 LESS 40%

l ¥42 500 less 12.5%

F GOODS TAX

Many countries have a **tax** on all goods and services. This tax is paid any time an item is bought, and also on services by electricians, plumbers, and so on.

This tax may be called a **goods and services tax (GST)** or a **value added tax (VAT)** or have some other title.

The percentage paid varies from 8% to 35% depending on the country and its taxation laws.

Example 11 **Self Tutor**

In order to make a profit a shop must sell an item for \$80. A goods tax of 10% must be added to this price.

a What is the goods tax amount?

b What price should the item be sold for?

a The goods tax is 10% of \$80

$= \frac{10}{100} \times \$80$

$= \$8$

b The shop sells the item for \$80 + \$8 = \$88.

EXERCISE 16F

1 If the goods tax is 10%, how much tax must be added to the following prices?

a

Selling price = \$20 + tax

b

Selling price = €800 + tax

c

Selling price = £56 + tax

2 If the goods tax is 15%, how much tax must be added to the following prices:

a €100 **b** \$10 **c** £16 **d** RMB 320?

3 Find the total price, including 18% goods tax, of a service which would otherwise cost:

a \$100 **b** €48 **c** \$2000 **d** \$640?

4 A shopkeeper needs to sell a pair of shoes for £160 to make a profit. A tax of 15% must be added on to this price.

a What is the tax she must add on? **b** What must she sell the shoes for?

5 Conrad buys a lounge chair for \$750 plus a government charge of 17.5%. Find:

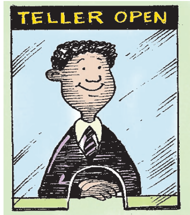
a the government charge **b** the final price of the lounge chair.

6 A bicycle shop sells bicycles for €250 plus a tax of $12\frac{1}{2}\%$.

a What is the tax amount to be added?

b How much will customers have to pay for a bicycle?

G SIMPLE INTEREST



When a person borrows money from a bank or a finance company, the borrower must repay the loan in full, and pay an additional charge which is called **interest**.

If the charge is calculated each year or *per annum* as a fixed percentage of the original amount borrowed, the charge is called **simple interest**.

For example, suppose \$8000 is borrowed for 4 years at 10% per annum simple interest.

The simple interest charged for each year is
$$\begin{aligned} & 10\% \text{ of } \$8000 \\ &= \tfrac{10}{100} \times \$8000 \\ &= \$800 \end{aligned}$$

So, the simple interest for 4 years is $\$800 \times 4 = \$3200$.

The borrower must repay $\$8000 + \$3200 = \$11\,200$.

Example 12 **Self Tutor**

Find the simple interest payable on a loan of €5000 for $3\frac{1}{2}$ years at 8% p.a.

The simple interest charged for 1 year $= 8\%$ of €5000
$= \frac{8}{100} \times$ €5000
$=$ €400

So, the simple interest for $3\frac{1}{2}$ years $=$ €400 $\times 3.5$
$=$ €1400

EXERCISE 16G

1 Find the simple interest when:

- **a** \$1000 is borrowed for 1 year at 15% per annum simple interest
- **b** £3500 is borrowed for 2 years at 10% per annum simple interest
- **c** \$5000 is borrowed for 4 years at 8% per annum simple interest
- **d** €20 000 is borrowed for $1\frac{1}{2}$ years at 12% per annum simple interest
- **e** \$140 000 is borrowed for $\frac{1}{2}$ year at 20% per annum simple interest.

2 Find the total amount to repay on a loan of:

- **a** \$2000 for 5 years at 8% p.a. simple interest
- **b** €6500 for 3 years at 10% p.a. simple interest
- **c** \$8000 for $4\frac{1}{2}$ years at 12% p.a. simple interest
- **d** ¥100 000 for 10 years at 10% p.a. simple interest.

DEMO

KEY WORDS USED IN THIS CHAPTER

- cost price
- goods tax
- percentage
- simple interest
- discount
- interest
- profit
- fraction
- loss
- selling price

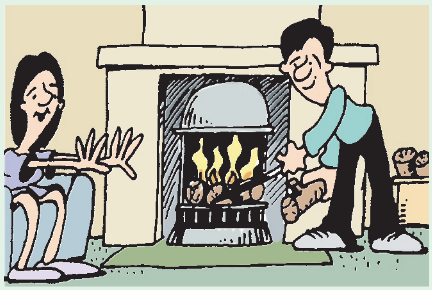
REVIEW SET 16A

1 Write 200 mL as a percentage of 4000 mL.

2 An airline offers a special of 30% off its normal prices during its off-peak flights to Madrid. If its normal price is €324 return, what is the special price?

3 About 8% of all students are left-handed. In a school of 375 students, how many left-handed students would you expect to find?

4 A small country town has 280 households. 45% use a wood burning fire to warm their homes, 30% use electricity, 15% use gas, and the rest use oil or kerosene. How many households use gas, oil or kerosene?



5 If the goods tax is 15%, how much must be added to the price of a shirt which would otherwise sell for $30?

6 A survey of 500 year 6 students showed that 55% always started their homework as soon as they arrived home from school, and 30% always started after tea. The rest had no regular pattern as to when they did their homework. How many students:

a had no regular pattern **b** started as soon as they arrived home?

7 In a town of 7200 people, 1800 were over 60 years of age and 3600 were under 40. Find the percentage of people aged from 40 to 60.

8 As a result of dieting, Alfred reduced his 90 kg weight by 10%. What was his reduced weight?

9 Maryanne received 12% p.a. simple interest on her $3500 investment.

a How much interest did she earn after 2 years?

b What was her new balance?

10 A salesman offered 20% discount on a holiday package costing £2100.

a Find the amount of discount.

b Find the new price of the holiday.

11 A table tennis set was bought for $40 and sold for $55. Find the profit:

a in dollars **b** as a percentage of the cost.

REVIEW SET 16B

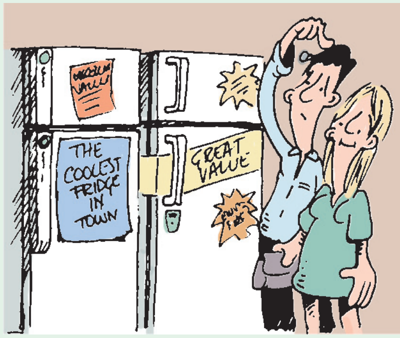
1 Express the first quantity as a percentage of the second.

a 13 goals from 25 shots   **b** 58 cm from 2 m

2 Anthony lost 6 marks in a test out of 25. What percentage did he score for the test?

3 What percentage is 650 kilometres of a 2000 km journey?

4 One hundred students agree to come to a fund raising school disco. The committee has determined that the DJ costs €180, and balloons and streamers will cost €20. If they want to make a 50% profit, how much should they charge each student?



5 A fridge has a marked price of €840, but a discount of 15% is given.

a Find the discount.

b What is the actual price paid for the fridge?

6 A telemarketing company offers a "100% Money Back Guarantee". If I return my $189 exercise machine, how much will I get back?

7 Jody has organised a loan of $7000 for 5 years at 8% p.a. simple interest.

a How much interest will she need to pay?

b Find the total amount of money that must be repaid.

8 A survey of 250 primary school students found that 24% usually have the TV on while they are doing their homework, 56% never have the TV on, and the rest sometimes have the TV on. How many students:

a usually have the TV on   **b** never have the TV on?

9 Klaus spent €15 from the €50 he was given for his birthday. What percentage of his money did he spend?

10 A dentist charges $270 for dental treatment. A 10% services tax must be added to this amount.

a What is the service tax that must be added?

b How much will the customer have to pay?

11 An item of jewellery was bought for £400 and later was sold at a loss of 15%. What was:

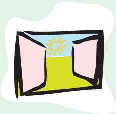
a the loss   **b** the selling price of the jewellery?

Chapter 17

Data collection and representation

Contents:

OPENING PROBLEM



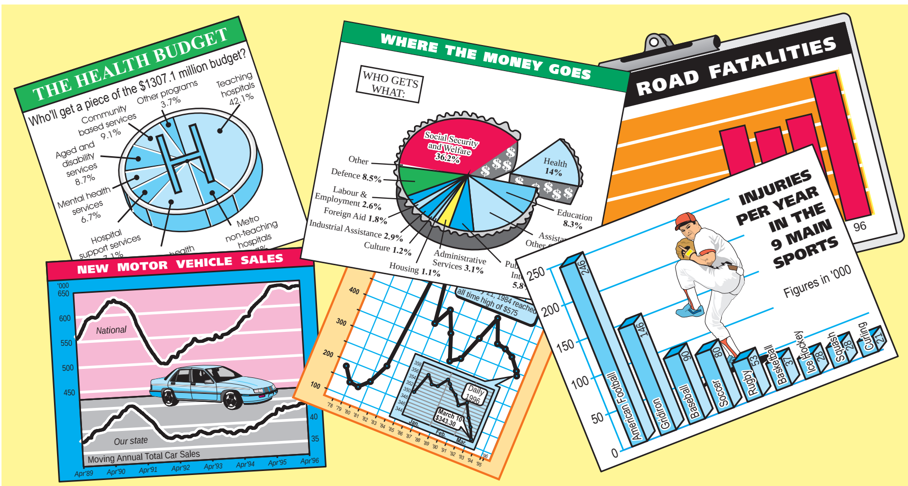
Tony and Carl play for the same basketball team. Due to an injury at practice Tony played only half of the season. The points scored by the players in each match were:

Tony:	17	21	15	8	18	12	27	15	22	31	28	8
Carl:	19	19	13	10	15	15	24	18	26	27	23	13
	20	24	18	26	19	25	8	36	21	23	26	19

Which player's performance was better?

Things to think about:

- Would it be fair to simply total the points each player scored for the season?
- How could we display the data in the meaningful way?
- What would be the 'best' way to solve the problem?



Statistics is about collection, organisation, display, analysis and interpretation of data.

Many groups such as schools, businesses and government departments collect information. The information is used to determine whether changes are needed, or whether changes that have been made have been successful.

Governments sometimes conduct a **census** in which they gather data from all of the nation's population. This information is used to help make decisions which will affect us in the future. For example, the government must consider how much money needs to be provided for health care in the years ahead because the number of elderly people is increasing.

Results of the collection and interpretation of data are displayed using **graphs**, **tables** and **diagrams**.

A SAMPLES AND POPULATIONS

These are important words used in statistics:

Population: The whole group of objects or people from whom we are collecting data.

Sample: A group chosen to take part in a survey or to be measured or tested.

Random sample: A sample selected so that any person or object has as much chance as any other of being selected.

Inference: A conclusion you make based on your survey or investigation.

For example, suppose we conduct a survey on how much chocolate students at your school eat. The *population* is the students at your school.

A *sample* is chosen by selecting 10 students at random from the school roll. An *inference* might be that most students eat chocolate at least once a week.

CENSUS OR SURVEY

When the government carries out a **census** it requires everyone in the **population** to take part. This process is very expensive and takes a lot of time.

Instead, the government may conduct a **survey** of a **sample** of the population. It is important that the results of a survey are typical of the whole population. To ensure this, the sample must be randomly chosen, and as large as is practical.

DISCUSSION

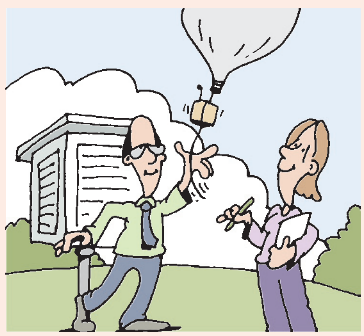
1 Discuss why:

a clothing manufacturers would like to know the body measurements of people in different age groups

b the manager of your school canteen would be interested in the types and quantities of food you eat

c your school keeps records of what is bought by the school population throughout the year

d meteorologists are interested in temperature, rainfall, and atmospheric pressure measurements throughout the country and throughout the world.



2 For each of the situations listed in question **1**, discuss how the information could be collected.

3 Discuss how you would gather data in each of the following situations:

a You wish to manufacture shoes and want to know how many of each size to make.

b As a private citizen you wish to make a case for traffic lights near the local school.

c You own a lawn mowing business and want to expand your business to a new area.

d You are an employer and you need to choose one person from 50 applicants.

4 Organisations and marketing researchers have many clever ways of gathering information by tempting us with offers. Discuss some of the ways in which information is collected from you. Collect samples from newspapers, magazines, packaging, and letterbox deliveries which invite you to provide data.

EXERCISE 17A

1 Suggest how to select a random sample of:

a 400 adults
b bottles of soft drink at a factory
c 30 students at a school
d words from the English language

Are there any advantages or disadvantages in the methods you have suggested?

2 How would you randomly select:

a one ticket out of 5 tickets
b one of the letters A or B
c one of the numbers 1, 2, 3, 4, 5 or 6
d a card from a pack of 52 playing cards?

Example 1 **Self Tutor**

From a school of 400 students, a random sample of 60 students was selected. 13 were found to have blue eyes.

a How many students are in the population?
b How many students are in the sample?
c What fraction of the sample has blue eyes?
d Estimate how many in the population have blue eyes.

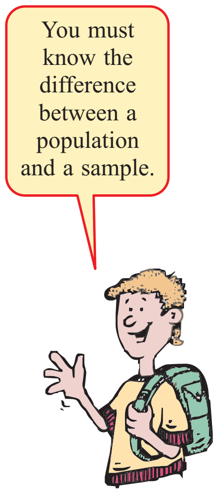
a There are 400 students in the population.
b There are 60 students in the sample.
c 13 out of 60 students in the sample have blue eyes so the fraction of the sample with blue eyes is $\frac{13}{60}$.
d $\frac{13}{60}$ of 400 $\quad \{\frac{13}{60}$ of the population have blue eyes$\}$

$= \frac{13}{60} \times 400$

≈ 87

Calculator: 13 [÷] 60 [×] 400 [=]

So, approximately 87 students in the school have blue eyes.



3 From a colony of 10 000 ants, 300 are collected and examined for red eye colour. 36 were found to have red eyes.

- **a** How many ants form the population?
- **b** How large was the sample?
- **c** What percentage of the sample had red eyes?
- **d** Estimate the total number of red-eyed ants.

4 50 people were randomly selected from the 750 who attended the opening night of a new play. Of the 50 people, 33 said that they liked the play.

- **a** What was the population size of people attending the play?
- **b** How large was the sample?
- **c** What percentage of the sample did *not* like the play?
- **d** Estimate the total number of people who did not like the play.

B CATEGORICAL DATA

Categorical data is data which can be placed in categories.

For example, suppose we stand at a street intersection and record the colour of each car going past.

We use the code R = red, B = blue, G = green, W = white, O = other colours to help us record the data efficiently.

The following results were observed in a sample of 50 cars:

BGWWR	OGWRW	OOBBG	OGRWR	WWWGB
BBGGW	WWWOG	WOBWW	RWWRB	OOBWR

Having collected our categorical data, we first **organise** it in groups. We can do this using either

- a **dot plot** or
- a **tally and frequency table**

Organisation of the data helps us to identify its features. For example:

The **mode** is the most frequently occurring category.

DOT PLOTS

A **dot plot** is a graph which displays data, where each dot represents one data value.

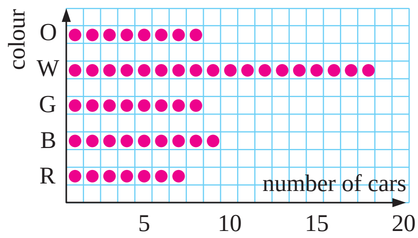
Dot plots are often used to record data initially and may be **horizontal** or **vertical**.

A dot plot for the car colour data is shown alongside:

Check that there is one dot for each car recorded in the data.

The mode is 'white' as W is the most frequently occurring category.

Horizontal dot plot of car colour data



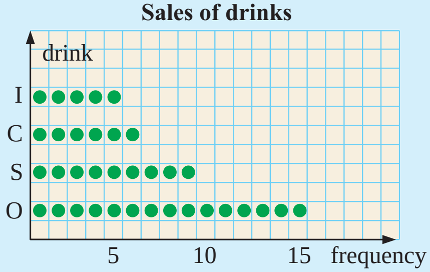
Example 2 — Self Tutor

At recess time the sales of drinks were recorded over a three minute period.
O = 100 plus, S = soy milk, C = cola, I = iced tea.

The data was: OSSCI OCISO IOCSO OOOSC SOCOS SOOCO OIOIS

a Draw a dot plot of the data. **b** What is the mode?

a **Sales of drinks**



b The mode is 100 plus.

EXERCISE 17B.1

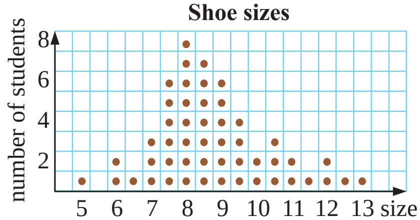
1 The dot plot shows shoe sizes for students in grade 6.

a How many students are in grade 6 at this school?

b How many have shoe size 9 or more?

c What percentage have shoe sizes 8 or more?

Shoe sizes



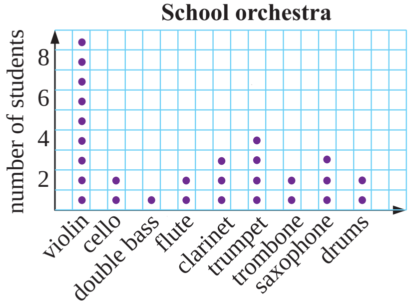
2 The dot plot shows the numbers of students playing various instruments in the school orchestra.

a How many play stringed instruments?

b How many students are in the orchestra?

c Find the mode of the data.

School orchestra



3 A class of students at a school in England were asked which summer sport they wanted to play. The choices were: T = tennis, S = swimming, C = cricket, B = basketball and F = football.

The data was: FFCTC CSFST TTBFS FFCSF TFTBC

a Draw a horizontal dot plot of the data.

b Find the mode of the data.

4 Students voted the most popular attractions at the local show to be the side shows (S), the farm animals (F), the ring events (R), the dogs and cats (D), and the wood chopping (W). The students in a class were then asked to name their favourite.

The data was: SRWSS WFDDS RRFWS RSRWS SRRRF

a Draw a vertical dot plot of the data.

b Find the mode of the data.

TALLY AND FREQUENCY TABLES

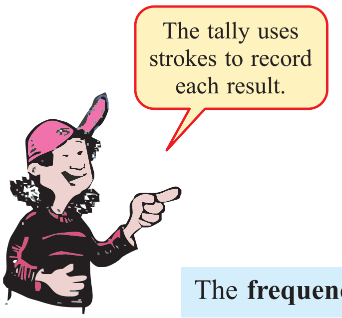
If there is a lot of data, a tally and frequency table is a useful way to collect the information.

The **tally** is used to count the data in each category. The **frequency** summarises the tally, giving the total number of each category.

Such a table is also called a **frequency distribution table** or simply a **frequency table**.

For the car colour data the frequency table is:

Colour	*Tally*	*Frequency*
Other	~~||||~~ |||	8
White	~~||||~~ ~~||||~~ ~~||||~~ |||	18
Green	~~||||~~ |||	8
Blue	~~||||~~ ||||	9
Red	~~||||~~ ||	7



The **frequency** of a category is the number of items in that category.

Example 3 Self Tutor

The data below records how students in a class travel to school on a particular day.
W = walk, Bi = bicycle, Bu = bus, C = car, T = train

The data is:

W Bi Bu T C Bi C W Bi Bu Bi C C Bi Bu W Bu Bu T C
Bi Bi Bu T C C Bi C C C W W Bu T C

a Draw a frequency table to organise the data.

b Find the mode of the data.

a

<table>
<tr><th>Method of Travel</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>Walk</td><td>卌</td><td>5</td></tr>
<tr><td>Bicycle</td><td>卌 |||</td><td>8</td></tr>
<tr><td>Bus</td><td>卌 ||</td><td>7</td></tr>
<tr><td>Car</td><td>卌 卌 |</td><td>11</td></tr>
<tr><td>Train</td><td>||||</td><td>4</td></tr>
<tr><td></td><td>Total</td><td>35</td></tr>
</table>

b The mode is 'car' as this category occurs most frequently.

EXERCISE 17B.2

1 The results of a survey of eye colour in a class of 28 year 6 students were:
Br Bl Gn Bl Gn Br Br Bl Gn Gr Br Gr Br Br Bl Br Bl Br Gr Gn
Br Bl Br Gn Gr Br Bl Gn
where Br = brown, Bl = blue, Gn = green, and Gr = grey.

a Complete a frequency table for the data.

b Find the mode of the data.

2 Students in a science class obtained the following levels of achievement:
D C C A A C C D C B C C C D B C C C C E B A C C B C B C

a Complete a frequency distribution table for the data above.

b Use your table to find the:

i number of students who obtained a C

ii fraction of students who obtained a B.

c What is the mode of the data?

3 Tourists staying in a city hotel were surveyed to find out what they thought about the service by the hotel staff. They were asked to choose E = excellent, G = good, S = satisfactory, or U = unsatisfactory.
The results were:
EGGSE USSGG SGUGG ESGUG SSEGG

a Complete a frequency table for the data.

b What is the mode of the data?

c Suggest a reason why this survey would be carried out.

C GRAPHS OF CATEGORICAL DATA

Categorical data is often displayed using **column graphs** and **pie charts**.

USING HAESE & HARRIS SOFTWARE

Click on the icon to load a statistical package which can be used to draw a variety of statistical graphs.

STATISTICS PACKAGE

Change to a different graph by clicking on a different icon. See how easy it is to change the labels on the axes and the title of the graph.

Type into the correct cells the information given on car colour.

Colour	*Frequency*
white	38
red	27
blue	19
green	18
other	11

Print off graphs of the data including a pie chart, a column graph, and a strip graph.

Use the software or a spreadsheet to reproduce some of the statistical graphs in the remaining part of this chapter. You can also use this software in any statistical project you may be required to do.

COLUMN GRAPHS

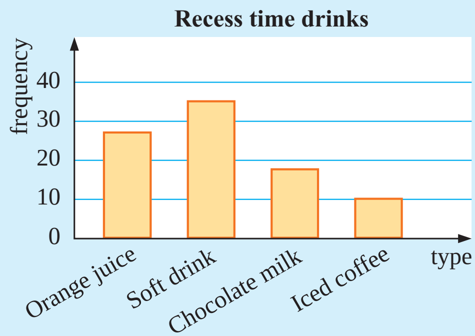
DEMO

Column graphs consist of rectangular columns of equal width. The height of each column represents the the frequency of the category.

Example 4 Self Tutor

The graph given shows the types of drink purchased by students at recess time.

- **a** What is the least popular drink?
- **b** What is the mode of the data?
- **c** How many students drink orange juice?
- **d** What percentage of students drink chocolate milk?



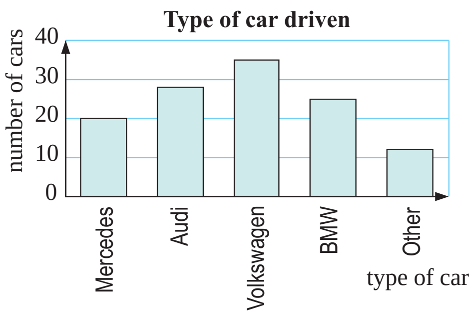
- **a** Iced coffee {shortest column}
- **b** 'Soft drink' is the mode.
- **c** 27 students drink orange juice.
- **d** The total number of students purchasing drinks $= 27 + 35 + 18 + 10 = 90$
  So, the percentage of students drinking chocolate milk is $\frac{18}{90} \times 100\% = 20\%$

EXERCISE 17C.1

1 The results of a survey of eye colour in a class of 28 year 6 students were:

Eye colour	Brown	Blue	Green	Grey
Frequency	11	7	6	4

- **a** Illustrate these results using a hand drawn column graph.
- **b** What is the most frequently occurring eye colour?
- **c** What percentage of the students have blue eyes?

2 

The column graph shows the type of vehicle driven by 120 randomly selected people in Munich.

a Use the graph to estimate the frequency of each type of car.

b Which make of car is the most popular?

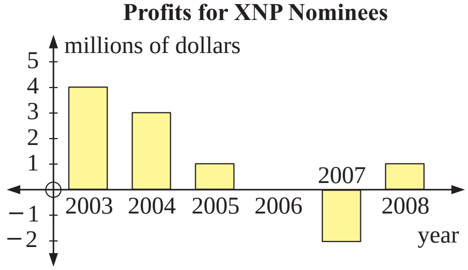
c What percentage of the surveyed people drive an Audi?

3 Yearly profit and loss figures for a business can be easily illustrated on a column graph. For the example given:

a in what years was a profit made

b what happened in 2006

c what was the overall profit or loss over the 6-year period?

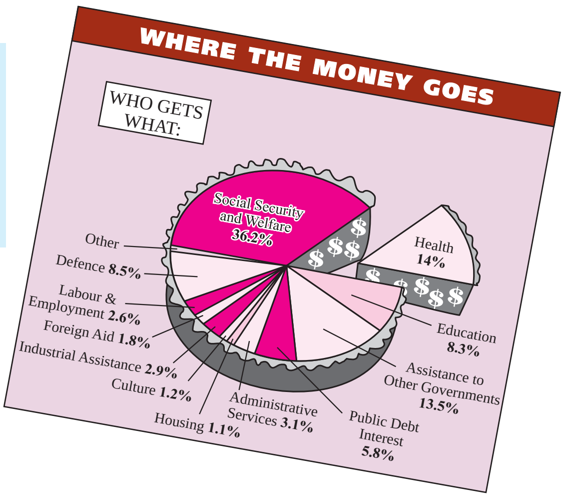


PIE CHARTS

A **pie chart** is a useful way of displaying how a quantity is divided up. A full circle represents the whole quantity.

We divide the circle into **sectors** or wedges to show each type or category.

For example, the pie chart alongside shows how the budget of a country is distributed.

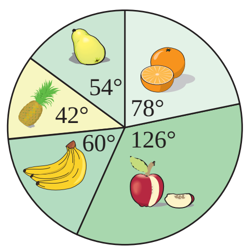


Fruit	*Frequency*
Orange	13
Apple	21
Banana	10
Pineapple	7
Pear	9
Total	60

The table opposite shows the results when a class of year 8 students were asked 'What is your favourite fruit?'.

There are 60 people in the sample, so each person is entitled to $\frac{1}{60}$th of the pie chart. $\frac{1}{60}$th of 360^o is 6^o, so we can calculate the sector angles on the pie chart:

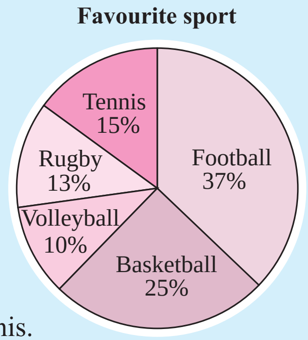
$13 \times 6^o = 78^o$ for the orange sector
$21 \times 6^o = 126^o$ for the apple sector
$10 \times 6^o = 60^o$ for the banana sector
$7 \times 6^o = 42^o$ for the pineapple sector
$9 \times 6^o = 54^o$ for the pear sector.



The completed pie chart is shown alongside.

Example 5 — Self Tutor

The pie chart shows the results of a survey of 120 year 7 students. The students were asked the question: "*What is your favourite sport*?"



Use the chart to determine:

a the most popular sport
b the least popular sport
c the number of students whose favourite sport is basketball
d the number of students whose favourite sport is tennis.

---

a The largest sector angle indicates the most popular sport, and this is football.
b Volleyball is the least popular sport as it has the smallest sector angle.
c 25% of students said basketball is their favourite sport.
So, the number of students whose favourite sport is basketball is

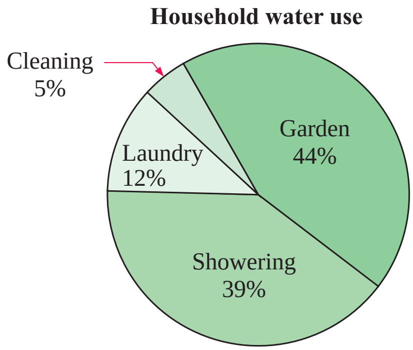
25% of 120
$= \frac{25}{100} \times 120 = 30$ *Calculator:* 25 [÷] 100 [×] 120 [=]

d The number of students whose favourite sport is tennis is

15% of 120
$= \frac{15}{100} \times 120 = 18$ *Calculator:* 15 [÷] 100 [×] 120 [=]

EXERCISE 17C.2

1

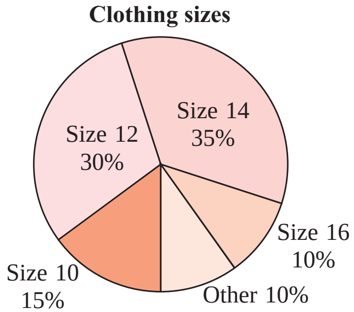


The pie chart alongside illustrates the proportion of water required for various household uses.

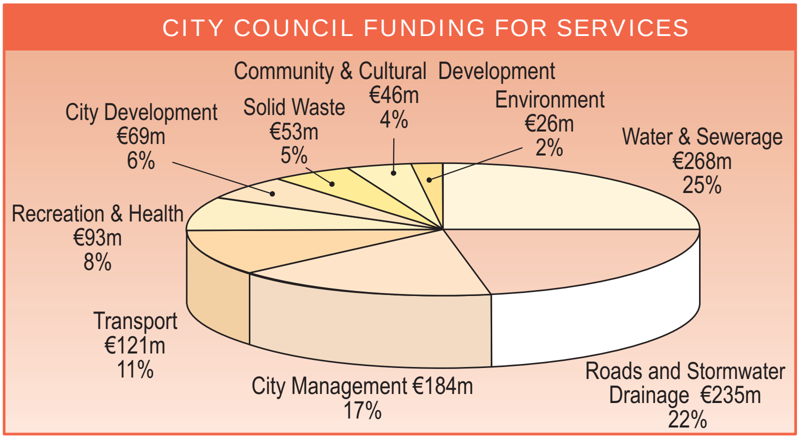
a For what purpose is the most water used?
b For what purpose is the least amount of water used?
c If the household used 400 kilolitres of water during a particular period, estimate the quantity of water used for:
 i showering **ii** cleaning.

2 The pie chart alongside shows the percentages of women in France who wear certain sizes of clothing.

a Find what size is most commonly worn.
b A group of 200 women attends a fashion parade. Estimate how many would wear size 10 clothing.



3 This pie chart shows both the percentages and the actual amounts a council spent in various areas.



a Briefly describe what the graph is about.

b Comment on the usefulness of having both percentages and amounts shown.

c What percentage of total funding is spent on:

i Recreation and Health **ii** Community and Cultural Development?

d How much money is spent on:

i Environment **ii** City Development?

e On what service is the largest amount spent?

f How much is spent in total?

D NUMERICAL DATA

Numerical data is data which is in number form.

Numerical data can be *organised* using a **stem-and-leaf plot** or a **tally and frequency table**. Numerical data is usually represented graphically by a **column graph**.

STEM-AND-LEAF PLOTS

A **stem-and-leaf plot** can be used to write a set of data in order.

For example, the weights (in kg) of army recruits are:

101, 91, 83, 84, 72, 93, 67, 85, 79, 87, 78, 89, 68,
80, 107, 70, 85, 64, 95, 76, 87, 74, 68, 59, 82, 77

For each data value, the units digit is used as the **leaf**, and the digits before it determine the **stem** on which the leaf is placed.

So the stem labels are 5, 6, 7, 8, 9, and 10, and they are written under one another in ascending order.

We now look at each data value in turn. The first data value is 101. Its stem label is 10 and its leaf is 1. We record 1 to the right of the stem label 10. The next data value is 91. Its stem label is 9 and its leaf is 1. We record 1 to the right of the stem label 9. Using this method we record all the data in an unordered stem-and-leaf plot.

Unordered stem-and-leaf display of weight data

unit = 1 (↑ scale)

Stem	Leaf
5	9
6	7 8 4 8
7	2 9 8 0 6 4 7
8	3 4 5 7 9 0 5 7 2
9	1 3 5
10	1 7

(a stem label → 9)

Ordered stem-and-leaf display of weight data

Stem	Leaf
5	9
6	4 7 8 8
7	0 2 4 6 7 8 9
8	0 2 3 4 5 5 7 7 9
9	1 3 5
10	1 7

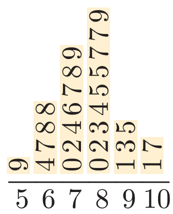
(a stem ← 6 | 4 7 8 8; a leaf ← 9)

The leaves on each stem are now written in ascending order. So for the stem 6 | 7 8 4 8 we write 6 | 4 7 8 8. This gives an ordered stem-and-leaf plot.

Notice that:

- 6 | 4 7 8 8 represents the four scores 64, 67, 68 and 68.
- The leaves are placed in ascending order.
- The scale (unit = 1) tells us the place value of each leaf. If the scale was 'unit = 0.1' then 6 | 4 7 8 8 would represent 6.4, 6.7, 6.8, 6.8
- Rotating the diagram, we see the shape of a column graph.

Stem	Leaf
5	9
6	4788
7	0246789
8	023455779
9	135
10	17



Example 6 — Self Tutor

A fisherman recorded the total weight of all schnapper he caught each day. Construct a stem-and-leaf plot for the data shown below (in kg):

11, 16, 07, 25, 39, 26, 14, 17, 18, 31
31, 25, 43, 32, 25, 19, 16, 08, 34, 21

Stem-and-leaf display for schnapper catch

Stem	Leaf
0	7 8
1	1 4 6 6 7 8 9
2	1 5 5 5 6
3	1 1 2 4 9
4	3

unit = 1 kg

EXERCISE 17D.1

1

The weights of 24 football players were recorded to the nearest kg as follows:

72	63	90	70	67	71	89	64	93	86
66	78	75	89	80	91	81	72	87	72
86	84	84	87						

Construct a stem-and-leaf plot to display this data.

2 The weights of 30 fifteen week old piglets were recorded to the nearest kg as follows:

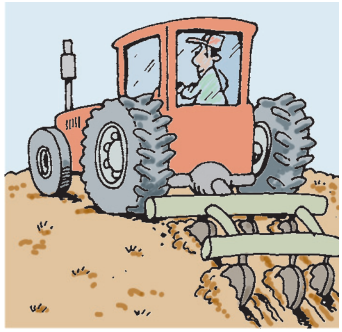
18	20	30	30	25	19	30	34	28	36	32	33	38	13	37
29	43	50	20	44	23	27	27	47	37	17	38	51	29	39

Construct a stem-and-leaf plot to display this data.

3 The time in hours taken by a farmer to plough, fertilise, and seed each of his paddocks is given below:

7	24	9	12	41	30	36
28	18	27	32	24	13	25

Construct a stem-and-leaf plot to display this data.



4 The time (in hours) taken by farmers to travel to their nearest town centre is given below:

1.0	2.4	0.9	1.2	3.6	3.0	0.7
0.8	1.8	2.7	0.2	2.4	1.3	0.5

Construct a stem-and-leaf plot to display the data, stating the scale used.

WORKING WITH NUMERICAL DATA

Example 7 **Self Tutor**

An exceptional hockey player scores the following number of goals over a 25 match period: 4 3 6 1 5 8 4 2 2 4 6 0 5 1 9 3 7 2 6 6 8 3 6 2 10

a Organise the data in a tally and frequency table.

b Graph the data on a column graph.

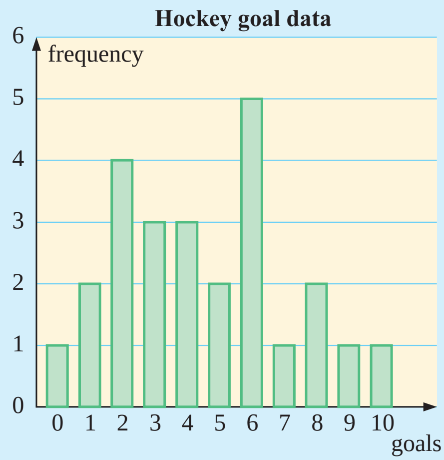
c On how many occasions did the player score 5 or more goals in a match?

d On what percentage of occasions did the player score 4 or more goals in a match?

a

Goals	*Tally*	*Frequency*
0	|	1
1	||	2
2	||||	4
3	|||	3
4	|||	3
5	||	2
6	~~||||~~	5
7	|	1
8	||	2
9	|	1
10	|	1

b



c $2+5+1+2+1+1=12$ times

d He scored 4 or more goals on 15 occasions.

So, the percentage $= \frac{15}{25} \times 100\%$

$= \frac{3}{5} \times 100\%$

$= 60\%$

EXERCISE 17D.2

1 **a** Complete a frequency distribution table for the number of children in 30 families:

0, 4, 6, 2, 1, 3, 2, 4, 0, 2, 1, 2, 5, 0, 2, 3, 1, 4, 2, 1, 2, 4, 3, 3, 0, 4, 5, 2, 2, 4

b Use your table to find the:

i number of families with two children

ii fraction of families with three children.

2 Following are the ages of children at a party:

12, 11, 17, 12, 14, 13, 11, 12, 15, 13, 12, 14, 11, 14, 12, 10, 12, 11, 13, 14

a Organise the data in a tally and frequency table.

b How many children attended the party?

c How many of the children were aged 12 or 13?

d What percentage were 13 or more years old?

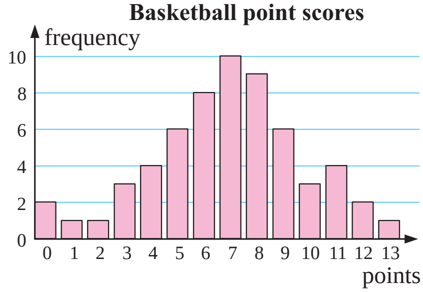
e Display the data on a column graph.

3 The given graph shows the number of points scored by a basketballer over a 60-match period.

a What point score occurred most frequently?

b On how many occasions were 10 or more points scored?

c In what percentage of matches were fewer than 5 points scored?



4 The numbers of goals kicked by a football player each match for the 2008 season were:

3 0 4 2 0 3 3 1 2 1 1 2 3 3 2 2 5 0 2 1 4 3

a Complete a frequency table for the given data.

b Use the table to find the number of games where the player kicked:

i exactly 3 goals **ii** at least 3 goals.

5 A record was kept of the number of goals scored by a goal shooter in netball games during the season. The results were:

10, 7, 8, 5, 8, 7, 10, 10, 6, 11, 5, 7, 7, 12, 7, 11, 6, 5, 8, 8, 7

a Complete a frequency table for the data above.

b Draw a column graph of the data.

c Find the number of games in which the shooter scored:

i exactly 8 goals **ii** at least 8 goals.

6 It is stated on match-boxes that the average contents is 50. When 40 boxes were sampled, the following numbers of matches were counted:

48, 51, 49, 50, 51, 52, 50, 48, 49, 51, 50, 53, 48, 49, 51, 50, 52, 49, 50, 52,
51, 48, 50, 49, 50, 51, 52, 50, 49, 48, 52, 50, 51, 49, 50, 50, 48, 53, 52, 49

- **a** Prepare a frequency table for this data.
- **b** How many boxes had exactly 50 matches?
- **c** How many boxes had 50 or more matches?
- **d** What fraction of boxes had less than 50 matches?
- **e** Do you think the manufacturer's claim is valid?

E MEAN OR AVERAGE

The **mean** or **average** of a set of numbers is an important measure of their middle. We talk about averages all the time. For example:

- the average speed of a car
- average height and weight
- the average score for a test
- the average wage or income.

The **mean** or **average** is the total of all scores divided by the number of scores.

For example, the mean of 2, 3, 3, 5, 6 and 11 is

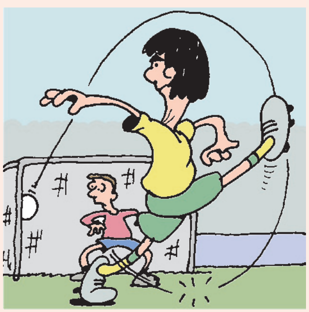
$$\frac{2+3+3+5+6+11}{6} \quad \{\text{there are 6 scores}\}$$

$$= \frac{30}{6}$$

$$= 5$$

DISCUSSION — COMPARING DATA

Discuss how averages can be used to compare different sets of data. You may wish to consider these statements:



- In the last World Cup, Brazil scored an average of 2.3 goals per match. Germany scored an average of 1.5 goals per match.
- The X8 model travels 11.6 km per litre of fuel, whereas the Z3 travels 12.7 km per litre.
- In American Football, why is the average height and weight of the players important?

Example 8 Self Tutor

Find the mean of 7, 11, 15, 6, 11, 19, 23, 0 and 7.

$$\text{Mean} = \frac{7+11+15+6+11+19+23+0+7}{9} = \frac{99}{9} = 11$$

EXERCISE 17E

1 Find the mean of 1, 2, 3, 4, 5, 6 and 7.

2 Calculate the mean of the scores 7, 8, 0, 3, 0, 6, 0, 11 and 1.

3 The weights of a group of newborn ducklings are: 60 g, 65 g, 62 g, 71 g, 69 g, 69 g
Find the average birthweight of the ducklings.

4 In a skijumping competition, Lars jumps the following distances: 110 m, 112 m, 118 m, 103 m, 122 m
Calculate the average length of Lars' ski jumps.

5 In a basketballer's last 12 games of a season he scored 23, 18, 36, 29, 38, 44, 18, 52, 47, 20, 50, and 42 points. What was his mean point score over this period?

6 Baseballers Sean and Rick each throw a set of baseball pitches. The speeds of their pitches, in kilometres per hour, are:
Sean: 130, 135, 131, 119, 125 Rick: 132, 125, 138, 121, 129

a Find the average speed of the pitches thrown by each baseballer.

b Who has the fastest average pitching speed?

7 Compare the performance of two groups of students in the same mental arithmetic test out of 10 marks.
Group X: 7, 6, 6, 8, 6, 9, 7, 5, 4, 7 *Group Y:* 9, 6, 7, 6, 8, 10, 3, 9, 9, 8, 9

a Calculate the mean of each group.

b There are 10 students in *group X* and 11 in *group Y*. Because of unequal numbers in each group it is unfair to compare their means. True or false?

c Which group performed better at the test?

8 The given data shows the goals scored by girls in the local netball association.

a Find the mean number of goals for each goal shooter.

b Which goal shooter has the best average performance?

Name	*Goals*	*Games*
Sally Brown	238	9
Jan Simmons	235	10
Jane Haren	228	9
Peta Piper	219	7
Lee Wong	207	8
Polly Lynch	199	7
Sam Crawley	197	6

ACTIVITY — A POSSIBLE STATISTICAL EXPERIMENT

In this activity you will grow wheat over a 21 day period in a controlled experiment. You will use 6 grains of wheat in each of 4 plots.

You will need: 4 saucers or coffee jar lids, cotton wool, 24 grains of wheat, measure, eye dropper, diluted liquid fertiliser.

What to do:

1. Layer the cotton wool three quarters of the way up each lid. Place 6 grains of wheat at equal distances apart in each lid.
2. Label the lids as plots 1, 2, 3 and 4. Saturate each plot with 15 mL of water.
3. Over a 3 week period, perform the following:
   - In plot 1 squeeze 2 drops of water onto each grain of wheat every weekday.
   - In plot 2 squeeze 2 drops of water onto each grain of wheat every Monday, Wednesday and Friday.
   - In plot 3 squeeze 2 drops of water and 1 drop of diluted fertiliser onto each grain every Monday, Wednesday and Friday.
   - In plot 4 squeeze 2 drops of water and 1 drop of diluted fertiliser onto each grain every weekday.
4. Place all the plots in the same safe, sheltered place with plenty of light.
5. Every Monday, Wednesday and Friday, record the mean height of any germinating seeds for each plot. Avoid handling any shoots. Make a table to summarise your results.
6. Use graphs and the language of statistics to comment on your results.

KEY WORDS USED IN THIS CHAPTER

- average
- bar graph
- categorical data
- dot plot
- frequency
- frequency table
- inference
- mean
- mode
- numerical data
- pie chart
- population
- random sample
- sample
- stem-and-leaf plot
- tally and frequency table

REVIEW SET 17A

1 The data below represents birth months in a year 7 class. January is represented by the number 1, February by the number 2, and so on up to December which is 12. Boys are shown in black and girls in blue.

6 7 3 9 5 5 9 12 10 4 1 12 6 3 5
7 7 4 10 3 7 1 9 5 9 4 8 7 11 4

a Prepare a tally and frequency table to show this data.

b Answer the following questions:

i How many students were in the class?

ii How many girls were in the class?

iii What fraction of the class was born in April?

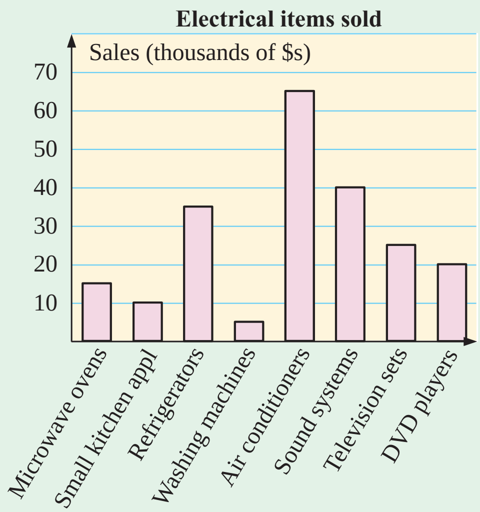
iv What percentage of the class was born in March?

2 In a diving competition, Sally's final dive was awarded the following scores:
8.8 9.1 8.9 9.0 9.2 8.6 8.8

a Find the mean of the 7 scores given.

b If the highest and lowest scores were left out, what would be the average of the 5 remaining scores?

3 The column graph represents the value of one month's sales at Stan's Super Savings Store.



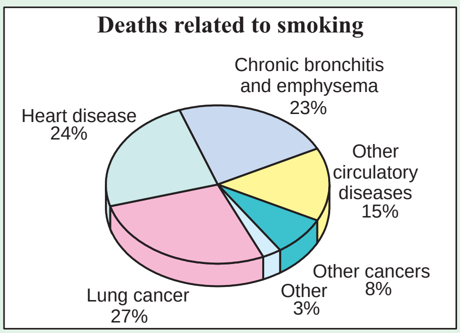
a **i** What goods represent the highest value of electrical items sold?

ii Give two reasons why this may have happened.

b What was the total value of goods sold?

c If 200 small kitchen appliances like kettles and toasters were sold, what was their average price?

4 Use the pie chart to answer the following questions:



a What was the major disease causing death as a result of smoking?

b What 2 groups of diseases made up 50% of all smoking related deaths?

c If 20 000 people died in one year as a result of smoking, estimate how many died from:

i heart disease **ii** lung cancer

iii other cancers?

REVIEW SET 17B

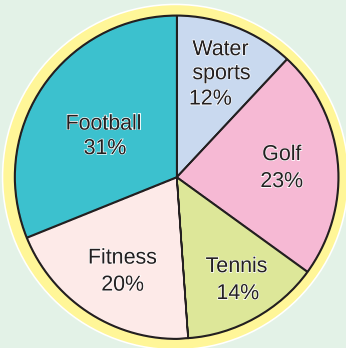
1 A medal is awarded to the best and fairest player in a national sporting competition. Umpires award 3 votes to the player they feel was the best and fairest in each game. 2 votes are awarded for second best, and 1 vote for the third best.

Listed below are the votes awarded to a recent winner. The first vote from the left was for the first game, the second vote was for the second game, and so on.

0 2 0 3 1 0 3 2 3 1 1 3 2 0 3 2 1 0 3 2

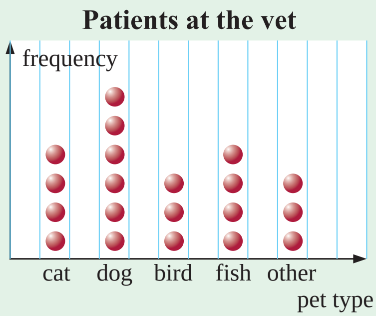
- **a** Construct a frequency table showing the votes awarded to the winner in each game.
- **b** Draw a column graph to show the frequency of the votes.
- **c** In how many games did the winner not receive votes?
- **d** What was the winner's total vote?
- **e** In what percentage of games did the winner receive votes?
- **f** What was the mean number of votes the winner received per game?

2 The given pie chart represents the sale of €100 000 worth of goods by a sports store during its February sale.



- **a** Gear for which sport sold best?
- **b** What value of goods for water sports and tennis was sold?
- **c** The same percentage of goods was sold in the sports store's $\frac{1}{2}$ million euro 'End of Year Sale'.
  - **i** What value of fitness gear was sold?
  - **ii** What was the total amount of tennis and water sports sales?

3 The dot plot shows the types of pets treated at a vet on one day.



- **a** How many pets were treated on this day?
- **b** Find the mode of the data.
- **c** What percentage of the pets treated were fish?

4 Bill's Bakery advertises a new variety in its range of pastries. The daily sales of the new variety are: 23, 25, 18, 21, 17, 14, 15, 19, 18, 11, 15, 12, 6, 9.
Find the mean of the daily sales.

5 The time in minutes taken for customers at a restaurant to receive their meals is given below:

15 28 31 8 22 18 35 24 15 9 28 17 21 20 13

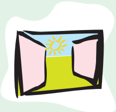
- **a** Construct a stem-and-leaf plot to display this data.
- **b** Find the average time for the customers to receive their meals.

Chapter 18

Algebra and patterns

Contents:

OPENING PROBLEM



Consider the following pattern created using matchsticks. We start with figure 1, add some matchsticks to make figure 2, add more to make figure 3, and so on.

Figure: 1 2 3

Figure 1 is made using 3 matchsticks, figure 2 is made using 7 matchsticks, and figure 3 is made using 11 matchsticks.

Things to think about:

- How many matchsticks do we need to add each time to move from one figure to the next?
- Can you write down a formula for the number of matchsticks used in figure n?
- How many matchsticks are needed to make figure 32?

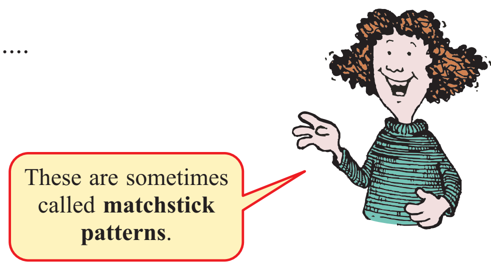
A PATTERNS

You have probably already seen some **number patterns** or **sequences**. For example:

- 2, 5, 8, 11, 14, 17, ... where 3 is added to one number to get the next one.
- 39, 35, 31, 27, 23, ... where 4 is subtracted from one number to get the next one.

Patterns also exist with geometric shapes. These are called **geometric patterns**. For example:

- 
- 



INVESTIGATION THE TRIANGLES PATTERN

Consider the pattern:

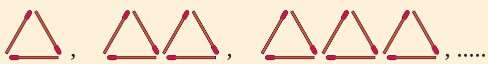


Figure: 1 2 3

What to do:

1. Copy the given pattern and draw the next 4 figures.
2. Copy and complete the table showing the number of matches required to make each figure.

Figure number	1	2	3	4	5	6	7
Matches needed	3	6					

3 Without drawing them, write down the number of matches needed to make the figures 8, 9, 10 and 11.

4 Predict the number of matchsticks needed to make figure:

a 30 **b** 50 **c** 200 **d** 1000.

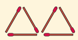
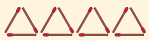
5 Can you predict the number of matchsticks required to make figure n?

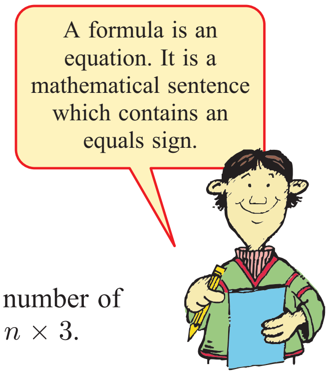
From the **Investigation** you should have made these discoveries about the triangle pattern

Figure number	1	2	3	4	5	6	7
Matches needed	3	6	9	12	15	18	21

+3 +3 +3 +3 +3 +3

To get the next figure we add 3 to the previous one. We can hence construct a table which shows the number of matches required for each figure.

Figure number	*Figure*	*Matches needed*
1		$3 = 1 \times 3$
2		$6 = 2 \times 3$
3		$9 = 3 \times 3$
4		$12 = 4 \times 3$



So, for figure 100, the number of matches needed is 100×3.

For figure n, where n could be any positive whole number, the number of matches M that we need is given by the **rule** or **formula** $M = n \times 3$.

Example 1 Self Tutor

Consider the matchstick pattern: 

a Draw the next *two* figures of the pattern.

b Copy and complete:

Figure number (n)	1	2	3	4	5	6
Matches needed (M)	1	3				

c Write a formula connecting the figure number n and the matches needed M.

a 

b

Figure number (n)	1	2	3	4	5	6
Matches needed (M)	1	3	5	7	9	11

+2 +2 +2 +2 +2

c We notice that each time the *figure number* n goes up by one, the *matches needed* M goes up by two.

We therefore compare "$2 \times n$" with M:

$2 \times n$	2	4	6	8	10	12
Matches needed (M)	1	3	5	7	9	11

From the table we can see that:

Matches needed $= 2 \times$ figure number $- 1$ or $M = 2 \times n - 1$

EXERCISE 18A

1 Consider the matchstick pattern:  , ...

a Draw the next two figures of the pattern.

b Copy and complete:

Figure number (n)	1	2	3	4	5
Matches needed (M)	1	3			

c As the value of n increases by 1, what happens to the value of M?

d Copy and complete:

$2 \times n$	2	4			
M	1	3			

e Write down the formula connecting M and n using **d**.

2 Consider the matchstick pattern:  , ...

a Draw the next two figures of the pattern.

b Copy and complete:

Figure number (n)	1	2	3	4	5
Matches needed (M)	4	7			

c As the value of n increases by 1, what happens to the value of M?

d Copy and complete:

$3 \times n$	3	6	9		
M	4	7			

e Write down the formula connecting M and n using **d**.

3 Consider the matchstick pattern:  , ...

a Draw the next two figures of the pattern.

b Copy and complete:

Figure number (n)	1	2	3	4	5
Matches needed (M)	5				

c As the value of n increases by 1, what happens to the value of M?

d Copy and complete:

$3 \times n$	3	6			
M	5				

e Write down the formula connecting M and n using **d**.

4 By following the steps used in questions **1** to **3**, find formulae connecting the number of matchsticks needed (M) to the figure number (n) in:

a  , ...

b  , ...

c  , ...

B VARIABLES AND NOTATION

In **Section A** we discovered **formulae** or **rules** for matchstick patterns. These formulae linked the two **variables**, *number of matchsticks needed* and *figure number*.

We call these quantities variables because they can take many different values. In **algebra** we use letters or symbols to represent variables. This allows us to write formulae more neatly. For example:

Writing the *number of matches needed* as M and the figure number as n, we obtained formulae such as $M = n \times 2 - 1$, $M = 3 \times n + 1$, and $M = 3 \times n + 2$.

PRODUCT NOTATION

Centuries ago mathematicans agreed that to make expressions easier, they would:

- omit the $\times$ sign wherever possible
- write the numbers in products before the variables.

For example:

- $M = n \times 2 - 1$ is written as $M = 2n - 1$
- $\underbrace{\text{the number of matchsticks}}_{M}$ $\underset{=}{\text{is}}$ $\underbrace{\text{three times the figure number}}_{3n}$ $\underbrace{\text{plus}}_{+}$ $\underbrace{\text{one}}_{1}$.

$M = 3n + 1$ is a much shorter statement and is easier to read than when given in words. We say the formula is written in **algebraic form**, as it uses symbols rather than words.

Example 2 Self Tutor

In a matchstick pattern, the number of matchsticks needed is seven times the figure number, minus four.

Rewrite this statement using variables M and n in simplest algebraic form.

If the number of matchsticks is M and the figure number is n, the statement in algebraic form is $M = 7n - 4$.

Where two or more variables are used in an algebraic product we agree to write them in **alphabetical order**.

So, $a \times b$ and $b \times a$ would both be written as ab.

Example 3 Self Tutor

Write using product notation:

a $x \times 2 \times y$ **b** $3 \times x - 2 \times y$

a $x \times 2 \times y = 2xy$ **b** $3 \times x - 2 \times y = 3x - 2y$

EXERCISE 18B

1 Write using product notation:

a $c \times d$ **b** $d \times c$ **c** $a \times b \times c$ **d** $a \times 5$
e $m \times 2 \times n$ **f** $b \times 3 \times a$ **g** $3 \times t + 2$ **h** $n \times 7 - 4$
i $b \times 7 \times a$ **j** $2 \times a \times 5 \times c$ **k** $5 + s \times 3$ **l** $6 - t \times p$
m $11 + q \times p$ **n** $3 \times r - 6$ **o** $b \times a + c \times a$ **p** $c \times 3 + d \times 2$

2 Write in algebraic form:

a The number of matches M is three times the figure number n.
b The number of shapes s is two more than the figure number n.
c The number of matches M is five times the figure number n, plus three.
d The number of shapes N is twice the figure number n, minus four.
e The number of matches M is two more than three times the figure number n.
f To obtain the number of shapes N you add one to the figure number n, then double the result.

C ALGEBRAIC FORM

Formulae are not the only things that can be written in a shorthand way using algebra.

A vital part of algebra is the ability to convert **word sentences** into algebraic form. You will learn this skill with practice.

The following table shows words commonly associated with the 4 operations $+$, $-$, $\times$, $\div$.

$+$	add, sum, exceeds, more than, plus
$-$	subtract, minus, take, less than, difference
$\times$	multiply, times, product, double, twice, treble, triple
$\div$	divide, quotient

Example 4 **Self Tutor**

Write in algebraic form:

a the sum of p and q
b a number 5 times larger than a
c a number which exceeds r by t
d twice the sum of k and 4

a Sum means add, so the sum of p and q is $p + q$.
b The number is $5 \times a$ which is written as $5a$.
c The number which exceeds 7 by 3 is $7 + 3$, so the number which exceeds r by t is $r + t$.
d The whole of $k + 4$ must be doubled, so the number is $2(k + 4)$.

QUOTIENT NOTATION

We have seen previously how $3 \div 4$ can be written as the fraction $\frac{3}{4}$.

In general, we can write any division in algebra using $\boldsymbol{a \div b = \frac{a}{b}}$

Example 5 Self Tutor

Write in algebraic form:

a the average of a and b

b a divided by the sum of b and c

a The average of 3 and 5 is $\frac{3+5}{2}$,
so the average of a and b is $\frac{a+b}{2}$.

b a is divided by the whole of $b+c$,
so we write $\frac{a}{b+c}$.

EXERCISE 18C

1 Write in algebraic form:

a m plus n
b 3 more than a
c b minus c
d a third of g
e double n
f treble y
g 4 more than a
h 2 less than d
i d more than a
j q less than r
k four times n
l twice n, add 5

2 Write in algebraic form:

a the average of m and n
b the sum of x and y, divided by 4
c 5 divided by the sum of r and s.

3 Copy and complete:

a The product of 3 and 8 is ......
The product of 3 and m is ......
The product of a and m is ......

b If I subtract 5 from 9 I get ......
If I subtract 5 from d I get ......
If I subtract c from d I get ......

c The number 2 less than 6 is ......
The number 2 less than a is ......
The number x less than a is ......

d The number which exceeds 7 by 5 is ......
The number which exceeds 7 by t is ......
The number which exceeds r by t is ......

e The number of cents in \$3 is ......
The number of cents in \$$D$ is ......

f The number of dollars in 400 cents is .....
The number of dollars in c cents is ......

4 **a** There are 11 people on an aeroplane and x get off. How many are left on the aeroplane?

b A train has 86 passengers and y more get on at the next station. How many are now on the train?

c A hotel has x floors with y apartments per floor. Each apartment contains 4 rooms. How many rooms does the hotel have in total?

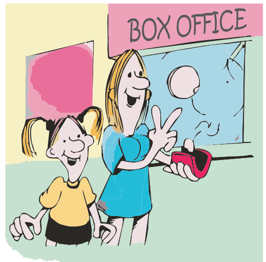
d A can of soft drink costs c cents. How much would 7 cans cost in dollars?

5 A man is n years old.

a How old was he 12 years ago?

b His wife is 6 years older than he is. How old is she?

c His mother is twice his age. How old is she?



6 Concert tickets cost €40 for adults and €15 for children. If a group of x adults and y children attend, write an expression for the total cost.

D THE VALUE OF AN EXPRESSION

Consider the algebraic expression $3x + 5$.

When $x = 2$,
$$3x + 5 = 3 \times 2 + 5 = 6 + 5 = 11$$

When $x = -4$,
$$3x + 5 = 3 \times (-4) + 5 = -12 + 5 = -7$$

Notice that first we multiplied by 3 and then added 5.
We did this to follow the BEDMAS order.

Example 6 Self Tutor

Find the value of:

a $4x - 11$ when $x = -2$

b $\frac{x}{3} + 2$ when $x = 12$

c $4(x + 3)$ when $x = 7$

d $\frac{x+2}{3}$ when $x = -8$

a
$$4x - 11 = 4 \times (-2) - 11 = -8 - 11 = -19$$

b
$$\frac{x}{3} + 2 = \frac{12}{3} + 2 = 4 + 2 = 6$$

c
$$4(x + 3) = 4(7 + 3) = 4(10) = 40$$

d
$$\frac{x+2}{3} = \frac{(-8) + 2}{3} = \frac{-6}{3} = -2$$

EXERCISE 18D

1 Find the value of:

a $5x - 3$ when **i** $x = 4$ **ii** $x = 0$ **iii** $x = -3$

b $5 - 2x$ when **i** $x = 3$ **ii** $x = 10$ **iii** $x = \frac{1}{2}$

c $\frac{x}{2} + 5$ when **i** $x = 6$ **ii** $x = -4$ **iii** $x = 3$

d $6 - \frac{x}{3}$ when **i** $x = 12$ **ii** $x = -6$ **iii** $x = 4$

e $\dfrac{x+1}{3}$ when **i** $x=2$ **ii** $x=-4$ **iii** $x=4$

f $\dfrac{2x-1}{5}$ when **i** $x=8$ **ii** $x=4$ **iii** $x=-3$

2 Evaluate:

a $3x+7$ for $x=1,\ 3$ and -2

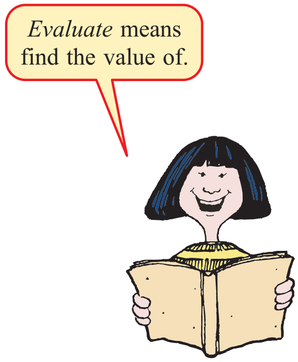
b $3-5x$ for $x=2,\ -3$ and -7

c $\dfrac{x}{3}-2$ for $x=6,\ 1$ and -4

d $5-\dfrac{x}{6}$ for $x=30,\ -6$ and 5

e $\dfrac{x+4}{3}$ for $x=5,\ -13$ and $\frac{1}{2}$

f $\dfrac{3x+1}{2}$ for $x=13,\ -2$ and $\frac{2}{3}$



E SUBSTITUTING INTO FORMULAE

Suppose we have a formula which connects two variables. If we are given the value of one of these variables, we can **substitute** it into the formula to find the value of the other variable.

For example, consider the pattern:  , ...

The formula for the number of matchsticks M in figure n is $M=2n-1$. We can substitute the value $n=10$ to find the number of matchsticks in figure 10.

When $n=10$, $M=2\times 10-1=19$

So, there are 19 matchsticks in figure 10.

Example 7 **Self Tutor**

Copy and complete the table by substituting into the formula $W=3t+2$:

t	1	3	6	15
W				

The formula is $W=3t+2$

When $t=1$,
$W=3\times 1+2$
$\therefore W=3+2$
$\therefore W=5$

When $t=3$,
$W=3\times 3+2$
$\therefore W=9+2$
$\therefore W=11$

When $t=6$,
$W=3\times 6+2$
$\therefore W=18+2$
$\therefore W=20$

When $t=15$,
$W=3\times 15+2$
$\therefore W=45+2$
$\therefore W=47$

We can now complete the table:

t	1	3	6	15
W	5	11	20	47

EXERCISE 18E

1 Use substitution to help complete the tables for the given formulae:

a $S = n + 3$

n	1	2	3	4	5
S					

b $L = 4b$

b	1	3	5	7	9
L					

c $C = 2d + 7$

d	1	4	8	10	15
C					

d $P = 3t - 4$

t	1	2	5	9	15
P					

2 Substitute the given values of x into the formulae to find the values for y in each case:

a $y = 5x + 3$ if **i** $x = 4$ **ii** $x = 6$ **iii** $x = 15$

b $y = 7x - 5$ if **i** $x = 2$ **ii** $x = 3$ **iii** $x = 10$

c $y = 3(x + 2)$ if **i** $x = 5$ **ii** $x = 12$ **iii** $x = 20$

d $y = 50 - 4x$ if **i** $x = 5$ **ii** $x = 8$ **iii** $x = 10$

e $y = 2(20 - x)$ if **i** $x = 2$ **ii** $x = 9$ **iii** $x = 14$

f $y = 4(x + 1) - 3$ if **i** $x = 3$ **ii** $x = 7$ **iii** $x = 24$

Example 8 **Self Tutor**

The cost of hiring a tennis court is given by the formula $C = 5h + 8$ where C is the cost in dollars and h is the number of hours the court is hired for. Find the cost of hiring the tennis court for: **a** 4 hours **b** 10 hours.

The formula is $C = 5h + 8$

a Substituting $h = 4$ we get

$C = 5 \times 4 + 8$
$= 20 + 8$
$= 28$

So, it costs \$28 for 4 hours.

b Substituting $h = 10$ we get

$C = 5 \times 10 + 8$
$= 50 + 8$
$= 58$

So, it costs \$58 for 10 hours.

3 The cost of staying at a hotel is given by the formula $C = 50d + 20$ where C is the cost in £ and d is the number of days a person stays. Find the cost of staying for:

a 3 days **b** 6 days **c** 2 weeks

4 For most medicines, the dose a child should take depends on the child's age, a years. One particular medicine has the following rule for calculating the dose D mL:

$$D = \frac{50 \times a}{a + 12}.$$

Find the dose for children aged:

a 4 years **b** 8 years **c** 12 years.

Example 9 **Self Tutor**

Examine the matchstick pattern: 

a Copy and complete:

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)	2				

b Find the rule connecting M and n.

c Find the number of matchsticks needed to make figure 90.

a

n	1	2	3	4	5
M	2	5	8	11	14

+3 +3 +3 +3

b The increases in M by 3 tell us to multiply n by 3.

$3n$	3	6	9	12	15
M	2	5	8	11	14

Since the M values are 1 less than the $3n$ values, $M = 3n - 1$.

c When $n = 90$, $M = 3 \times 90 - 1$
$= 270 - 1$
$= 269$

So, 269 matches are needed to make figure 90.

5 Examine the matchstick pattern: 

a Copy and complete:

Figure number (n)	1	2	3	4	5	6
Matchsticks needed (M)						

b Find the rule connecting M and n.

c Find the number of matchsticks needed to make figure 63.

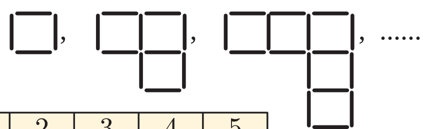
6 Examine the matchstick pattern: 

a Copy and complete:

Figure number (n)	1	2	3	4	5	6
Matchsticks needed (M)						

b Find the rule connecting M and n.

c Find the number of matchsticks needed to make figure 75.

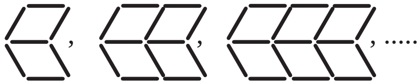
7 Look at the following pattern: 

a Copy and complete:

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)					

b Find the rule connecting M and n.

c Find the number of matchsticks needed to make figure 57.

8 Look at the following pattern: 

a Copy and complete:

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)					

b Find the rule connecting M and n.

c Find the number of matchsticks needed to make figure 80.

9 Examine the following matchstick pattern: 

a Copy and complete:

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)					

b Find the rule connecting M and n.

c Find the number of matchsticks needed to make figure 29.

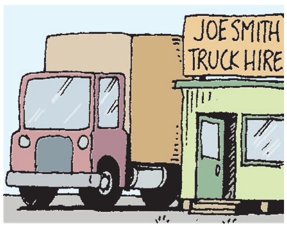
F PRACTICAL PROBLEMS USING FORMULAE

We can construct formulae to help us with many practical situations that involve patterns.

For example:

Joe Smith hires small trucks for moving furniture.

He charges an initial fee of £14 plus £3 per kilometre travelled.



Distance travelled	*Hire fee*
0 km	£14
1 km	£17
2 km	£20
3 km	£23
⋮	

We can construct a table to show Joe's fees such as the one above. Notice that each time the distance travelled increases by 1 km, the fee increases by £3. So, the fee for just the distance d kilometres is £$3d$.

We then need to add on the £14 initial fee.

So, the total hire fee H for travelling d kilometres is $H = 3d + 14$ pounds.

If Erin travels 38.7 km, she would have to pay Joe $3 \times 38.7 + 14$ pounds

$= £130.10$

DISCUSSION

Why is Joe's formula very easy to write down, unlike the geometric matchstick patterns we have seen previously?

Example 10 **Self Tutor**

A taxi company charges €3 'flagfall' and €1.80 for each kilometre travelled.
Suppose the total charge is €C for travelling n kilometres. Find:

a the cost just for travelling the distance n kilometres
b the formula connecting C and n
c the total charge for travelling 21.6 km.

a The charge for each kilometre is €1.80, so the cost for just the distance n kilometres is €$1.80n$.

b To find the total charge C we need to add on the 'flagfall' which is €3.
So, the total charge $C = 1.80n + 3$ euros.

c When $n = 21.6$,
$$C = 1.80 \times 21.6 + 3$$
$$= 41.88$$
So, the total charge for travelling 21.6 km is €41.88 .

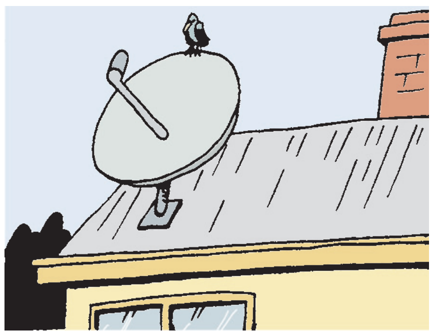
EXERCISE 18F

1 On weekends a mechanic charges a \$40 callout fee plus \$30 for every hour he spends fixing a car.
So, for a breakdown taking two hours to fix, the total cost would be $\$40 + 2 \times \$30 = \$100$.

a What is the charge for doing h hours work (excluding the callout fee)?
b If C is the total charge (in dollars) for a job taking h hours, what is the formula connecting C and h?
c Find the total charge for a callout taking:
i 1 hour **ii** $3\frac{1}{4}$ hours **iii** 4 hours 12 min.

2 A satellite TV company charges €75 installation and €42.00 per month from then on.

a How much will the monthly fee amount to after m months?
b If the total cost is €C for m months, what formula connects C to m?
c Use the formula to find the cost to install and use the satellite TV for a period of:
i 7 months **ii** $4\frac{1}{2}$ years.



3 On a rainy day the flow of a river increases by 2 cumecs each hour. When the rain starts, the river flow is 8 cumecs.

a Calculate the flow after 9 hours of rain.
b Write a formula to show how you got your answer.
c Construct a table to show the flow each hour for 9 hours.

4 Below is a table of fees which an electrician charges for jobs of different length:

Number of hours (h)	1	2	3	4	5
Cost of job (C)	\$70	\$110	\$150	\$190	\$230

a Find the rule connecting C and h.

b Find the cost of a job taking: **i** 23 hours **ii** $17\frac{1}{2}$ hours.

5 A runner in a 14 km road-race starts quickly but then slows. She runs the first kilometre in 5 minutes but then takes an extra 12 seconds for each kilometre afterwards.

a Write a formula which gives the time for the nth kilometre of the race.

b How long does she take to run the:

i 6th kilometre **ii** 13th kilometre?

c How long does it take the runner to finish the race?

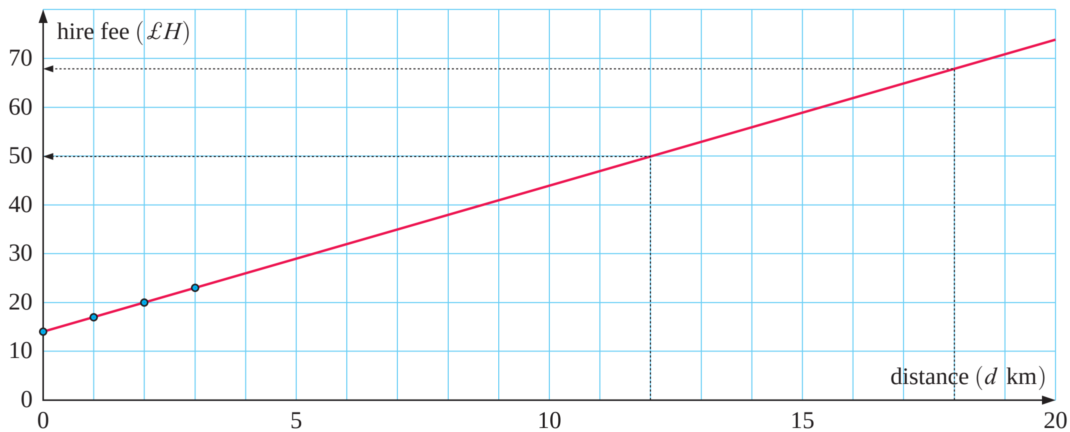
G LINEAR GRAPHS

Consider again Joe's fees for hiring small trucks on page **342**. We saw that the hire fee for travelling d kilometres was $H = 3d + 14$ pounds.

We saw how this formula was related to the table of values:

d (km)	0	1	2	3
H (£)	14	17	20	23

We can also display the hire fees using a graph. We do this by plotting points from the table of values: (0, 14), (1, 17), (2, 20), (3, 23) and connecting them with a straight line. The points lie in a straight line because the fee increases by the same fixed amount for each kilometre driven.



It can easily be read from the graph that when $d = 12$, $H = £50$ and when $d = 18$, $H = £68$.

A **linear graph** has points which lie in a straight line.

EXERCISE 18G

1 **a** Use only the graph above to find the cost of hiring one of Joe's trucks to travel a distance of:

i 7 km **ii** 14 km **iii** 17.7 km

b Why is it easier to use the formula $H = 3d + 14$ for trips of more than 20 km?

2 Draw the graph of P against n from the table which follows:

n	0	1	2	3	4	5	6
P	7	10	13	16	19	22	25

P is the profit in dollars for selling n spanner sets. Make sure that your graph can be extended to $n = 25$.

a Find the profit in selling:

i 10 spanner sets **ii** 18 spanner sets **iii** 22 spanner sets

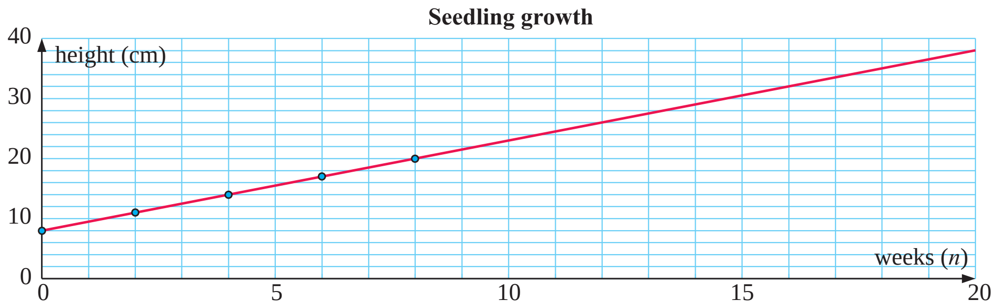
b Tania found a formula for calculating the profit. Her formula was $P = 3 \times n + 7$.

i Check that this formula fits the tabled values.

ii Check your answers to **a**.

iii Find the profit when selling 35 spanner sets.

3 The following graph shows the growth of a seedling over a period of weeks.



a Find the height of the seedling:

i when planted **ii** after 4 weeks **iii** after 16 weeks.

b If the linear trend continues, how long will it take for the seedling to reach a height of:

i 20 cm **ii** 26 cm **iii** 33.5 cm?

KEY WORDS USED IN THIS CHAPTER

- algebra
- quotient
- equation
- linear graph
- product
- rule
- increase
- sum
- number pattern
- substitution
- variables
- geometric pattern
- notation
- formula

REVIEW SET 18A

1 Write in algebraic form:

a $x \times 2 \times y$ **b** $M = 3 \times n + d$ **c** $b \times a + c \times 3$

d $n \div 3$ **e** $(a + b) \div c$ **f** $100 \div (x - 3)$

2 Write in algebraic form:

a the product of c and 2 **b** treble the sum of a and 6

c 5 more than t **d** d less than n

3 Toothpaste costs £x a tube. What will be the total cost of y tubes?

4 Theatre tickets cost $\$x$ per adult, $\$y$ per senior, and $\$z$ per child. What will be the total cost for 2 adults, 1 senior, and 5 children?

5 If $y = 39 - 4x$, find y when: **a** $x = 2$ **b** $x = 7$.

6 The following pattern is built out of matchsticks:  , ...

a Draw the next 2 figures in the pattern.

b Copy and complete:

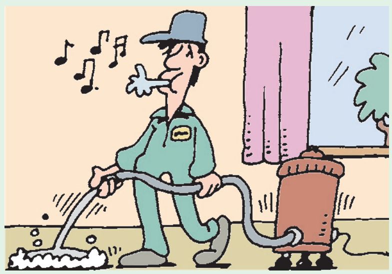
Figure number (n)	1	2	3	4	5
Matchsticks needed (M)					

c Find the rule connecting M and n.

d How many matchsticks are needed to build:

i 7 squares **ii** 101 squares?

7 TLC Carpet Cleaning Company charges a \$20 callout fee and then \$15 for each room it cleans. Copy and complete the table of values for the charge C dollars for cleaning n rooms:



n	0	1	2	3	4	5
C						

a What is TLC's fee formula?

b Find how much TLC would charge for cleaning a mansion with 27 rooms.

c Draw a line graph of C against n with n on the horizontal axis and $n = 0, 1, 2, 3, ..., 10$.

REVIEW SET 18B

1 Write in algebraic form:

a $x \times 5$ **b** $N = 5 \times g - 6$ **c** $m \div 10$

d $c \times a \times 4$ **e** $a + b \div c$ **f** $(a + b) \div m$

2 Write in algebraic form:

a the sum of a and twice b **b** twice the sum of a and b

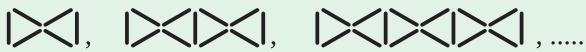
c d more than 3 **d** n less than treble c

3 What is the cost of 8 golf clubs at €x each?

4 If $M = \dfrac{5x+15}{10}$, find M when: **a** $x = 3$ **b** $x = 8$

5 What is the change from £100 when n items are bought costing £d each?

6 The following pattern is built out of matchsticks:



a Copy and complete the table of values:

Figure number (n)	1	2	3	4	5	6
Number of matchsticks (M)						

b Find the rule connecting M and n.

c Write down the rule in sentence form.

d Use the rule to find the number of matchsticks needed for figure 80.

7 Joe sells television sets. He is paid \$400 per week plus \$80 for every television set he sells.

a Copy and complete the following table of values showing the amount Joe earns (E dollars) for selling n television sets:

n	0	1	2	3	4	5
E						

b Write a formula for the amount that Joe earns.

c How much would Joe earn if he sold 8 television sets in a week?

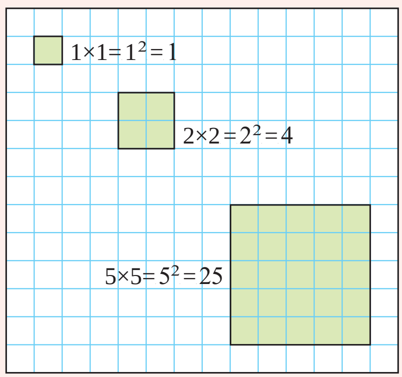
d Draw a line graph of E against n with n on the horizontal axis.

e Use your graph to check your answer to **c**.

ACTIVITY SQUARE NUMBERS

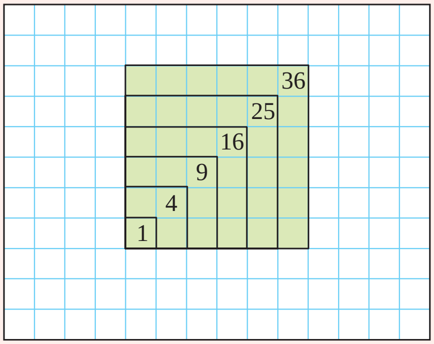
You will need: some sheets of graph paper

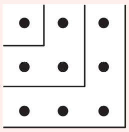
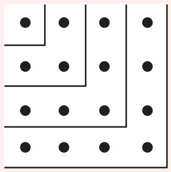
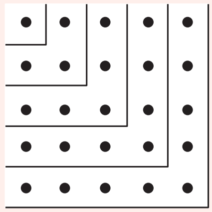
What to do:



1 Near the top left hand corner of the page, shade in one square region.
Leave two squares of space and shade a 2×2 square region. $2 \times 2 = 2^2 = 4$ squares.
Continue this pattern to construct squares 3×3, 4×4, and so on until you cannot fit any more on the page without overlapping.
Write a rule which explains how the square number was produced.

2 Construct overlapping squares as shown.
On the top right corner of each square, write down the **total** number of squares enclosed by the larger square.
On your piece of paper, what is the largest number of smaller squares that you can enclose with a larger square?
List your square numbers in ascending order.



3 **A**  **B**  **C** 

A shows that the sum of the first 3 odd numbers is 3^2. $1+3+5=9$
B shows that the sum of the first 4 odd numbers is 4^2. $1+3+5+7=16$
C shows that the sum of the first 5 odd numbers is 5^2. $1+3+5+7+9=25$

- **a** Draw the next two diagrams in this pattern.
- **b** Write down the next three lines in this pattern.
- **c** Add the first and last number in each sum then divide it by 2. What do you find?
- **d** What numbers when squared are equal to these sums?
  - **i** $1+3+5+7+ ..... +21 =$
  - **ii** $1+3+5+7+ ..... +23 =$
  - **iii** $1+3+5+7+ ..... +35 =$
  - **iv** $1+3+5+7+ ..... +39 =$

4 A square number results from one being added to the product of any four consecutive whole numbers.
For example:
$1\times2\times3\times4+1 = 25 = 5^2$ $\qquad 2\times3\times4\times5+1=121 = 11^2$
Copy and complete:
- **a** $3\times4\times5\times6+1 = ... = 19^2$
- **b** $4\times5\times6\times7+1 = ... = \square^2$
- **c** $5\times6\times7\times8+1 = ... = n^2$
- **d** $6\times7\times8\times9+1 = ... = \Delta^2$
- **e** $7\times8\times9\times10+1 = ... = \bigcirc^2$

Use any four consecutive whole numbers to show that the rule is true.

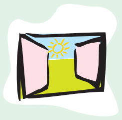
5 Find other patterns and formulae using square numbers to share with the class.

Chapter 19

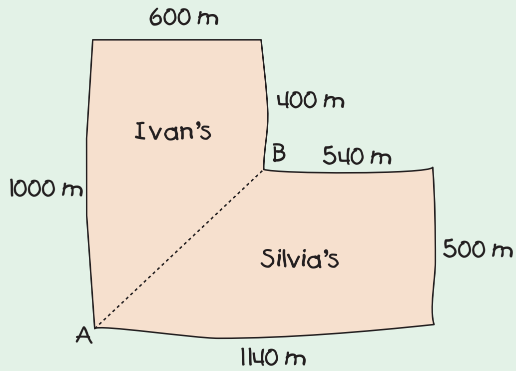
Area, volume and capacity

Contents:

OPENING PROBLEM



Hector owns a farm. Its dimensions are shown in the diagram alongside. Hector has decided to divide his property using a straight fence from A to B. He gives his son Ivan the land on one side of the fence, and his daughter Silvia the land on the other side.



Things to think about:

- How much land did Hector own?
- Who gets more land, Ivan or Silvia?

A AREA

In any house or apartment there are **surfaces** such as carpets, walls, ceilings, and shelves. These surfaces have **boundaries** which define the **shape** of the surface.

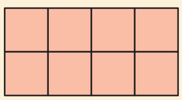
As in the **Opening Problem**, people need to measure the amount of surface within a boundary, whether it be land, a wall, or an amount of dress material.

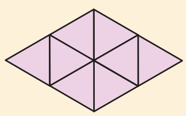
Area is the amount of surface inside a region.

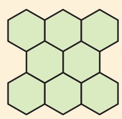
Descriptions on cans of paint, insect surface spray, and bags of fertiliser refer to the area they can cover. Garden sprinklers are designed to spray water over a particular surface area.

INVESTIGATION 1 CHOOSING UNITS OF AREA

Some identical shapes can be placed together to cover a surface with no gaps. For example:


squares


equilateral triangles

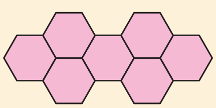

regular hexagons

We can use these shapes to compare different areas.

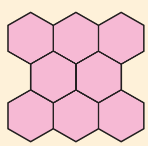
What to do:

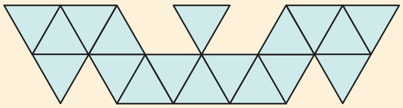
1 Compare the areas of these shapes. For each pair, which has the bigger area?

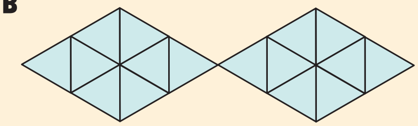
a **A**



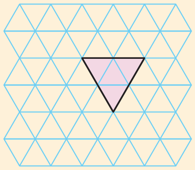
B

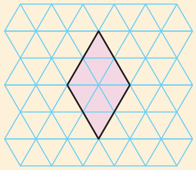


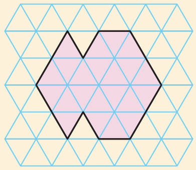
b **A** 

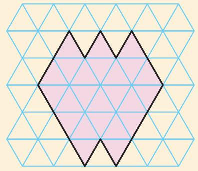
B 

2 Use  as a unit of area to measure the area of each of the shapes:

a 

b 

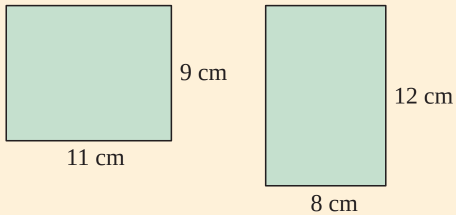
c 

d 

3 Explain why a circle would be an inappropriate choice as a unit of area.

4 Suppose you wish to compare the areas of the two illustrated rectangles.

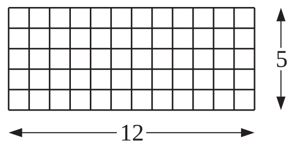
- Give a reason why circular units would not be acceptable.
- Which of the shapes above would be best to use as a measure of area? Give *two* reasons why you think this is so.



SQUARE UNITS

As you have seen, it is possible to compare area using a variety of shapes. Some shapes have advantages over others.

The square has been chosen as the universal unit used to measure area.

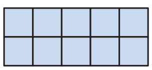


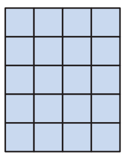
$5 \times 12 = 60$ squares

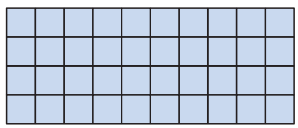
The **area** of a closed figure, no matter what shape, is the number of square units (unit^2 or u^2) it encloses.

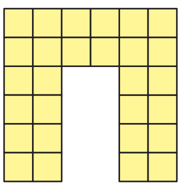
EXERCISE 19A.1

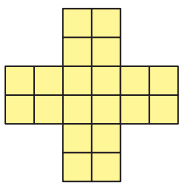
1 Find the area in square units of each of the following shapes:

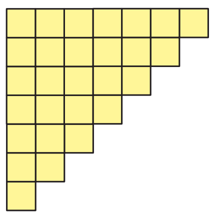
a 

b 

c 

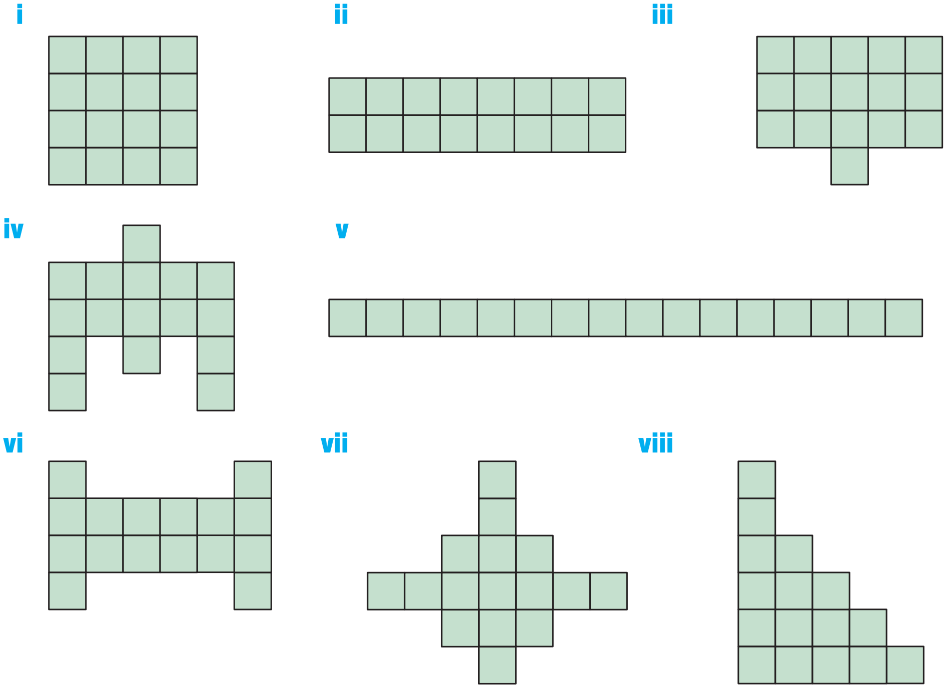
d 

e 

f 

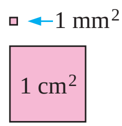
2 **a** Check to see that all of the following shapes have the same area.

b What is the perimeter of each?



c What does this exercise tell you about the area and the perimeter of a shape?

METRIC AREA UNITS



In the metric system, the units of measurement used for area are related to the units we use for length.

> 1 **square millimetre** (mm^2) is the area enclosed by a square of side length 1 mm.
>
> 1 **square centimetre** (cm^2) is the area enclosed by a square of side length 1 cm.
>
> 1 **square metre** (m^2) is the area enclosed by a square of side length 1 m.
>
> 1 **hectare** (ha) is the area enclosed by a square of side length 100 m.
>
> 1 **square kilometre** (km^2) is the area enclosed by a square of side length 1 km.

EXERCISE 19A.2

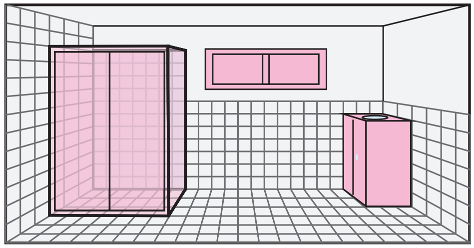
1 What units of area would most sensibly be used to measure the area of the following?

- **a** the floor space in a house
- **b** an envelope
- **c** wheat grown on a farm
- **d** carpet for a doll's house
- **e** a freckle on your skin
- **f** a large island
- **g** a micro chip for a computer
- **h** a bathroom mirror
- **i** a postage stamp
- **j** a page of a book

2 **a** How many tiles have been used for:

i the floor **ii** the walls?

Do not forget tiles behind and under the sink cabinet and in the shower.

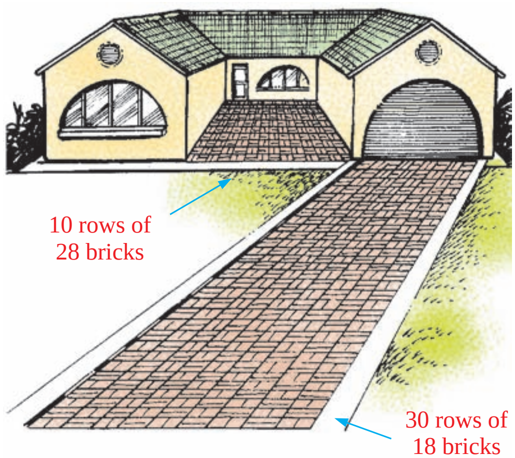


b There are 25 tiles for each square metre. How many square metres of tiles were used?

c The tiles cost €36.90 per square metre and the tiler charged €18.00 per square metre to glue them. What was the total cost of tiling?

3 **a** In the given picture, how many pavers were used for:

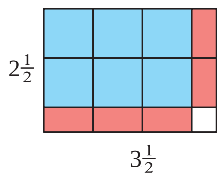
i the driveway **ii** the patio?



b The pavers in the patio are the same as the pavers in the driveway. If there are 50 pavers for every square metre, how many square metres of paving were laid?

c If the cost of the pavers is \$16.90 per m^2, and the cost of laying them is \$14 per m^2, what is the total cost of the paving?

4 Look at the given diagram. It is $3\frac{1}{2}$ units long and $2\frac{1}{2}$ units wide.



a How many blue full units are there?

b How many red $\frac{1}{2}$ units are there?

c What fraction of a unit is the small white square?

d What is the total area of the rectangle?

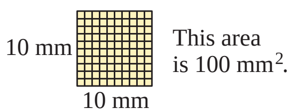
e Calculate $3\frac{1}{2} \times 2\frac{1}{2}$. What do you notice?

B CONVERSION OF AREA UNITS

We can convert from one unit of area to another using length relationships.

For example: $1 \text{ cm} = 10 \text{ mm}$

so $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm}$
$= 10 \text{ mm} \times 10 \text{ mm}$
$= 100 \text{ mm}^2$



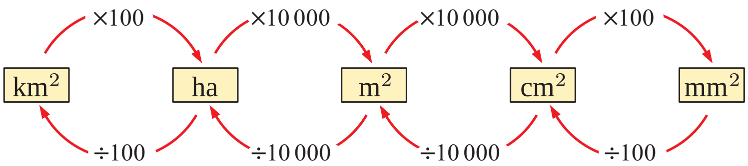
$1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm}$
$= 10\,000 \text{ cm}^2$

$1 \text{ km}^2 = 1000 \text{ m} \times 1000 \text{ m}$
$= 1\,000\,000 \text{ m}^2$

A **hectare** is an area $100 \text{ m} \times 100 \text{ m}$ or $10\,000 \text{ m}^2$.

$1\text{ cm}^2 = 100\text{ mm}^2$	$1\text{ ha} = 10\,000\text{ m}^2$
$1\text{ m}^2 = 10\,000\text{ cm}^2$	$1\text{ km}^2 = 100\text{ ha}$

AREA UNIT CONVERSIONS

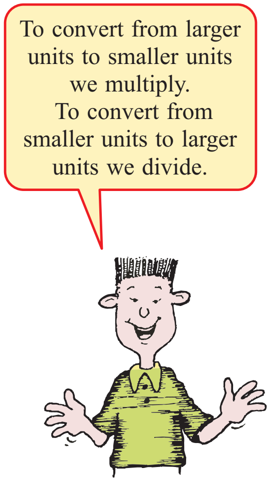


Example 1 **Self Tutor**

Convert: **a** 4.2 m^2 to cm^2 **b** $350\,000\text{ m}^2$ to ha

a 4.2 m^2
$= (4.2 \times 10\,000)\text{ cm}^2$
$= 42\,000\text{ cm}^2$

b $350\,000\text{ m}^2$ to ha
$= (350\,000 \div 10\,000)\text{ ha}$
$= 35\text{ ha}$



EXERCISE 19B

1 What operation needs to be done to convert:

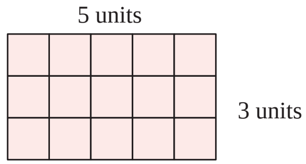
a cm^2 to mm^2 **b** m^2 to cm^2 **c** ha to m^2 **d** km^2 to ha
e m^2 to mm^2 **f** km^2 to m^2 **g** mm^2 to cm^2 **h** cm^2 to m^2
i m^2 to ha **j** ha to km^2 **k** mm^2 to m^2 **l** m^2 to km^2

2 Convert:

a 452 mm^2 to cm^2 **b** 7.5 m^2 to cm^2 **c** 5.8 ha to m^2
d 3579 cm^2 to m^2 **e** 6.3 km^2 to ha **f** 36.5 m^2 to mm^2
g $550\,000\text{ mm}^2$ to m^2 **h** 5.2 cm^2 to mm^2 **i** 6800 m^2 to ha
j 4400 mm^2 to cm^2 **k** 0.6 ha to m^2 **l** 200 ha to km^2
m 0.7 cm^2 to mm^2 **n** 480 ha to km^2 **o** 25 cm^2 to mm^2
p 0.8 m^2 to cm^2 **q** 8800 mm^2 to cm^2 **r** 6600 cm^2 to m^2
s 0.5 km^2 to ha **t** 550 ha to km^2 **u** 10 cm^2 to m^2

C THE AREA OF A RECTANGLE

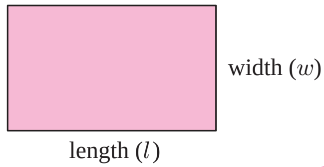
Consider a rectangle 5 units long and 3 units wide.



Clearly the area of this rectangle is 15 units^2, and we can find this by multiplying $5 \times 3 = 15$.

This leads to the general rule:

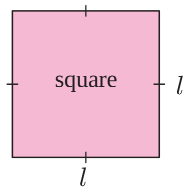
Area of rectangle = length × width
$A = l \times w$



DEMO

Since a **square** is a rectangle with equal length and width:

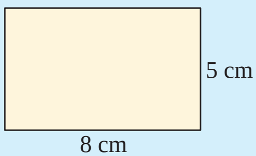
$$A = \text{length} \times \text{length}$$
$$= l \times l$$
$$= l^2$$



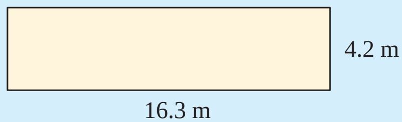
Example 2 **Self Tutor**

Find the areas of the following rectangles:

a



b



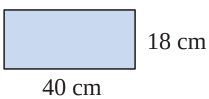
a Area = length × width
$= 8 \text{ cm} \times 5 \text{ cm}$
$= 40 \text{ cm}^2$

b Area = length × width
$= 16.3 \text{ m} \times 4.2 \text{ m}$
$= 68.46 \text{ m}^2$

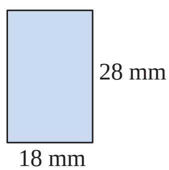
EXERCISE 19C

1 Find the areas of the following:

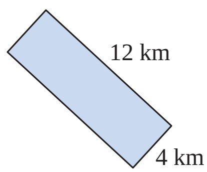
a



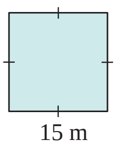
b



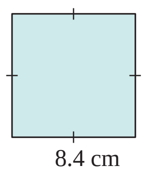
c



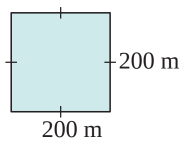
d



e



f

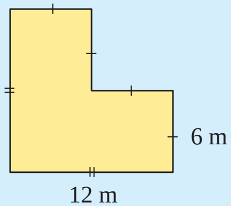


in hectares

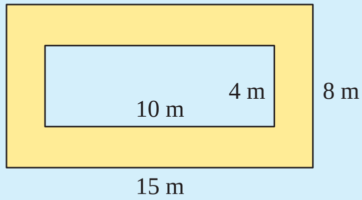
Example 3 — Self Tutor

Find the shaded area of:

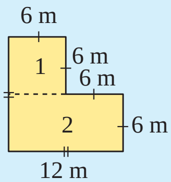
a



b



a The shape can be split into two rectangles.

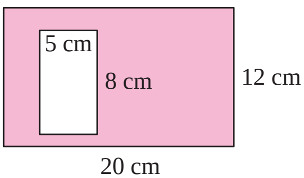


Shaded area $= \text{Area } 1 + \text{Area } 2$
$= 6 \text{ m} \times 6 \text{ m} + 12 \text{ m} \times 6 \text{ m}$
$= 36 \text{ m}^2 + 72 \text{ m}^2$
$= 108 \text{ m}^2$

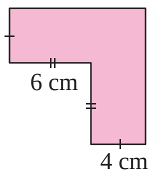
b Shaded area
$=$ area large rectangle $-$ area small rectangle
$= 15 \text{ m} \times 8 \text{ m} - 10 \text{ m} \times 4 \text{ m}$
$= 120 \text{ m}^2 - 40 \text{ m}^2$
$= 80 \text{ m}^2$

2 Find the shaded areas:

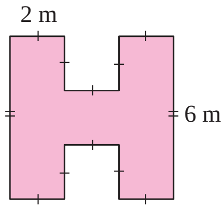
a



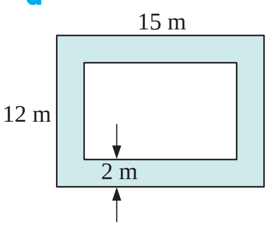
b



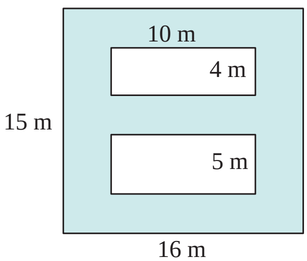
c



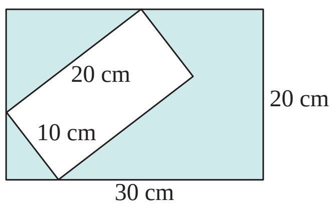
d



e



f



3 A rectangular garden bed 3 m by 5 m is dug into a lawn 10 m by 8 m. Find the area of lawn remaining.

4 A rectangular wheat field is 450 m by 600 m.

a Find the area of the field in hectares.

b Find the cost of seeding the field if this process costs $180 per hectare.

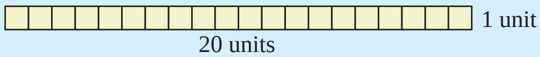
5 A floor 3.5 m by 5 m is to be covered with floor tiles 25 cm by 25 cm square.

- **a** Find the area of each tile.
- **b** Find the area of the floor.
- **c** Find the number of tiles required.
- **d** Find the total cost of the tiles if each costs £3.50 .

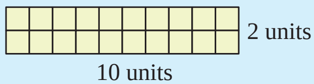
Example 4 **Self Tutor**

Using only whole units of measurement, write all the possible lengths, widths, and perimeters of a rectangle of area 20 units^2. Use scale drawings to represent your answer.

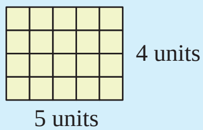
The possible pairs of factors which multiply to give 20 are: 20×1, 10×2, 5×4.



$\text{Perimeter} = 2 \times 20 + 2 \times 1$
$= 42$ units



$\text{Perimeter} = 2 \times 10 + 2 \times 2$
$= 24$ units



$\text{Perimeter} = 2 \times 5 + 2 \times 4$
$= 18$ units

6 Using only whole units, write all the possible lengths, widths and perimeters of the following rectangular areas:

a 6 units^2 **b** 8 units^2 **c** 12 units^2 **d** 16 units^2

7 Using only whole numbers for sides, write all possible areas which can be found from rectangles or squares with perimeters of:

a 12 m **b** 20 m **c** 36 km

Illustrate the possible answers for **a**.

ACTIVITY WORKING WITH AREA

What to do:

1. Draw a square metre with chalk.
   Estimate how many members of your class can stand on the square metre with feet entirely within it. Check your estimate.
2. Use a measuring tape to measure the dimensions of any *two* rectangular regions such as the floor area of a classroom, a tennis court, a basketball court, or the sides of a building. Calculate the area in each case.

D THE AREA OF A TRIANGLE

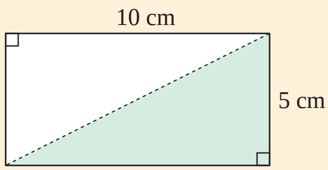
INVESTIGATION 2 THE AREA OF A TRIANGLE

You will need:

scissors, ruler, pencil and square centimetre graph paper.

What to do:

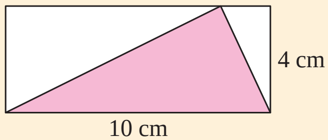
1 Draw a 10 cm by 5 cm rectangle using the graph paper. Draw in the dashed diagonal and colour one triangle green.



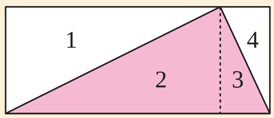
Cut out the two triangles, then place one on top of the other so you can see they have identical shape.

Copy and complete: The areas of the two triangles are ....
The area of each triangle is a .... the area of the rectangle.

2 Draw a 10 cm by 4 cm rectangle using the graph paper. Construct the triangles shown alongside and colour in the pink region.

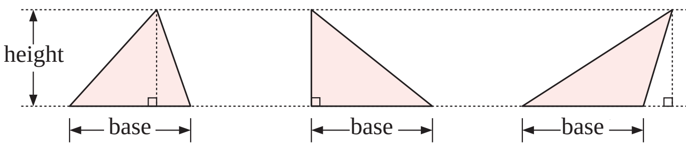


Now divide the pink triangle along the dashed line so you form four regions.



Using what you found in **1**, copy and complete:
The areas of regions 1 and 2 are ......
The areas of regions 3 and 4 area ......
So, area 2 + area 3 = area 1 + area 4.
The total area of the pink triangle is a ...... the area of the rectangle.

From the **Investigation** you should have found that the area of a triangle is half the area of a rectangle which has the *same base and height* as the triangle.



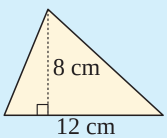
Area of triangle $= \frac{1}{2} \times$ **base** $\times$ **height** or $\dfrac{\textbf{base} \times \textbf{height}}{\textbf{2}}$.

DEMO

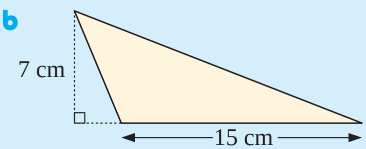
Example 5 Self Tutor

Find the area of the following triangles:

a



b

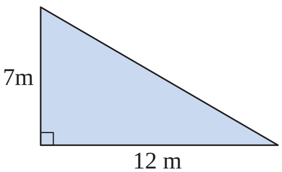


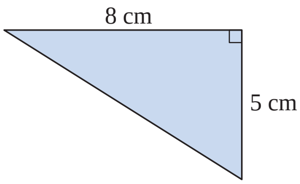
a Area of triangle
$= \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm}$
$= 48 \text{ cm}^2$

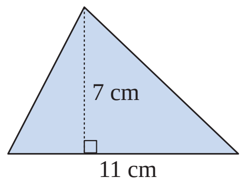
b Area of triangle
$= \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 15 \text{ cm} \times 7 \text{ cm}$
$= 52.5 \text{ cm}^2$

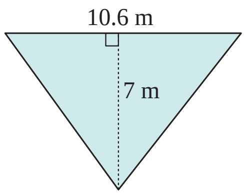
EXERCISE 19D

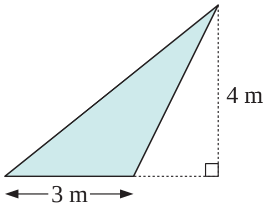
1 Find the areas of the following triangles:

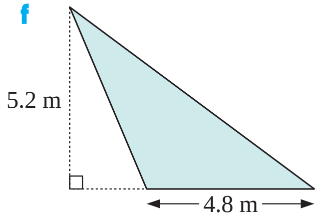
a


b


c


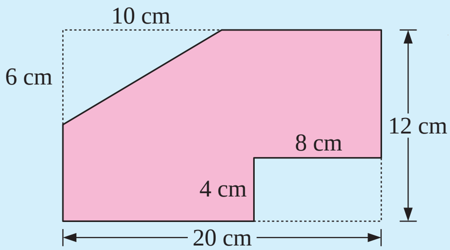
d


e


f


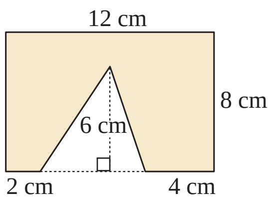
Example 6 Self Tutor

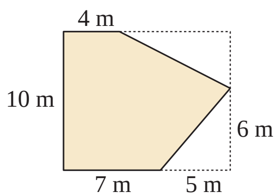
Find the shaded area:

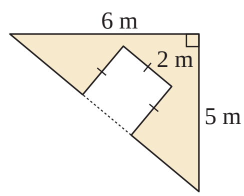


Shaded area $=$ area large rectangle $-$ area triangle $-$ area small rectangle
$= (20 \times 12 - \frac{1}{2} \times 6 \times 10 - 8 \times 4) \text{ cm}^2$
$= (240 - 30 - 32) \text{ cm}^2$
$= 178 \text{ cm}^2$

2 Find the shaded area:

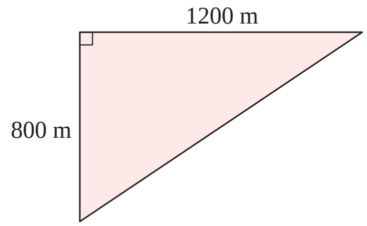
a


b


c


3 **a** Find the area in hectares of the triangular field shown.

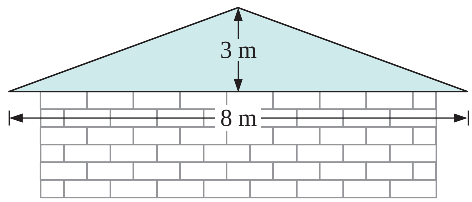
b How much would it cost to fertilise the field if this process costs €360 per hectare?



4 The area shaded green is covered with weatherboard on both sides of this house.

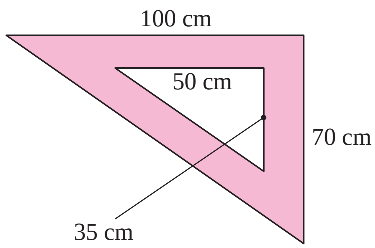
a What is the area of weatherboard used?

b What was the total cost if weatherboard costs £13.90 per m^2?

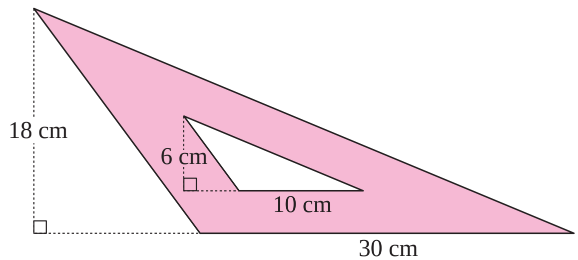


5 Find the shaded area:

a

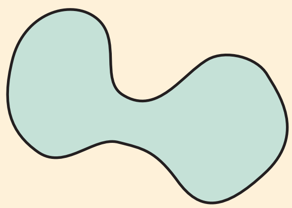


b



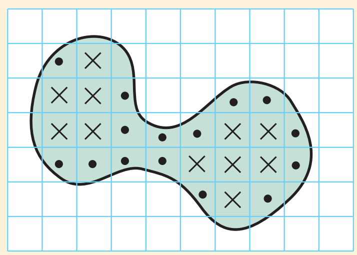
INVESTIGATION 3 AREAS OF IRREGULAR SHAPES

Have you ever thought how you could determine the area of a shape which is not regular? For example, consider the figure alongside:



At best we can only estimate the answer, and one method of doing this is to draw grid lines across the figure.

We count all the full squares, and as we do so we cross them out.



Now we have to make a decision about the part squares inside the shape.

For a good approximate answer, we can count squares which are more than half full as 1, and those less than half full as 0.

We hope that errors will cancel each other out when we add all of these together.

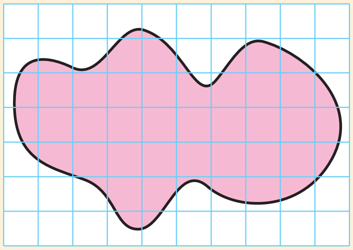
Thus our estimate for the total area is 26 square units.

What to do:

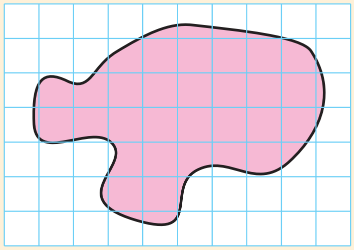
PRINTABLE GRID PAPER

1 Estimate the areas of:

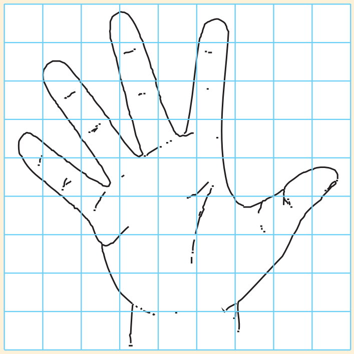
a



b



2 Place your hand on cm^2 grid paper and trace around the outside. What is the difference in area between your hand with its fingers together and when the fingers are apart? Why is this?

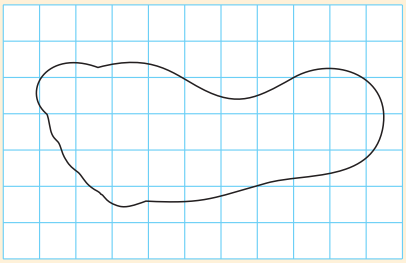


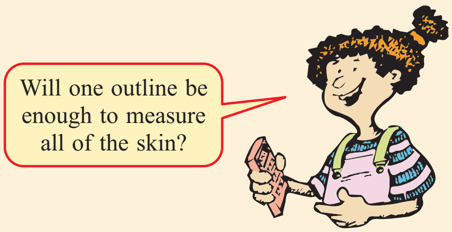
a Estimate the area of your hand in cm^2.

b Do you think your estimate will be more or less accurate if your fingers are together or apart? Explain your answer.

3 **a** Estimate the area of the sole of your shoe.

b Estimate the area of your bare foot.





E VOLUME

This stone

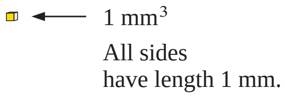
occupies more space than this pebble.

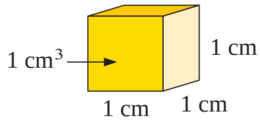
We say that the stone has greater *volume* than the pebble.

The **volume** of a solid is the amount of space it occupies.
This space is measured in **cubic units**.

As with area, the units used for the measurement of volume are related to the units used for the measurement of length.

1 **cubic millimetre** (mm^3) is the volume of a cube with a side of length 1 mm.
1 **cubic centimetre** (cm^3) is the volume of a cube with a side of length 1 cm.
1 **cubic metre** (m^3) is the volume of a cube with a side of length 1 m.



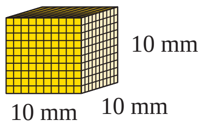


VOLUME UNIT CONVERSIONS

Converting from one unit of volume to another unit of volume can be done by considering a cube of side unit length.

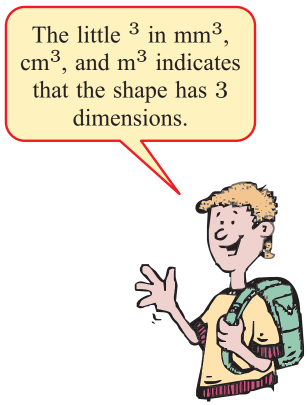
For example, 1 cm = 10 mm so

$$\begin{aligned} 1\text{ cm}^3 &= 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} \\ &= 10\text{ mm} \times 10\text{ mm} \times 10\text{ mm} \\ &= 1000\text{ mm}^3 \end{aligned}$$

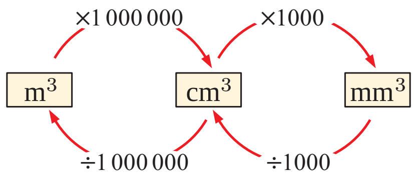


A similar conversion can be performed for other volume units.

$$\begin{aligned} 1\text{ m}^3 &= 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} \\ &= 1\,000\,000\text{ cm}^3 \end{aligned}$$



CONVERSION DIAGRAM



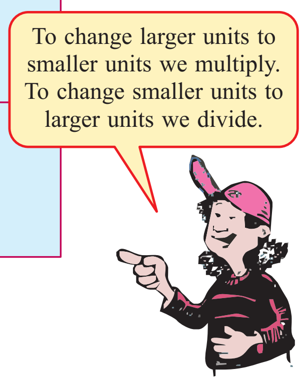
Example 7 **Self Tutor**

Convert:

a 0.163 m^3 to cm^3 **b** 7953 mm^3 to cm^3

a
$$\begin{aligned} &0.163\text{ m}^3 \\ &= (0.163 \times 1\,000\,000)\text{ cm}^3 \\ &= 163\,000\text{ cm}^3 \end{aligned}$$

b
$$\begin{aligned} &7953\text{ mm}^3 \\ &= (7953 \div 1000)\text{ cm}^3 \\ &= 7.953\text{ cm}^3 \end{aligned}$$



EXERCISE 19E.1

1 Perform the following conversions:

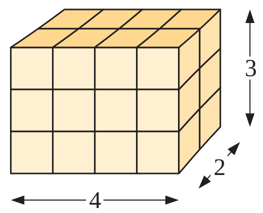
a $8\ mm^3$ to cm^3
b $0.06\ m^3$ to cm^3
c $11.8\ cm^3$ to mm^3
d $0.64\ cm^3$ to mm^3
e $3\ m^3$ to mm^3
f $0.0075\ m^3$ to mm^3

2 Perform the following conversions:

a $500\ mm^3$ to cm^3
b $7000\ mm^3$ to cm^3
c $5\,000\,000\ cm^3$ to m^3
d $450\,000\ cm^3$ to m^3
e $2\,000\,000\ mm^3$ to m^3
f $5\,400\,000\,000\ mm^3$ to m^3

RECTANGULAR PRISMS

A **rectangular prism** is a 3-dimensional solid with 6 rectangular faces. It has the same rectangular cross-section along its entire length.



For example, a $4 \times 2 \times 3$ prism is shown alongside.

Clearly there are 3 layers and each of these layers contains $4 \times 2 = 8$ cubes.

So, there are $8 \times 3 = 24$ cubes altogether. The volume is $4 \times 2 \times 3 = 24$ units^3.

This leads to the following rule for volume:

Volume of a rectangular prism = length × width × height

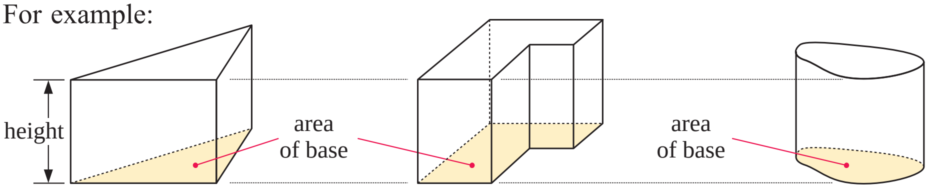
Since length × width = area of base, we can also write

Volume of rectangular prism = area of base × height

SOLIDS OF UNIFORM CROSS-SECTION

A solid of uniform cross-section has the same size and shape along its entire length.

For example:



The volume can be found by multiplying the base area and the height.

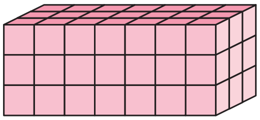
Volume of solid of uniform cross-section = area of base × height

EXERCISE 19E.2

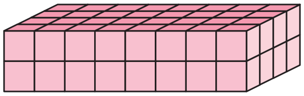
DEMO

1 Find the number of cubic units in each of the following solids:

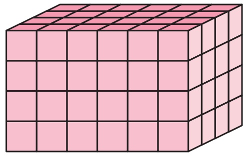
a



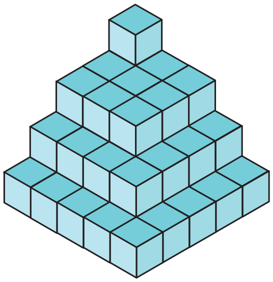
b



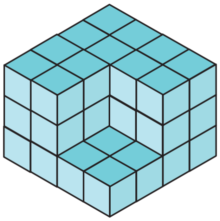
c



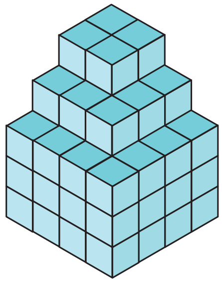
d



e

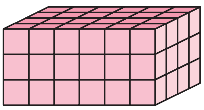


f

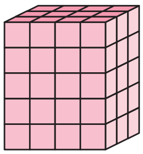


2 Arrange the rectangular prisms with dimensions as given, in ascending order of volumes, from the lowest number of cubic units to the highest:

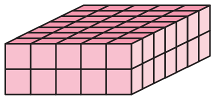
A



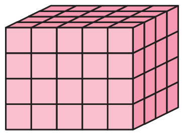
B



C



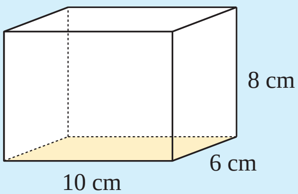
D



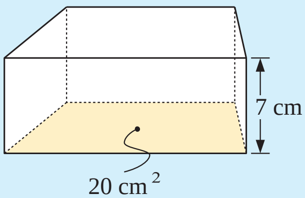
Example 8 **Self Tutor**

Find the volume of the following prisms:

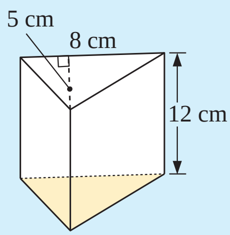
a



b



c



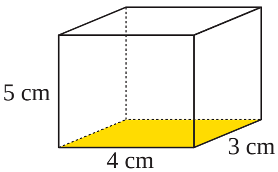
a Volume
$= \text{area of base} \times \text{height}$
$= 10 \text{ cm} \times 6 \text{ cm} \times 8 \text{ cm}$
$= 480 \text{ cm}^3$

b Volume
$= \text{area of base} \times \text{height}$
$= 20 \text{ cm}^2 \times 7 \text{ cm}$
$= 140 \text{ cm}^3$

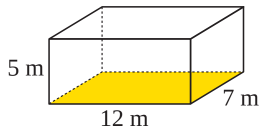
c Volume $= \text{area of base} \times \text{height}$ $= \frac{1}{2} \times 8 \text{ cm} \times 5 \text{ cm} \times 12 \text{ cm}$
$= 20 \text{ cm}^2 \times 12 \text{ cm}$
$= 240 \text{ cm}^3$

3 Find the volume of the following rectangular prisms:

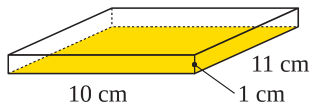
a



b

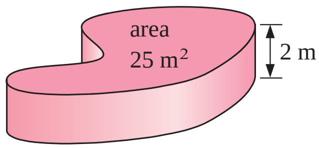


c

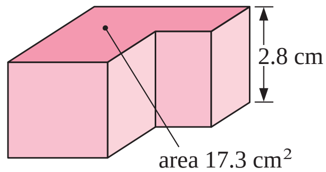


4 Find the volume of the following solids of uniform cross-section:

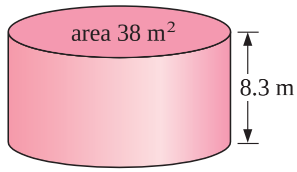
a



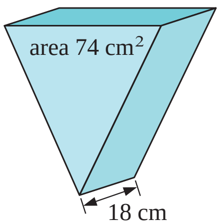
b



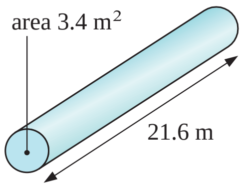
c



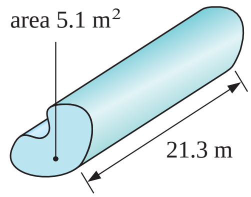
d



e

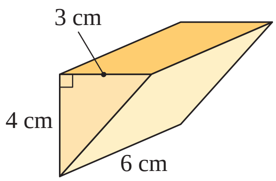


f

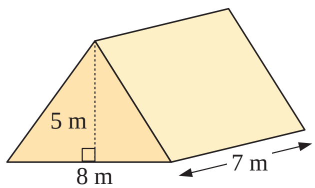


5 Find the volumes of the prisms shown:

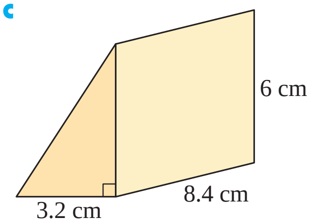
a



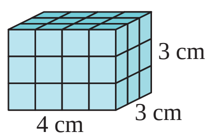
b



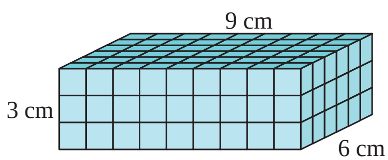
c



6 The rectangular prism alongside has a volume of 36 cm^3.
Show that there are exactly 8 different rectangular prisms with whole number sides that have a volume of 36 cm^3.
There is no need to draw them.



7



Find:

a the volume of this prism

b the sum of the areas of its 6 faces.

8 Find the volume of this book to the nearest cm^3.

F CAPACITY

Volume and capacity are very similar terms.

The word capacity is usually used when referring to either a liquid or gas.

> The **capacity** of a container is a measure of the amount of fluid it can contain.

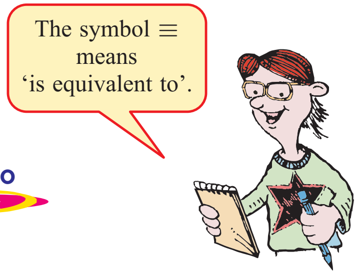
The units for capacity are closely related to those of volume.

The most commonly used units of capacity are **litre** (L) and **millilitre** (mL). For larger capacities such as reservoirs and swimming pools, the units of kilolitre (kL) and megalitre (ML) are used.

> 1 L = 1000 mL
> 1 kL = 1000 L
> 1 ML = 1 000 000 L

The relationship between capacity units and volume units is:

> $1 \text{ mL} \equiv 1 \text{ cm}^3$
> $1 \text{ L} \equiv 1000 \text{ cm}^3$
> $1 \text{ kL} \equiv 1\,000\,000 \text{ cm}^3 \equiv 1 \text{ m}^3$



DEMO

Example 9 — Self Tutor

Convert:

a 8 L to mL **b** 12.4 kL to L **c** 3400 cm^3 to L

a 8 L
= (8 × 1000) mL
= 8000 mL

b 12.4 kL
= (12.4 × 1000) L
= 12 400 L

c 3400 cm^3
$\equiv (3400 \div 1000)$ L
$\equiv 3.4$ L

EXERCISE 19F

1 What units of capacity are most suitable for measuring:

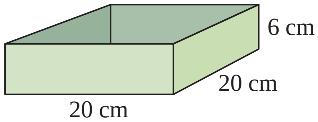
a a perfume bottle
b a thermos flask
c an Olympic pool
d a 6 cylinder car engine
e a drinking glass
f household water use
g a model aeroplane engine
h an oil refinery
i an ocean tanker
j a reservoir
k domestic gas use
l a baby’s bottle?

2 Convert:

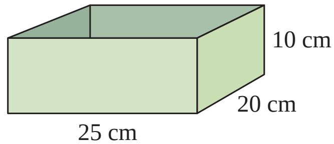
a 5.6 kL to L	**b** 3540 mL to L	**c** 760 000 L to ML
d 7200 cm^3 to L	**e** 6.3 kL to m^3	**f** 12.4 kL to mL
g 0.0625 L to mL	**h** 400 cm^3 to mL	**i** 3.5 ML to kL

3 Find the capacity of the following containers. Express your answers using appropriate units.

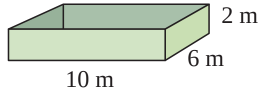
a



b

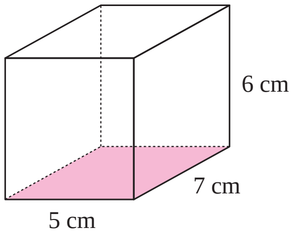


c

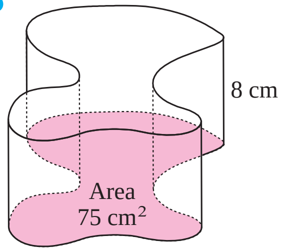


4 Find the capacity in mL of these containers:

a



b



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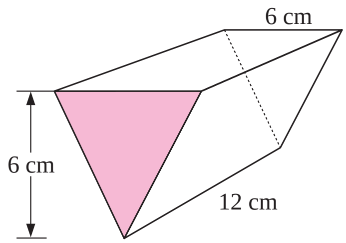
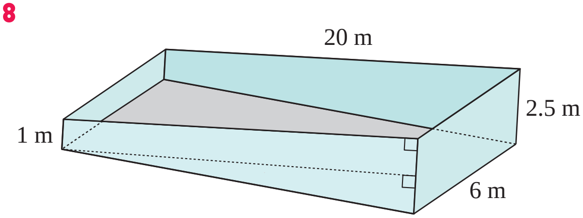


5 Find the capacity in litres of a rectangular box 80 cm by 60 cm by 15 cm.

6 How many times could a water container 15 cm by 8 cm by 5 cm be filled from a 40 L container?

7 How many 30 cm by 20 cm by 90 cm fuel tanks can a car manufacturer fill from its 27.54 kL storage tank?

8



Find the amount of water (in kL) required to fill the swimming pool shown alongside.

Hint: Use the dashed line to divide the side of the pool into a rectangle and a triangle.

RESEARCH

In and around your home, look for clues which tell you the capacity of the:

- fridge
- bath tub
- watering can
- freezer compartment
- washing machine
- cistern flush
- car engine
- garbage bin
- fuel tank.

If you cannot find the clues, describe ways that you could measure the capacity.

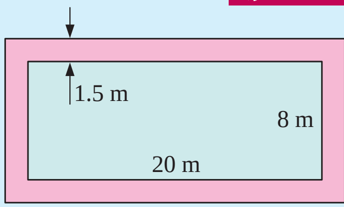
G PROBLEM SOLVING

Example 10 Self Tutor

A concrete path 1.5 m wide is to be laid around a 20 m by 8 m swimming pool. Concrete of the required depth costs \$41 per square metre.

a Find the area to be concreted.

b Find the cost of the concrete.



a The length of the large rectangle $= (20 + 1.5 + 1.5)$ m $= 23$ m.
The width of the large rectangle $= (8 + 1.5 + 1.5)$ m $= 11$ m.

Area of path = area of large rectangle − area of small rectangle
$= (23 \times 11 - 20 \times 8)\ \text{m}^2$
$= (253 - 160)\ \text{m}^2$
$= 93\ \text{m}^2$

b Cost of path $= 93 \times \$41$
$= \$3813$

EXERCISE 19G

1 A room 5 m by 6 m by 3 m high is to have its walls timber panelled.

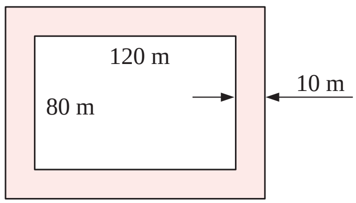
a Find the area of timber panelling required.

b If the timber panelling costs €67 per square metre, find the total cost of the panelling.

2 A rectangular playing field 120 m by 80 m is to be surrounded by a 10 m wide strip of bitumen.

a Find the area of bitumen.

b If each truckload of bitumen covers 50 m^2, how many truckloads of bitumen will be required?

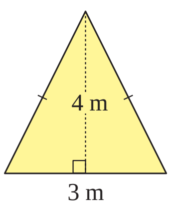


3 If each page of a book is 25 cm by 15 cm, find the total area (in m^2) of paper used in a book of 420 pages.

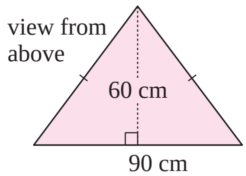
4 The area of a rectangle is 1 hectare. Find the width of the rectangle if it has a length of:

a 100 m	**b** 1 kilometre	**c** 250 metres	**d** 2000 metres
e 800 metres	**f** 1.25 km	**g** 500 metres	**h** 12.5 metres

5 How much canvas is needed for a tent which has three identical sides like this?



6 To celebrate her 3 years in business, a baker bakes a large triangular cake with the dimensions shown. How much icing must she make if she covers the top of the cake to a depth of 5 mm with icing?



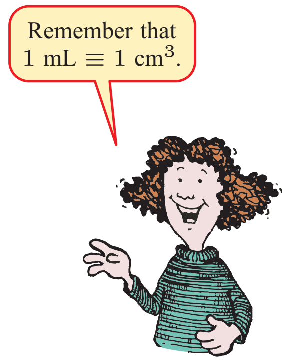
Example 11 **Self Tutor**

How many 80 mL oil bottles can be filled from a rectangular container 20 cm by 16 cm by 40 cm?

$$\begin{aligned}\text{Capacity of container} &= 20 \text{ cm} \times 16 \text{ cm} \times 40 \text{ cm}\\ &= 12\,800 \text{ mL}\end{aligned}$$

$$\begin{aligned}\text{The number of bottles} &= \frac{\text{capacity of container}}{\text{capacity of bottle}}\\ &= \frac{12\,800 \text{ mL}}{80 \text{ mL}}\\ &= 160\end{aligned}$$

So, 160 bottles can be filled.

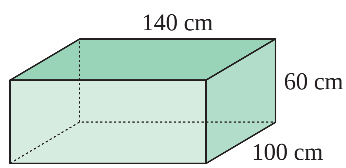


7 How many 300 mL spring water bottles can be filled from a rectangular container 3 m × 2 m × 1.5 m?

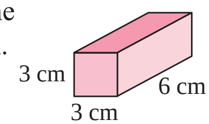
8 Engineers dug a 150 metre × 80 metre × 17 metre deep hole to dump the town's rubbish. How much compacted rubbish can be dumped if the engineers need a depth of at least 2 metres of soil on top once the hole is filled?

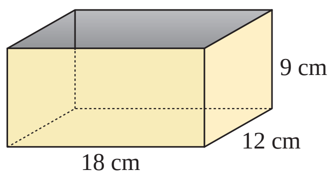
9 **a** How much water is in this rainwater tank if it is $\frac{3}{4}$ full?

b How many 8 litre buckets full would it take to empty it?



10 Illustrate the best way to pack the smaller prisms into the larger box. How many can be packed?





KEY WORDS USED IN THIS CHAPTER

- area
- cubic unit
- rectangular prism
- triangle
- capacity
- hectare
- square unit
- uniform
- cross-section
- rectangle
- surface
- volume

LINKS click here

HOW MANY BRICKS ARE NEEDED TO BUILD A HOUSE?

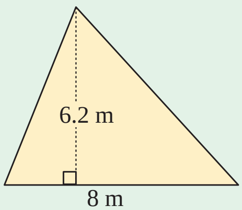
Areas of interaction:
Approaches to learning

REVIEW SET 19A

1 Convert: **a** 3.56 ha to m^2 **b** 357 000 mm^2 to m^2 **c** 7.2 cm^3 to mm^3

2 Find the shaded area:

a



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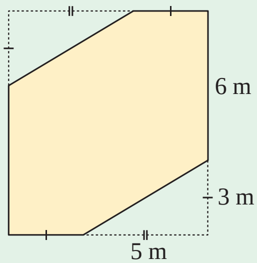
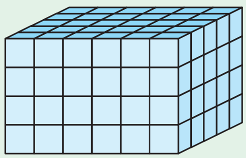


3 Find the volume of:

a



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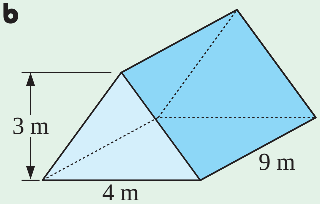

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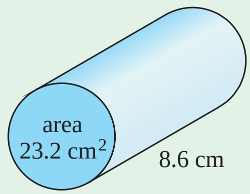


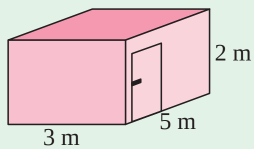
4 Convert the following:

a 380 mL to L **b** 5.4 kL to m^3 **c** 7528 cm^3 to L

5 The outside of a shed with the dimensions shown is to be painted.

a Find the total area to be painted (including the roof).

b If a litre of paint covers 15 m^2, what quantity of paint will be required?



6 **a** How many posters 120 cm long by 90 cm high can Lotus stick on her 3.6 m by 3 m high bedroom wall?

b Lotus wants the space between the posters equal. What is the area of each space if the top row of posters touches the ceiling and the bottom row touches the floor?

7 **a** How many 2 cm by 3 cm stamps can fit on a sheet 200 mm by 300 mm?

b If each stamp costs 45 pence, what is the cost of half a sheet?

8 Determine the capacity of a rectangular rainwater tank 5 m by 3 m by 4.5 m.

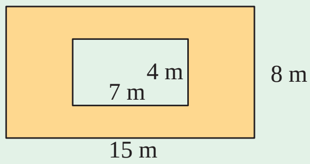
REVIEW SET 19B

1 Convert:

a 3400 m^2 to ha **b** 3.2 cm^2 to mm^2 **c** 7.2 m^2 to mm^2

2 Find the shaded area:

a



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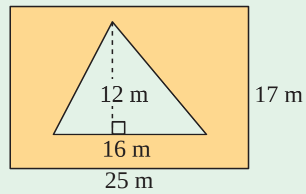
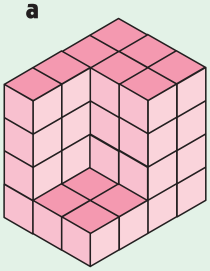


3 Convert the following:

a 45 000 L to kL **b** 8900 mm^3 to cm^3 **c** 4.6 kL to L

4 Find the volume of:

a



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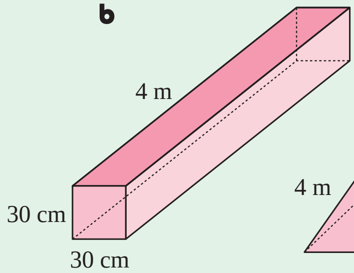
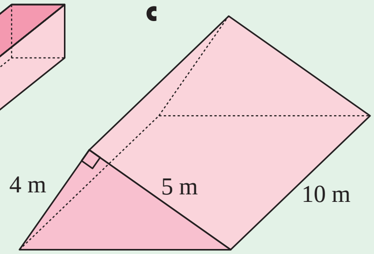


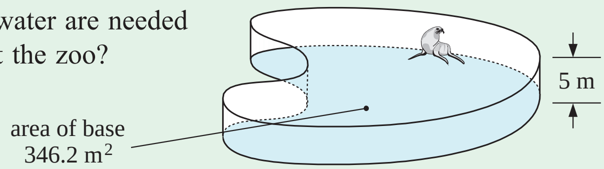
c



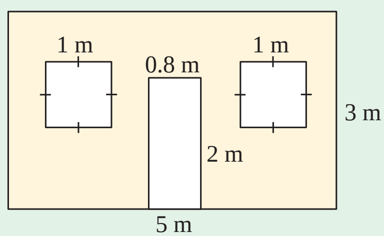
5 How many $10\text{ cm} \times 5\text{ cm} \times 10\text{ cm}$ containers can be filled from a container with dimensions $1\text{ m} \times 1\text{ m} \times \frac{1}{2}\text{ m}$?

6 Using only whole units, how many different rectangular prisms can be made with volume 63 cm^3?

7 How many kilolitres of sea water are needed to fill this seal's enclosure at the zoo?



8 A wall of a house has two windows and a door with the dimensions illustrated. If the wall is wallpapered and the wallpaper costs £8.75 per square metre, find the cost of papering the wall.



9 Find the area of exposed floorboards if a 5 m by 3 m carpet is placed on the floor of a 6.5 m by 8 m room.

ACTIVITY — MEASUREMENT MESSAGE

Match each metric measurement in the first column with the item in the second column which would best be measured in that unit. Transfer the letter to the table below to find the message.

PRINTABLE WORKSHEET

1	mm
2	cm
3	m
4	km
5	mm^2
6	cm^2
7	m^2
8	hectare
9	mm^3
10	cm^3
11	m^3
12	mL
13	L
14	ML
15	second
16	minute
17	hour
18	day
19	year
20	g
21	kg
22	t

N	your age
C	the time between meals
U	the time until the weekend
S	the volume of a small teaspoon of sand
R	the time to spell your name
I	the volume of a block of chocolate
M	the capacity of a can of soft drink
M	the area of a small toenail
F	the length of an ant
S	the mass of a train
N	the volume of a truckload of soil
K	the area of a paddock
I	the mass of your pencil
E	the capacity of a bucket
R	the area of your driveway
T	the capacity of a lake
T	your own mass
A	the area of a handkerchief
L	the distance from home to school
I	the time to swim 1500 m
U	the length of a pencil
L	the width of your house

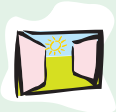
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

Chapter 20

Equations

Contents:

OPENING PROBLEM



Ilse and six friends go out to have icecreams. Ilse's mum has given her \$11 to help pay the bill, and the 7 girls decide to divide the remainder of the bill between them. The total bill is \$25.

Things to think about:

- If each of the girls pays $\$x$, can you explain why the total amount paid is $(7x+11)$ dollars?
- Can you explain why $7x + 11 = 25$?
- Given the equation $7x + 11 = 25$, how can we find the exact value of x?
- How much does each girl pay?

In this chapter we look at algebraic **equations** and methods used to solve them.

A WHAT ARE EQUATIONS?

An **equation** is a mathematical sentence which indicates that two expressions have the same value. An equation always contains an *equal* sign $=$.

A simple equation may be a true numerical statement like $3 \times 5 \quad = \quad 7 + 8.$

3×5	$=$	$7+8$
↑	↑	↑
LHS	**equals**	**RHS**

Notice that an equation has a **left hand side (LHS)** and a **right hand side (RHS)** and these are separated by the **equal sign**.

An **algebraic equation** like $3x + 2 = 11$ has an **unknown** or **variable** in it, in this case x. To **solve** an equation is to find the value of the variable which makes the equation true.

If we were to replace x by a variety of numbers, most of them would make the equation false.

For example,	if $x = 1$	the LHS is	$3 \times 1 + 2 = 3 + 2 = 5$	but	the RHS $= 11$
	if $x = 5$	the LHS is	$3 \times 5 + 2 = 15 + 2 = 17$	but	the RHS $= 11$
However,	if $x = 3$	the LHS is	$3 \times 3 + 2 = 9 + 2 = 11$	and	the RHS $= 11$.

So, $x = 3$ makes the equation $3x + 2 = 11$ true,
and we say $x = 3$ is the **solution** of the equation $3x + 2 = 11$.

Example 1 **Self Tutor**

What number can replace $\square$ to make the equation true?

a $3 + \square = 10$ **b** $3 \times \square = 18$ **c** $20 \div \square = 4$ **d** $\square - 8 = 4$

a $3 + 7 = 10$
so $\square = 7$

b $3 \times 6 = 18$
so $\square = 6$

c $20 \div 5 = 4$
so $\square = 5$

d $12 - 8 = 4$
so $\square = 12$

EXERCISE 20A

1 State whether each of the following is an equation or an expression:

- **a** $x - 3 = 7$
- **b** $2(x + 4)$
- **c** $3 \div 7 + x - 1$
- **d** $x - 2 = 7 - x$
- **e** $2(x - 1) = 3$
- **f** $3 - 2(1 + x)$

2 What number can be used to replace $\square$ to make the equation true?

- **a** $5 + \square = 15$
- **b** $\square + 9 = 22$
- **c** $15 - \square = 2$
- **d** $\square - 9 = 10$
- **e** $5 \times \square = 30$
- **f** $\square \div 3 = 8$
- **g** $75 \div \square = 15$
- **h** $\square \times 4 = 22 + 2$
- **i** $\square \times 2 + 1 = 11$

3 Find each of the following, suppose x is the number. Use the statement to write an equation involving x.

- **a** Seven added to a number is equal to ten.
- **b** Five subtracted from a number is equal to eleven.
- **c** A number multiplied by four is equal to twelve.
- **d** A number when divided by ten is equal to two.

B SOLVING SIMPLE EQUATIONS

In this chapter we will be dealing with equations which have **one unknown**.

Remember that in algebra:

- the $\times$ sign is omitted where possible. For example, $5 \times x$ is written $5x$.
- the $\div$ sign is usually written as a fraction. For example, $x \div 3$ is written $\frac{x}{3}$.

In **Chapter 18** we saw that given an expression involving x, we can substitute a value for x to evaluate the expression.

For example, consider the expression $4x - 3$.

When $x = 2$, $4x - 3 = 4 \times 2 - 3 = 5$.

In this chapter we are now presented with equations such as $4x - 3 = 5$. Our task is to work out that x must be 2.

$\therefore$ is read as *therefore*. We use it to show that the next line of work follows from the previous line.

SOLVING BY INSPECTION

Some simple equations can be solved by **inspection**.

For example, for the equation $x + 2 = 8$
we notice that since $6 + 2 = 8$, x must be 6.

We write: $x + 2 = 8$
$\therefore \quad x = 6$

Example 2 — Self Tutor

Solve by inspection:

a $a+6=11$ **b** $\frac{b}{3}=8$

c $14-x=8$ **d** $7p=49$

a $a+6=11$
$\therefore\ a=5$ {as $5+6=11$}

b $\frac{b}{3}=8$
$\therefore\ b=24$ {as $24\div 3=8$}

c $14-x=8$
$\therefore\ x=6$ {as $14-6=8$}

d $7p=49$
$\therefore\ p=7$ {as $7\times 7=49$}

SOLVING BY TRIAL AND ERROR

Another method of solving simple equations is to use **trial and error**. This involves substituting different numbers in place of x until the correct solution is obtained.

For example, to solve $4x-13=23$ we substitute different values for x and summarise our trials in a table.

x	$4x-13$	
1	-9	← much too small
5	7	← getting larger
8	19	← almost
9	23	✓

So, $x=9$ is the solution.

EXERCISE 20B

1 Solve *by inspection*:

a $7+a=15$ **b** $48\div p=6$ **c** $18=25-n$
d $t\div 4=10$ **e** $*-14=38$ **f** $3\times d=18$
g $n+7=14$ **h** $8a=200$ **i** $b\div 7=9$
j $t+3=3$ **k** $7+m=19$ **l** $t+9=4$
m $x-7=-2$ **n** $6+\square=9$ **o** $y\times 2=-6$
p $x\times x=0$ **q** $3-x=7$ **r** $5t=-15$
s $3x=60$ **t** $4x=-12$ **u** $7x=91$
v $\frac{6}{n}=2$ **w** $6=\frac{x}{8}$ **x** $\frac{55}{t}=11$

2 One of the numbers in the brackets is the correct solution to the equation. Find it using *trial and error*.

a $3x+8=23$ $\{2, 3, 5, 9\}$ **b** $4x+11=29$ $\{3, 4, 4\frac{1}{2}, 5\}$
c $5x-1=34$ $\{6, 7, 8, 9\}$ **d** $3x-5=13$ $\{6, 7, 8, 9\}$
e $7x+4=-3$ $\{3, 2, 1, -1\}$ **f** $11x+6=-16$ $\{4, 0, -4, -2\}$

3 Solve by *trial and error*:

a $3x + 11 = 32$ **b** $4x - 7 = 33$ **c** $5x - 22 = 23$

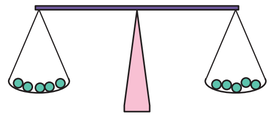
d $4x + 11 = 21$ **e** $8x = 10$ **f** $2 - 5x = -18$

C MAINTAINING BALANCE

The **balance** of an **equation** can be likened to the **balance** of a **set of scales**. Changing one side of the equation without doing the same thing to the other side will upset the balance.

PERFORMING OPERATIONS ON EQUATIONS

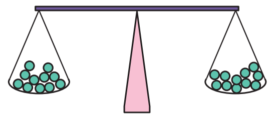
The equal sign represents the balancing point of the equation. The left hand side must balance the right hand side.



For example, $5 = 5$

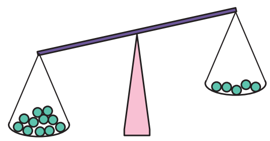
If 6 is added to both sides, the statement remains true:

$$5 + 6 = 5 + 6$$
$$\therefore \quad 11 = 11$$



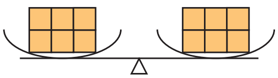
If 6 were added to one side only, then the statement would become false:

$$5 + 6 \neq 5$$
$$\therefore \quad 11 \neq 5$$

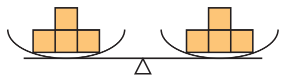


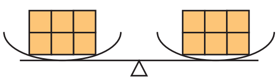
To maintain the balance, whatever is done on one side of the *equal* sign must also be done on the other side.

Imagine a set of scales with six identical blocks on each side. The scale is **balanced**.

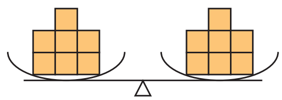


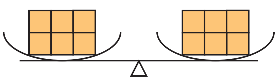
If we subtract 2 blocks from each side we get:



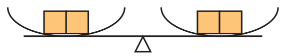


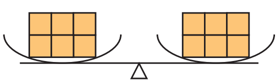
If we add 1 block to each side we get:



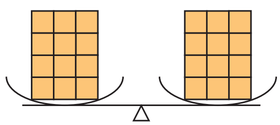


If we divide the number of blocks on each side by 3 we get:





If we multiply the number of blocks on each side by 2 we get:

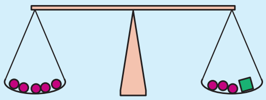


Notice that the scales are still balanced in each case!

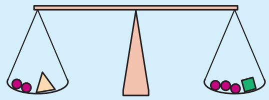
Example 3 **Self Tutor**

If the bar is perfectly balanced, find the relationship or connection between the objects:

a



b



a By taking 3 ● from both sides we can see that 1 ■ is equal to 2 ●

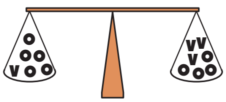
b By taking 2 ● from both sides we can see that 1 △ is equal to 1 ● plus 1 ■

So, △ = ● + ■

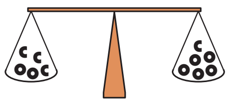
EXERCISE 20C.1

1 These scales are perfectly balanced. Find the relationship between the objects.

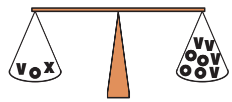
a



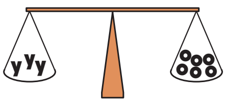
b



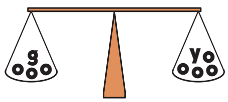
c



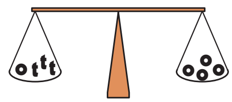
d



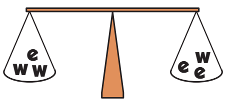
e



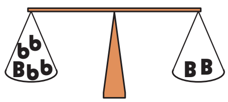
f



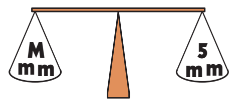
g



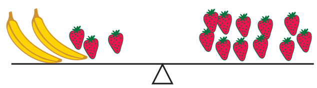
h



i



2 The set of scales is balanced with two bananas and three strawberries on one side and 11 strawberries on the other.

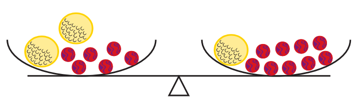


a If three strawberries are taken from the left side, what must be done to the right side to keep the scales balanced?

b There are now two bananas on the left hand side. How many strawberries balance their weight?

c How heavy is one banana in terms of strawberries?

3 The set of scales is balanced with two golf balls and six marbles on the left and one golf ball and nine marbles on the right.



a If 6 marbles are taken from the left side, what must be done to the right side to keep the scales balanced?

b If the golf ball on the right side is removed, what must be done to the left side to keep the scales balanced?

c Redraw the scales if both **a** and **b** occur.

d How heavy is one golf ball in terms of marbles?

BALANCE

The **balance** of an equation will be maintained if we:

- add the same amount to both sides
- subtract the same amount from both sides
- multiply both sides by the same amount
- divide both sides by the same amount.

Example 4 — Self Tutor

Consider the equation $x+5=10$. What equation results when we perform the following on both sides of the equation:

a add 3 **b** subtract 3 **c** divide by 2 **d** multiply by 4?

a
$$x+5=10$$
$$\therefore\ x+5+3=10+3$$
$$\therefore\ x+8=13$$

b
$$x+5=10$$
$$\therefore\ x+5-3=10-3$$
$$\therefore\ x+2=7$$

c
$$x+5=10$$
$$\therefore\ \frac{x+5}{2}=\frac{10}{2}$$
$$\therefore\ \frac{x+5}{2}=5$$

d
$$x+5=10$$
$$\therefore\ 4(x+5)=4\times 10$$
$$\therefore\ 4(x+5)=40$$

EXERCISE 20C.2

1 Find the equation which results from *adding*:

a 3 to both sides of $x=4$ **b** 5 to both sides of $x+7=5$
c 5 to both sides of $x-5=8$ **d** 7 to both sides of $2x-7=3$

2 Find the equation which results from *subtracting*:

a 2 from both sides of $x=8$ **b** 5 from both sides of $x+5=-2$
c 5 from both sides of $5-x=9$ **d** 6 from both sides of $3x+6=-1$

3 Find the equation which results from *multiplying* both sides of:

a $x=6$ by 2 **b** $2x=1$ by 3 **c** $\frac{x}{2}=5$ by 2
d $x+1=9$ by 7 **e** $\frac{x+1}{2}=-1$ by 2 **f** $\frac{1-x}{3}=4$ by 3

4 Find the equation which results from *dividing* both sides of:

a $2x=6$ by 2 **b** $3(x+2)=6$ by 3
c $2x+6=0$ by 2 **d** $3x+9=15$ by 3
e $3x=14$ by 3 **f** $6(x-1)=18$ by 6
g $4x-16=-4$ by 4 **h** $8(x+2)=24$ by 8

D INVERSE OPERATIONS

Imagine starting with \$50 in your pocket. You find \$10 and then pay someone \$10. You still have \$50.

This can be illustrated by a **flowchart** such as \$50 $\xrightarrow{+10}$ \$60 $\xrightarrow{-10}$ \$50

Observe that adding 10 and subtracting 10 have the opposite effect. One undoes the other.

We say that addition and subtraction are **inverse operations**.

Now imagine you start with \$50, and your friend gives you the same amount. Your money is now doubled. If you decide to give half to your brother, you will be back to your original \$50.

We again illustrate the process by a flowchart: \$50 $\xrightarrow{\times 2}$ \$100 $\xrightarrow{\div 2}$ \$50

Observe that multiplying by 2 and dividing by 2 undo each other.

We say that multiplication and division are **inverse operations**.

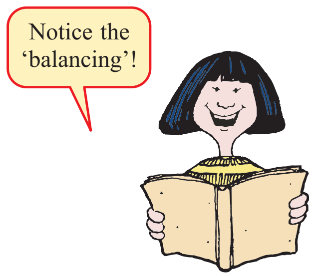
We can solve simple equations using *inverse operations*, but we must remember to keep the equation *balanced* by performing the same operation on *both sides* of the equation.

For example,

consider $x+3=7$ where 3 has been added to x.

$\therefore\ x+3-3=7-3$ {subtracting 3 is the inverse of adding 3}

$\therefore\ x=4$ {simplifying}



EXERCISE 20D

1 State the inverse of each of the following operations:

a $\times 3$ **b** $+5$ **c** -4 **d** $\div 7$

e $-\frac{3}{4}$ **f** $\times\frac{2}{3}$ **g** $+10$ **h** $\div\frac{1}{3}$

2 Simplify the following expressions:

a $x+7-7$ **b** $x-3+3$ **c** $x\div 2\times 2$ **d** $3x\div 3$

e $\frac{x}{5}\times 5$ **f** $\frac{2x}{2}$ **g** $\frac{2x}{3}\div\frac{2}{3}$ **h** $\frac{2x}{5}\times 5$

Example 5 Self Tutor

Solve for x using a suitable inverse operation: $x+5=11$

$x+5=11$

$\therefore\ x+5-5=11-5$ {The inverse of $+5$ is -5, so we take 5 from both sides.}

$\therefore\ x=6$

3 Find x using an inverse operation:

a $x+7=10$ **b** $x+15=6$ **c** $x+3=0$

d $x+11=-4$ **e** $7+x=9$ **f** $8+x=14$

Example 6 Self Tutor

Solve for y using a suitable inverse operation: $y-6=-2$

$y-6=-2$

$\therefore\ y-6+6=-2+6$ {The inverse of -6 is $+6$, so we add 6 to both sides.}

$\therefore\ y=4$

4 Find y using an inverse operation:

a $y-7=4$ **b** $y-2=0$ **c** $y-6=-1$

d $y-11=32$ **e** $y-8=-8$ **f** $y-15=-32$

Example 7 Self Tutor

Solve for t using a suitable inverse operation: $3t=-12$

$3t=-12$

$\therefore\ \dfrac{3t}{3}=\dfrac{-12}{3}$ {The inverse of $\times 3$ is $\div 3$, so we divide both sides by 3.}

$\therefore\ t=-4$

5 Find t using an inverse operation:

a $4t=8$ **b** $6t=30$ **c** $2t=4$

d $3t=15$ **e** $5t=20$ **f** $3t=-9$

g $7t=-56$ **h** $7t=56$ **i** $8t=-56$

Example 8 Self Tutor

Solve for d using a suitable inverse operation: $\dfrac{d}{7}=8$

$\dfrac{d}{7}=8$

$\therefore\ \dfrac{d}{7}\times 7=8\times 7$ {The inverse of $\div 7$ is $\times 7$, so we multiply both sides by 7.}

$\therefore\ d=56$

6 Find d using an inverse operation:

a $\frac{d}{2} = 3$ **b** $\frac{d}{4} = 7$ **c** $\frac{d}{2} = 8$

d $\frac{d}{5} = 6$ **e** $\frac{d}{3} = -4$ **f** $\frac{d}{7} = -1$

7 Find the unknown using a suitable inverse operation:

a $x + 7 = 0$ **b** $x - 5 = 6$ **c** $d + 9 = -1$

d $p - 6 = 8$ **e** $3g = 15$ **f** $\frac{x}{4} = 8$

g $7m = 28$ **h** $\frac{y}{2} = 4$ **i** $k + 6 = -2$

j $11s = -44$ **k** $t - 4 = 0$ **l** $4t = -36$

m $p - 15 = 23$ **n** $y + 11 = 7$ **o** $\frac{k}{7} = -2$

p $9n = -72$ **q** $\frac{e}{13} = 1$ **r** $n + 13 = 4$

s $\frac{d}{-6} = 12$ **t** $w - 19 = -6$ **u** $\frac{y}{-7} = -7$

E SOLVING EQUATIONS

So far we have solved simple equations by:

- inspection
- trial and error
- using one inverse operation.

To solve more complicated equations it is important to understand how expressions are **built up**. We can then **undo** them using **inverse operations**.

ALGEBRAIC FLOWCHARTS

Algebraic flowcharts help us to see how expressions are built up. By reversing the flowchart we can *undo* the expression and find the value of the unknown.

Example 9 Self Tutor

Complete the following flowcharts:

a x →(×2) [ ] →(+5) [ ]

b x →(+5) [ ] →(×2) [ ]

c x →(−3) [ ] →(÷2) [ ]

d x →(÷2) [ ] →(−3) [ ]

a $x \xrightarrow{\times 2} 2x \xrightarrow{+5} 2x+5$

b $x \xrightarrow{+5} x+5 \xrightarrow{\times 2} 2(x+5)$

c $x \xrightarrow{-3} x-3 \xrightarrow{\div 2} \dfrac{x-3}{2}$

d $x \xrightarrow{\div 2} \dfrac{x}{2} \xrightarrow{-3} \dfrac{x}{2}-3$

EXERCISE 20E.1

1 Copy and complete the following flowcharts:

a $x \xrightarrow{\times 3} \square \xrightarrow{-5} \square$

b $x \xrightarrow{-5} \square \xrightarrow{\times 3} \square$

c $x \xrightarrow{-4} \square \xrightarrow{\div 2} \square$

d $x \xrightarrow{\div 2} \square \xrightarrow{-4} \square$

2 Copy and complete the following flowcharts by inserting the missing operations:

a $x \xrightarrow{?} 4x \xrightarrow{?} 4x-7$

b $x \xrightarrow{?} x+5 \xrightarrow{?} \dfrac{x+5}{3}$

c $x \xrightarrow{?} \dfrac{x}{6} \xrightarrow{?} \dfrac{x}{6}+1$

d $x \xrightarrow{?} x-2 \xrightarrow{?} 8(x-2)$

Example 10 — Self Tutor

Use a flowchart to show how $5x+2$ is 'built up'.
Reverse it to 'undo' the expression.

Build up: $x \xrightarrow{\times 5} 5x \xrightarrow{+2} 5x+2$

Undoing: $5x+2 \xrightarrow{-2} 5x \xrightarrow{\div 5} x$

Example 11 — Self Tutor

Use a flowchart to show how $\dfrac{x+3}{2}$ is 'built up'.

Reverse it to 'undo' the expression.

Build up: $x \xrightarrow{+3} x+3 \xrightarrow{\div 2} \dfrac{x+3}{2}$

Undoing: $\dfrac{x+3}{2} \xrightarrow{\times 2} x+3 \xrightarrow{-3} x$

3 Use a flowchart to show how to 'build up' and then 'undo' the following expressions:

a $3x+4$	**b** $2x-5$	**c** $7x+11$	**d** $8x-15$
e $12x+5$	**f** $23x+10$	**g** $\dfrac{x}{2}+1$	**h** $\dfrac{x+1}{2}$
i $\dfrac{x}{3}-2$	**j** $\dfrac{x-2}{3}$	**k** $\dfrac{x}{4}+5$	**l** $\dfrac{x+5}{4}$
m $2x+5$	**n** $2(x+5)$	**o** $3x-1$	**p** $3(x-1)$

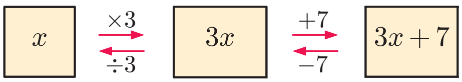
SOLVING BY ISOLATING THE UNKNOWN

We have already seen how an expression is 'built up' from an unknown.

When we are given an equation to solve which contains a *built up* expression, we need to do the reverse.

We use **inverse operations** to undo the build-up of the expression in the **reverse order** and thus **isolate** the unknown.

For example, to solve $3x+7=19$ we look at $3x+7$ and its 'build up'



We $\times 3$ and then $+7$ in the build up, so we -7 and then $\div 3$ to isolate x.

Example 12 — Self Tutor

Solve for x: $5x+7=19$

$5x+7=19$

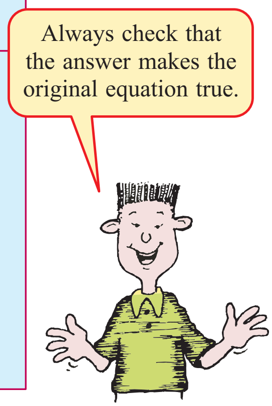
$\therefore\ 5x+7-7=19-7$ {subtracting 7 from both sides}

$\therefore\ 5x=12$

$\therefore\ \dfrac{5x}{5}=\dfrac{12}{5}$ {dividing both sides by 5}

$\therefore\ x=\frac{12}{5}$

Check: LHS $=5\times\frac{12}{5}+7=12+7=19=$ RHS



EXERCISE 20E.2

1 Solve the following equations:

a $4x-3=9$	**b** $3x+7=-11$	**c** $5x-5=0$
d $7x+6=-15$	**e** $10x-6=19$	**f** $2x+1=0$
g $5x+7=17$	**h** $2x+3=3$	**i** $5x-7=13$
j $3x+14=-1$	**k** $4x-6=-2$	**l** $7x+9=-10$

Example 13 **Self Tutor**

Solve for x: $\frac{x}{3} - 4 = -3$

$$\frac{x}{3} - 4 = -3$$

$$\therefore \quad \frac{x}{3} - 4 + 4 = -3 + 4 \qquad \{\text{adding 4 to both sides}\}$$

$$\therefore \quad \frac{x}{3} = 1$$

$$\therefore \quad \frac{x}{3} \times 3 = 1 \times 3 \qquad \{\text{multiplying both sides by 3}\}$$

$$\therefore \quad x = 3$$

Check: LHS $= \frac{3}{3} - 4 = 1 - 4 = -3 =$ RHS

2 Solve for x:

a $\frac{x}{2} + 3 = 8$ **b** $\frac{x}{3} - 1 = 4$ **c** $\frac{x}{5} + 2 = -3$

d $\frac{x}{6} + 3 = -4$ **e** $\frac{x}{7} - 2 = 4$ **f** $\frac{x}{10} - 6 = -1$

Example 14 **Self Tutor**

Solve for x: $\frac{x-3}{7} = -3$

$$\frac{x-3}{7} = -3$$

$$\therefore \quad 7\left(\frac{x-3}{7}\right) = 7 \times -3 \qquad \{\text{multiplying both sides by 7}\}$$

$$\therefore \quad x - 3 = -21$$

$$\therefore \quad x - 3 + 3 = -21 + 3 \qquad \{\text{adding 3 to both sides}\}$$

$$\therefore \quad x = -18$$

Check: LHS $= \frac{x-3}{7} = \frac{-18-3}{7} = \frac{-21}{7} = -3 =$ RHS

3 Solve for x:

a $\frac{x+3}{5} = 8$ **b** $\frac{x-2}{7} = -8$ **c** $\frac{x+4}{2} = 0$

d $\frac{x+4}{-2} = 3$ **e** $\frac{x-2}{5} = 1$ **f** $\frac{x+6}{8} = -2$

g $\frac{x+8}{-7} = -12$ **h** $\frac{x-5}{-6} = 3$ **i** $\frac{x-11}{15} = -4$

Example 15 — Self Tutor

Solve for x: $3(x-4)=39$

$$3(x-4)=39$$

$$\therefore \frac{3(x-4)}{3}=\frac{39}{3} \quad \{\text{dividing both sides by } 3\}$$

$$\therefore x-4=13$$

$$\therefore x-4+4=13+4 \quad \{\text{adding 4 to both sides}\}$$

$$\therefore x=17$$

Check: LHS $=3(x-4)=3(17-4)=3\times 13=39=$ RHS.

4 Solve for x:

- **a** $4(x+1)=12$
- **b** $3(x+5)=24$
- **c** $5(x-2)=35$
- **d** $2(x+11)=14$
- **e** $7(x-4)=63$
- **f** $8(x-7)=0$
- **g** $2(x-1)=8$
- **h** $3(x+2)=15$
- **i** $3(x-4)=-30$
- **j** $4(x+11)=48$
- **k** $5(x-7)=-80$
- **l** $3(x-2)=40$

5 Solve for x:

- **a** $11x-3=19$
- **b** $3x+11=-9$
- **c** $5x-15=0$
- **d** $4x-6=8$
- **e** $\frac{x}{2}=-6$
- **f** $\frac{x}{3}-1=7$
- **g** $\frac{x}{4}+2=-3$
- **h** $\frac{x}{8}-1=4$
- **i** $\frac{x+2}{3}=8$
- **j** $\frac{x-3}{4}=7$
- **k** $\frac{x+4}{-5}=2$
- **l** $\frac{x-6}{-2}=11$
- **m** $\frac{x+2}{8}=-4\frac{1}{2}$
- **n** $2(x+5)=26$
- **o** $3(x-2)=-15$
- **p** $2(x+7)=14$
- **q** $12(x-4)=9$
- **r** $5(x-6)=0$

F PROBLEM SOLVING WITH EQUATIONS

Worded problems can often be translated into an algebraic equation. We then solve the equation to solve the original problem.

For example, in the **Opening Problem** on page **374**, each of the girls pays the same amount. If we suppose this amount is $\$x$, then together they pay $\$7x$.

When we add on the $11 from Ilse's mother, we have $(7x+11)$ dollars.

This amount must equal the total bill, so $7x+11=25$.

We now have an equation which describes the worded problem.

$$7x + 11 = 25$$
$$\therefore \quad 7x + 11 - 11 = 25 - 11 \qquad \{\text{subtracting 11 from both sides}\}$$
$$\therefore \quad 7x = 14$$
$$\therefore \quad \frac{7x}{7} = \frac{14}{7} \qquad \{\text{dividing both sides by 7}\}$$
$$\therefore \quad x = 2$$

So, each of the girls pays $2.

Example 16 **Self Tutor**

Callum has a collection of badges. His aunt gives him 9 which she finds in a box at home. Then, while Callum is on holidays, he collects enough to double his collection. He now has 132 in total. How many badges did Callum have to start with?

Let x be the number of badges Callum has to start with. He is given 9 by his aunt, so the number is now $x + 9$.

Callum doubles his collection, so he now has $2(x + 9)$.

So, $2(x + 9) = 132$

$$\therefore \quad \frac{2(x+9)}{2} = \frac{132}{2} \qquad \{\text{dividing both sides by 2}\}$$
$$\therefore \quad x + 9 = 66$$
$$\therefore \quad x + 9 - 9 = 66 - 9 \qquad \{\text{subtracting 9 from both sides}\}$$
$$\therefore \quad x = 57$$

Callum had 57 badges to start with.

EXERCISE 20F

1. A packet of lollies is shared equally between six friends. Debbie eats three of her lollies and has four lollies left. How many lollies were in the packet?
2. Mrs Jones lives by herself, but she has a number of pet cats. There are a total of 30 legs in the house. How many cats does Mrs Jones own?
3. Pino won a sum of money in a lottery. He spent $50 on a new shirt, and shared the rest equally between his five children. Each child received $40. How much did Pino win in the lottery?
4. Yvonne bought some cartons of eggs and there were 6 eggs in every carton. When she got home from the shop, Yvonne realised that 7 of the eggs were broken. She still had 17 eggs that were not broken. How many cartons of eggs did Yvonne buy?
5. Don takes a certain amount of money to the horse races. He finds £5 on the ground, and then doubles his money by winning a bet. He now has £40. How much money did Don take to the races?
6. The average of two numbers if 14. If one of the numbers is 9, find the other number.

7 In a basketball match, Vince scored some field goals worth two points each, and also one goal worth three points. He scored a total of 19 points for the game. How many field goals did he score?

8 In a class of students, 13 are girls. The class is split into 4 equal teams for a hockey competition. If each team consists of 8 players, how many boys are in the class?

9 Each day a salesman is paid €50 plus one fifth of the sales he makes for the day. If the salesman is paid €110 one day, what value of sales did he make that day?

10 Simone sold cakes at her school fete for \$4 each. At the end of the day she had sold all but 3 of the cakes she baked. Simone received \$48 in total. How many cakes did she bake?

11 Lucien is training to be a cyclist. There is a training course 2 km from his house. Every day he cycles to the training course, completes a lap of the training course, then cycles home again. Over the course of one week he cycles 84 km. How long is the training course?

KEY WORDS USED IN THIS CHAPTER

- balance
- expression
- inspection
- trial and error
- build up
- flowchart
- inverse operation
- undo
- equation
- identity
- solution
- unknown

LINKS
click here

HOW ARE DIVING SCORES CALCULATED?

Areas of interaction:
Human ingenuity

REVIEW SET 20A

1 A number multiplied by three is equal to eighteen. Find the number.

2 **a** Is $2x + 5y = 7$ an equation or an expression?

b State the inverse of $\times 6$.

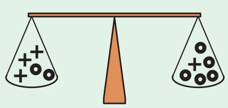
c Find the result of adding 8 to both sides of $3x - 8 = 5$.

d Solve $2x = -4$ by inspection.

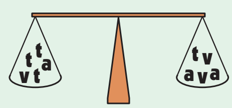
3 One of the numbers $\{1, 2, 5, 8\}$ is the solution to the equation $3x + 7 = 22$. Find the solution by trial and error.

4 The following scales are perfectly balanced. Find the relationship between the objects:

a



b



5 Find x using an inverse operation:

a $5+x=11$ **b** $x-8=-2$ **c** $4x=-32$ **d** $\frac{x}{3}=-1$

6 Copy and complete the following flowcharts:

a $\boxed{x} \xrightarrow{?} \boxed{3x} \xrightarrow{?} \boxed{3x+8}$ **b** $\boxed{x} \xrightarrow{-3} \boxed{} \xrightarrow{\div 4} \boxed{}$

7 Use a flowchart to show how the following expressions are built up from x:

a $\frac{x+4}{6}$ **b** $4x-5$

8 Use a flowchart to show how to isolate x in the following expressions:

a $\frac{x}{5}+8$ **b** $3(x-9)$

9 Solve for x:

a $4x+5=12$ **b** $\frac{x}{3}=-4$ **c** $\frac{x}{3}-5=7$

d $11x-6=2$ **e** $4(x-2)=20$ **f** $6(x+3)=54$

10 Anneke has €13. She is promised the same amount for washing the dishes each night. After seven nights of dishwashing she has €55. How much was she paid each night?

REVIEW SET 20B

1 What number can be used to replace $\square$ to make the equation true?

a $\square\div 9=5$ **b** $7\times\square=25+3$

2 **a** Solve by inspection: $a\div 6=7$.

b Find the equation which results from adding 6 to both sides of $3x-6=11$.

c State the inverse of dividing by 7.

d Solve $\frac{x}{8}=-3$ using a suitable inverse operation.

3 One of the numbers $\{2, 3, 5, 10\}$ is the solution to the equation $3x+5=14$. Find the solution by trial and error.

4 Find the equation which results from multiplying both sides of $\frac{3-x}{2}=5$ by 2.

5 State the inverse of the following operations: **a** -5 **b** $\times\frac{1}{2}$ **c** $\div 6$.

6 Find t using an inverse operation:

a $t+9=5$ **b** $t-6=0$ **c** $4t=20$ **d** $\frac{t}{-3}=8$

7 Copy and complete the following flowcharts:

a $\boxed{x} \xrightarrow{?} \boxed{\frac{x}{4}} \xrightarrow{?} \boxed{\frac{x}{4}-7}$ **b** $\boxed{x} \xrightarrow{+6} \boxed{} \xrightarrow{\times 5} \boxed{}$

8 Use a flowchart to show how the following expressions are built up from x:

a $2(3x-7)$ **b** $\dfrac{2x+3}{6}$

9 Use a flowchart to show how to isolate x in the following expressions:

a $\dfrac{5x-3}{4}$ **b** $6(2x+1)$

10 Solve for x:

a $4x-11=25$ **b** $5+4x=11$ **c** $\dfrac{x}{3}-5=8$

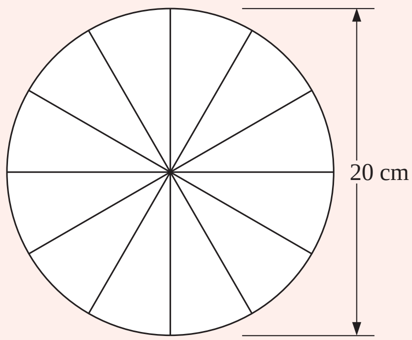
d $\dfrac{x}{5}+11=9$ **e** $3(x+7)=30$ **f** $4(x-8)=52$

11 Julian has been given a bag of chocolate truffles for his birthday. He decides to eat them all by himself. After eating 6 of them, however, he starts feeling ill and does not want any more. He shares the rest with his three sisters. If each of Julian's sisters is given 4 truffles and there is one left over, how many truffles were originally in the bag?

ACTIVITY POLYGONAL SPIRALS

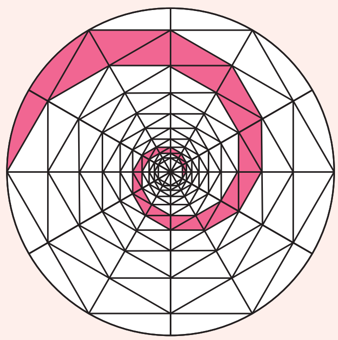
What to do:

1 Start with a large circle at least 20 cm across, and divide it into 12 sectors of equal size.



2 Make a series of hexagons as shown in the diagram below.
Continue the pattern for as long as you can.

3 Six spirals can be coloured. One of these is shown. On your figure colour all six spirals in different colours.



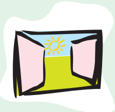
4 Repeat the above steps by first dividing the spiral into 16 sectors. Form octagons and octagonal spirals.

Chapter 21

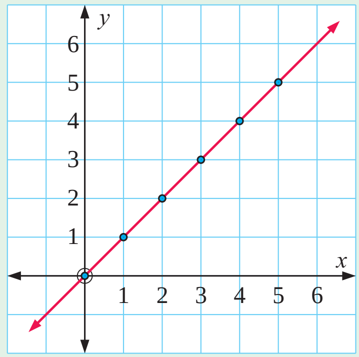
Coordinates and lines

Contents:

OPENING PROBLEM



The graph alongside shows the points (0, 0), (1, 1), (2, 2), (3, 3) and (4, 4). A straight line has been drawn through them.

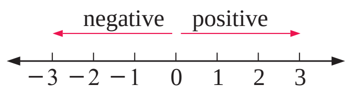


Things to think about:

- What is the point on the line with x-coordinate 5?
- How can we represent points on the line with x-coordinates that are negative?
- Can we write an equation which all points on the line satisfy?

A THE NUMBER PLANE

In **Chapter 13** we saw how the **number line** was extended in two directions to represent positive and negative numbers.



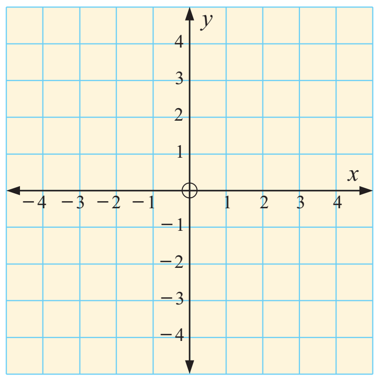
To extend the **number plane** we extend both the x-axis and the y-axis in two directions.

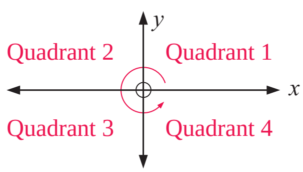
In the centre of the number plane is the origin O.

The x-axis is positive to the right of O and negative to the left of O.

The y-axis is positive above O and negative below O.

This number plane is called the **Cartesian plane**, which takes its name from **René Descartes**.





The Cartesian plane is divided into four **quadrants** by the x and y axes.

The four quadrants are numbered in an anticlockwise direction.

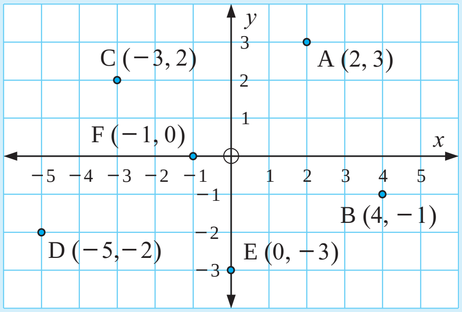
The first quadrant is the upper right hand region in which x and y are both positive.

We can now describe and plot points in any of the four quadrants or on either axis. As always, the **x-coordinate** is given first and the **y-coordinate** is given second.

Example 1 	Self Tutor

Plot the following points on a Cartesian plane:

A(2, 3), B(4, −1), C(−3, 2), D(−5, −2), E(0, −3), F(−1, 0).



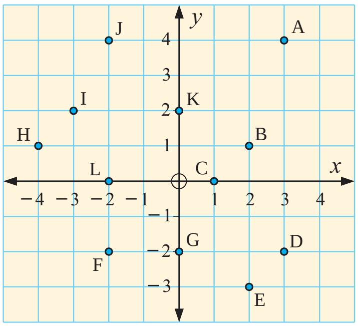
EXERCISE 21A

1 Draw a set of axes, then plot and label the following points:

a A(2, 3) **b** B(5, 1) **c** C(−1, 0) **d** D(−3, −3)
e E(0, −2) **f** F(5, 0) **g** G(3, −1) **h** H(0, 4)
i I(−4, 3) **j** J(4, 0) **k** K(3, −4) **l** L(6, 1)

2 Write down:

a the x-coordinates of A, D, E, G, H, J, L and O
b the y-coordinates of B, C, F, G, I, K, L and O
c the coordinates of all points.



3 Which of the points in question **2** lie:

a in the first quadrant
b in the second quadrant
c in the third quadrant
d in the fourth quadrant
e on the x-axis
f on the y-axis?

4 On a set of axes plot the points with coordinates given below. Join the points by straight line segments in the order given:
(0, 3), (6, 1), (6, 0), (0, 0), (0, −4), (2, −5), (2, −6), (−3, −6), (−3, −5), (−1, −4), (−1, 0), (−7, 0), (−7, 1), (−1, 3), (−1, 5), $(-\frac{1}{2}, 6)$, (0, 5), (0, 3).

5 In which quadrant would you find a point where:

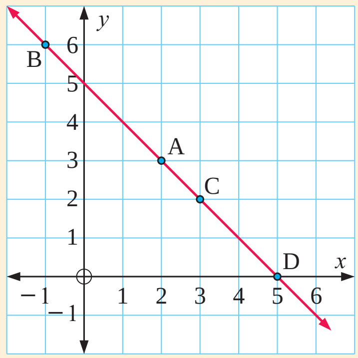
a both x and y are positive **b** both x and y are negative
c x is negative and y is positive **d** x is positive and y is negative?

B POINTS ON A STRAIGHT LINE

INVESTIGATION — POINTS WHICH FORM A LINE

Consider the points A(2, 3), B(−1, 6), C(3, 2) and D(5, 0).

These points are plotted on the number plane. We notice that they lie in a **straight line**.



If we look carefully at the points we can see that the x-coordinate and y-coordinate of each point add to 5.

This means that the **equation** of the line is $x + y = 5$.

What to do:

1. Give the coordinates of *four* other points which lie on the line. Check that $x + y = 5$ for each of these points.
2. Plot each of the following sets of points on a separate set of axes and draw a line which passes through the points:
   - **a** A(1, 2), B(3, 0), C(2, 1) and D(4, −1)
   - **b** A(1, 1), B(2, 2), C(5, 5) and D(−2, −2)
   - **c** A(−1, 1), B(−2, 2), C(−4, 4) and D(3, −3)
   - **d** A(1, 3), B(2, 4), C(3, 5) and D(5, 7)
   - **e** A(2, 4), B(3, 6), C(4, 8) and D(−1, −2)
   - **f** A(2, 1), B(4, 2), C(−6, −3) and D(8, 4).
3. For each set of points in **2**, find the equation of the line.

Suppose we know the equation of a straight line. We can **substitute** a value of x to find the point on the line with that x-coordinate.

Example 2 — Self Tutor

A line has equation $y = x + 2$. Find the coordinates of the point on the line with x-coordinate:

a 1 **b** 4 **c** −2 **d** 1.6

a When $x = 1$,
$y = 1 + 2 = 3$.
So, the point is (1, 3).

b When $x = 4$,
$y = 4 + 2 = 6$.
So, the point is (4, 6).

c When $x = -2$,
$y = -2 + 2 = 0$.
So, the point is (−2, 0).

d When $x = 1.6$,
$y = 1.6 + 2 = 3.6$.
So, the point is (1.6, 3.6).

EXERCISE 21B

1 A line has equation $y = x - 1$. Find the coordinates of the point on the line with x-coordinate:

a 1 **b** 3 **c** 0 **d** -2 **e** $5\frac{1}{2}$

2 A line has equation $y = 2x - 3$. Find the coordinates of the point on the line with x-coordinate:

a 3 **b** 0 **c** -1 **d** $\frac{1}{2}$ **e** 1.7

3 A line has equation $y = \frac{1}{2}x + 3$. Find the coordinates of the point on the line with x-coordinate:

a 4 **b** -2 **c** 100 **d** 5 **e** 2.6

4 A line has equation $y = -2x + 5$. Find the coordinates of the point on the line with x-coordinate:

a 0 **b** -4 **c** 3 **d** $-\frac{1}{2}$ **e** $\frac{3}{5}$

C GRAPHING STRAIGHT LINES

$y = 2x + 3$, $y = 3x - 1$, $y = \frac{1}{2}x + 5$, $y = -\frac{1}{4}x - 2$ are the equations of four different straight lines. In fact, any equation of the form $y = mx + c$ where m and c are numbers is the equation of a straight line.

For any straight line graph:

- the ***y*-intercept** is the value of y when the graph crosses the y-axis
- the ***x*-intercept** is the value of x when the graph crosses the x-axis.

Example 3 **Self Tutor**

For $y = x + 2$:

a Copy and complete the table of values:

b Graph the straight line.

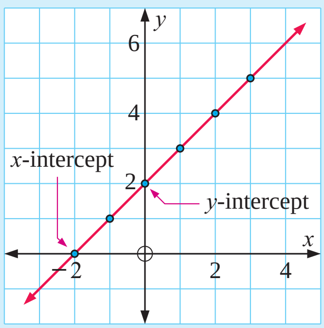
c Find the x and y-intercepts.

x	-2	-1	0	1	2	3
y						

a When $x = -2$, $y = -2 + 2 = 0$
when $x = -1$, $y = -1 + 2 = 1$
when $x = 0$, $y = 0 + 2 = 2$
when $x = 1$, $y = 1 + 2 = 3$
when $x = 2$, $y = 2 + 2 = 4$
when $x = 3$, $y = 3 + 2 = 5$

x	-2	-1	0	1	2	3
y	0	1	2	3	4	5

b



c The x-intercept is -2. The y-intercept is 2.

EXERCISE 21C

1 For $y = 2x - 1$, copy and complete:
Hence draw the graph of $y = 2x - 1$.

x	-2	-1	0	1	2	3
y						

2 For $y = -2x + 1$, copy and complete:
Hence draw the graph of $y = -2x + 1$.

x	-2	-1	0	1	2	3
y						

3 Construct a table of values and hence draw the graph of:

a $y = x - 2$ **b** $y = 2x + 2$ **c** $y = -x + 1$
d $y = -2x + 4$ **e** $y = 3x + 3$ **f** $y = \frac{1}{2}x$
g $y = \frac{1}{2}x - 2$ **h** $y = -\frac{1}{2}x + 1$ **i** $y = -4x + 2$

4 For each graph in **3** find the x and y-intercepts.

5 The following tables of values are all for straight line graphs. Complete each table, drawing a graph to help you if necessary.

a

x	-2	-1	0	1	2	3
y	5	7	9	11		15

b

x	-2	-1	0	1	2	3
y	8		4	2		

c

x	-1	0	1	2	3	4
y	8			2	0	

d

x	-3	-2	-1	0	1	2
y	-2				10	

e

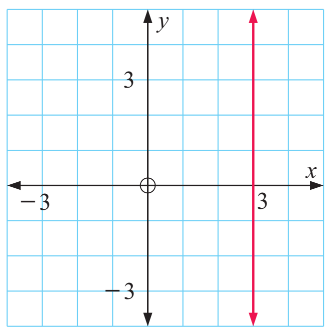
x	-2	-1	0	1	3	5
y	4	2	0	-2		

f

x	-3	-1	1	2	3	4
y			1	3	5	7

D SPECIAL LINES

The illustrated line is **vertical** and it cuts the x-axis at 3.

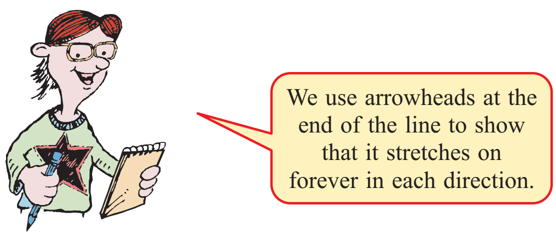


Every point on the line has x-coordinate 3, so the line has equation $x = 3$.

y is not mentioned in this equation as y can take any value.

Points such as (3, 2.791) and (3, −0.678) also lie on this line.

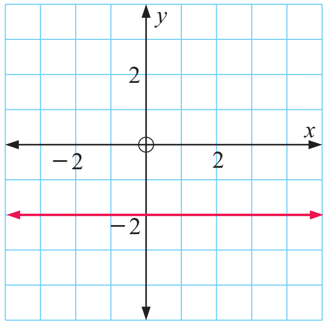
There are *infinitely many* points on the line.



We can describe **horizontal** lines in a similar way.

For example, every point on the line illustrated has y-coordinate -2.

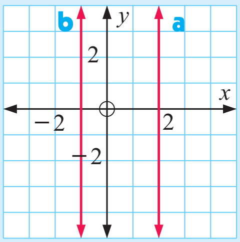
The equation of the line is therefore $y = -2$.



A **vertical line** which cuts the x-axis at k has equation $x = k$.
A **horizontal line** which cuts the y-axis at k has equation $y = k$.

Example 4 Self Tutor

Find the equations of the illustrated lines:

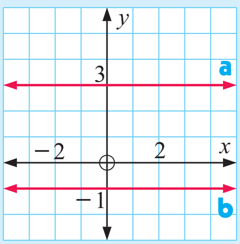


a Every point on the line has x-coordinate 2
$\therefore$ the equation is $x = 2$.

b Every point on the line has x-coordinate -1
$\therefore$ the equation is $x = -1$.

Example 5 Self Tutor

Find the equations of the illustrated lines:



a Every point on the line has y-coordinate 3
$\therefore$ the equation is $y = 3$.

b Every point on the line has y-coordinate -1
$\therefore$ the equation is $y = -1$.

EXERCISE 21D

1 On the same set of axes, graph the lines with equations:

a $x = 4$ **b** $x = -2$ **c** $x = 0$ **d** $x = 1\frac{1}{2}$.

2 On the same set of axes, graph the lines with equations:

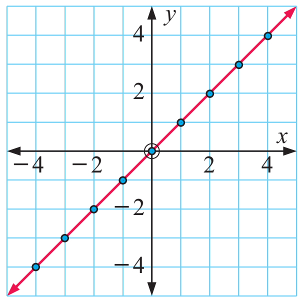
a $y = 2$ **b** $y = 0$ **c** $y = -3$ **d** $y = -\frac{1}{2}$.

3 **a** Copy and complete the table of values for this line:

x	-4	-3	-2	-1	0	1	2	3	4
y									

b Find the equation of the line.

c State the y-coordinate of the point on the line with x-coordinate 1.4.

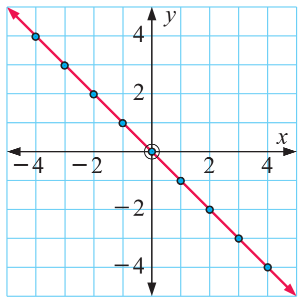


4 **a** Copy and complete the table of values for this line:

x	-4	-3	-2	-1	0	1	2	3	4
y									

b Find the equation of the line.

c A point on the line has x-coordinate -2.31. What is its y-coordinate?



5 Copy and complete:

a Any vertical line has an equation of the form ....

b Any horizontal line has an equation of the form ....

c The line at 45^o to the axes in quadrants (1) and (3) has equation ....

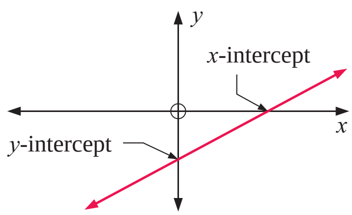
d The line at 45^o to the axes in quadrants (2) and (4) has equation ....

E THE x AND y-INTERCEPTS

The x and y-intercepts for the graph of a straight line can be found without actually graphing the line.

We use algebra to do this, noting that:

- the y-intercept occurs when $x = 0$
- the x-intercept occurs when $y = 0$.



Example 6 **Self Tutor**

Find the x and y-intercepts of the line with equation $y = 2x - 3$.

The line cuts the x-axis when $y = 0$

$\therefore \quad 2x - 3 = 0$

$\therefore \quad 2x = 3$

$\therefore \quad x = \frac{3}{2}$

So, the x-intercept is $1\frac{1}{2}$.

The line cuts the y-axis when $x = 0$

$\therefore \quad y = 2(0) - 3$

$\therefore \quad y = 0 - 3$

$\therefore \quad y = -3$

So, the y-intercept is -3.

EXERCISE 21E

1 Find the y-intercept of the line with equation:

a $y = 2x - 4$ **b** $y = x - 5$ **c** $y = 2x + 6$
d $y = 3x - 9$ **e** $y = 2x - 1$ **f** $y = 2x + 1$
g $y = -2x + 3$ **h** $y = -x + 7$ **i** $y = 7x - 10$
j $y = \frac{1}{2}x + 5$ **k** $y = -\frac{1}{2}x - 2$ **l** $y = \frac{1}{3}x + \frac{3}{2}$

2 What is the y-intercept of the line with equation $y = mx + c$?

3 Find the x-intercept of the line with equation:

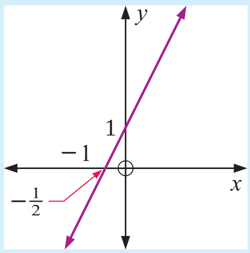
a $y = 2x - 4$ **b** $y = x - 5$ **c** $y = 2x + 6$
d $y = 3x - 9$ **e** $y = 2x - 1$ **f** $y = 2x + 1$
g $y = -2x + 3$ **h** $y = -x + 7$ **i** $y = 7x - 10$
j $y = \frac{1}{2}x + 5$ **k** $y = -\frac{1}{2}x - 2$ **l** $y = \frac{1}{3}x + \frac{3}{2}$

Example 7 Self Tutor

a Find the x and y-intercepts of the line with equation $y = 2x + 1$.
b Use the axes intercepts to draw the graph of $y = 2x + 1$.

a When $x = 0$, $y = 2(0) + 1 = 1$
$\therefore$ the y-intercept is 1
When $y = 0$, $2x + 1 = 0$
$\therefore$ $2x = -1$
$\therefore$ $x = -\frac{1}{2}$
$\therefore$ the x-intercept is $-\frac{1}{2}$

b



4 For each of the following lines:

i find the y-intercept
ii find the x-intercept
iii draw the graph using the axes intercepts.

a $y = 2x - 2$ **b** $y = 4x - 1$ **c** $y = 5x - 2$ **d** $y = -2x + 3$
e $y = -x + 4$ **f** $y = -3x + 5$ **g** $y = \frac{1}{2}x - 2$ **h** $y = -\frac{1}{2}x + 2$

KEY WORDS USED IN THIS CHAPTER

- Cartesian plane
- number plane
- vertical line
- horizontal line
- quadrant
- x-intercept
- line graph
- straight line
- y-intercept

REVIEW SET 21A

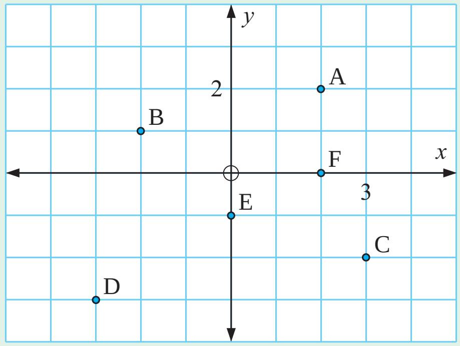
1 On the same set of axes, plot and label the following points:
A(3, −2), B(2, 4), C(−4, 1), D(0, −3).

2 Write down:

a the x-coordinates of A and D

b the y-coordinates of B and C

c the coordinates of A, B, E and F.



3 In which quadrants do the following points lie:

a (−2, 3) **b** (5, 7) **c** (0, −3)?

4 Are (2, 3) and (3, 2) the same point on the number plane? Illustrate your answer.

5 Find the equation connecting x and y given the following set of points:
A(−1, −2), B(1, 0), C(2, 1), D(3, 2).

6 On the same set of axes, graph the lines with equations:
$x = 3$, $y = 5$, $x = -1$ and $y = -1\frac{1}{2}$.

7 For the line with equation $y = 3x - 2$, copy and complete:
Hence draw the graph of $y = 3x - 2$.

x	−2	−1	0	1	2	3
y						

8 A line has equation $y = 3x - 1$. Find the coordinates of the point on the line with x-coordinate:

a 0 **b** 5 **c** −2 **d** $\frac{3}{5}$

9 Copy and complete the table of values for this straight line:

x	−2	−1	0	1	2	3
y	2		−1			$-5\frac{1}{2}$

10 Sketch the graph of the line with equation $y = x$.

11 For the line with equation $y = 2x - 5$:

a find the y-intercept

b find the x-intercept

c graph the line using the axes intercepts only.

REVIEW SET 21B

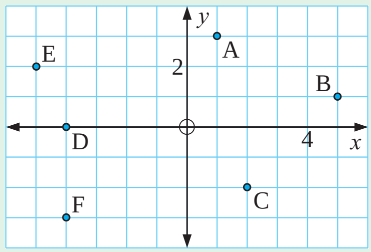
1 On a set of axes, plot and label the following points:
A(−4, 2), B(5, 7), C(6, −3), D(−2, 0).

2 Write down the:

a x-coordinates of D and F

b y-coordinates of C and E

c coordinates of A, B, C and D.



3 How many points have x-coordinate 4 and y-coordinate 5?

4 In which quadrant would I find a point with:

a positive x-coordinate **b** negative x and y-coordinates?

5 Write an equation describing all points with y-coordinate 2.

6 The points on a line all obey the rule $y = x - 5$.
Find the y-coordinate of the points on the line with x-coordinate:

a 2 **b** 0 **c** −2

7 On the same set of axes, graph the lines with equations $y = 1$, $x = 2\frac{1}{2}$, $y = -3$ and $x = -4$.

8 Construct a table of values and hence draw the graph of $y = \frac{1}{2}x + 3$.

9 Fill in the missing values for this straight line graph:

x	−2	−1	0	1	2	3
y	−9			−3	−1	

10 Sketch the graph of the line with equation $y = -x$.

11 For the line with equation $y = 3x + 2$:

a find the y-intercept

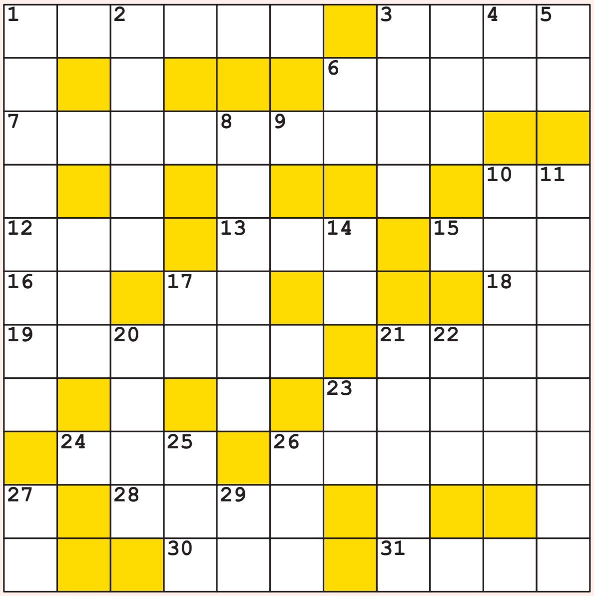
b find the x-intercept

c graph the line using the axes intercepts only.

PUZZLE — ROMAN CROSSWORD

Solve this puzzle, writing all answers in Roman numerals:

PRINTABLE TEMPLATE



Across

1. One thousand six hundred and fifty six
3. 1509
6. One thousand subtract 404
7. $(8 \times 10^2) + (6 \times 10^1) + (7 \times 10^0)$
10. 2×10^3
12. The difference between 936 and 531
13. 9×5
15. The sum of 800 and six hundred
16. The product of fifty and four
17. Half of 1020
18. 999 999 − 999 909
19. Twelve times seven
21. Ten lots of (eleven times four)
23. 937, 948, ..., 970, 981
24. Nine more than one thousand
26. 44 000 subtract 43 466
28. The dividend when the divisor is 2 and the quotient is 47
30. The quotient of 63 and 9
31. 12×2^3

Down

1. MMM minus CXL
2. (II times X^2) minus VI
3. Double DVII
4. XXXVI ÷ IX
5. V + VI
6. CCCII + CCIII
8. From C subtract XI
9. C divided by X
10. M + CX + XII
11. MM − CCCXXXIII
14. C − XC − IV
20. VII^2
21. The difference between M and LXXXI
22. The product of LXX and VIII
25. LXXX, LXXXV, XC, ..., C, CV
26. The sum of CCXCIX, CXLIII and LXIV
27. The product of CI and X
29. $I^2 + I^2$

Chapter 22

Transformations

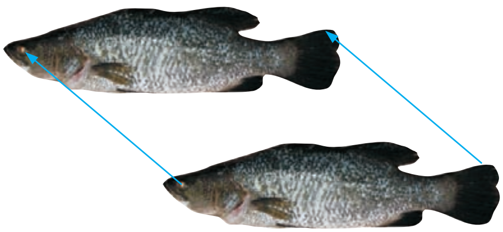
Contents:

TRANSFORMATIONS

Translation, reflection, rotation, and enlargement are all **transformations**.

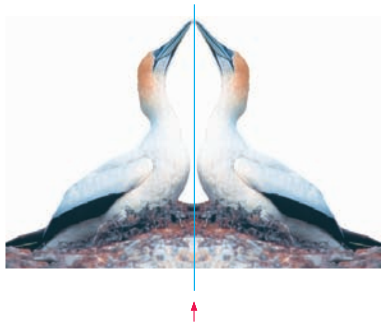
For example:

a **translation**



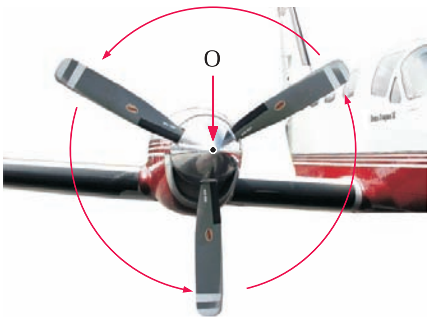
shift a particular distance in a particular direction

a **reflection**



mirror line

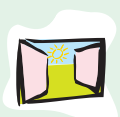
a **rotation** about O



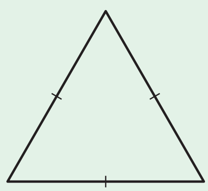
an **enlargement**

When we perform a transformation, the original shape is called the **object**. The shape which results from the transformation is called the **image**.

OPENING PROBLEM



Consider an equilateral triangle.



Things to think about:

- Can you draw a mirror line on an equilateral triangle? The figure must fold onto itself along that line, so it matches exactly.

  How many of these mirror lines can you draw?
- Can you find the centre of rotation of an equilateral triangle? The figure must rotate about this point and fit exactly onto itself in less than one full turn.
  How many times would an equilateral triangle fit onto itself in one full turn?
- Make a pattern using equilateral triangles so there are no gaps between the triangles and the edges meet exactly.

CONGRUENT FIGURES

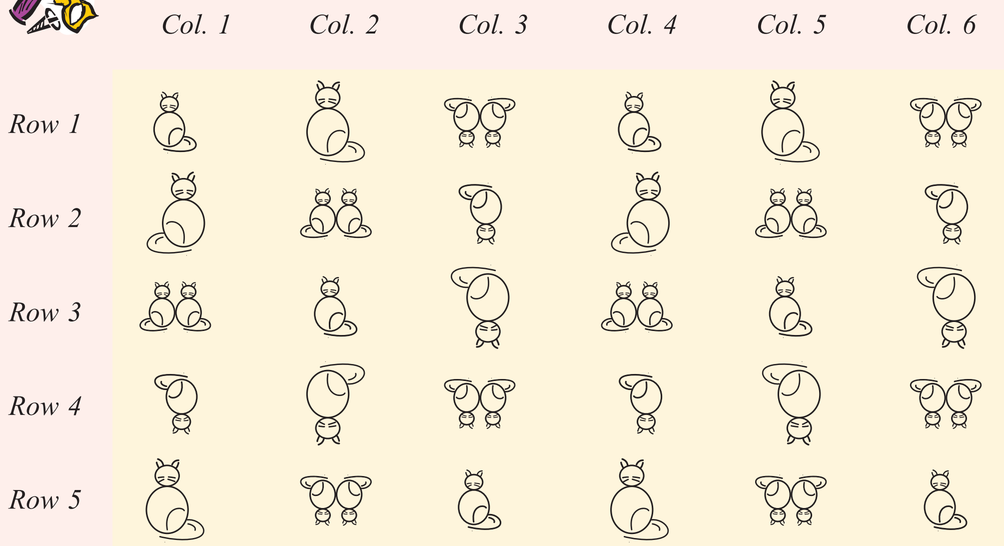
Two figures are **congruent** if they have exactly the same size and shape.

If one figure is cut out and it can be placed exactly on top of the other, then these figures are congruent.

The image and the object for a translation, rotation, or reflection are always congruent. The image and the object for an enlargement are not congruent because they are not the same size.

ACTIVITY 1 — TRANSFORMING CATS

Following is a fabric pattern which features cats and pairs of cats. The rows and columns have been numbered to identify each individual picture.



What to do:

1 Start with row 1 and column 1 cat.

- **a** Give the row and column numbers for *translations* of this cat.
- **b** Give the row and column numbers for *rotations* of this cat.
- **c** Give the row number in column 1 for an *enlargement* of this cat.
- **d** Give the row numbers in column 3 for cats *congruent* to this cat.

2 Start with row 1 column 2 cat.

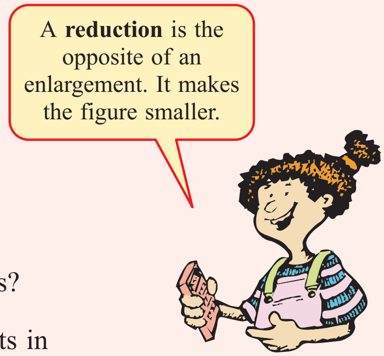
- **a** Give the row and column numbers for *translations* of this cat.
- **b** Discuss why row 4 column 2 is not a rotation of this cat.
- **c** Give the row and column numbers for *rotations* of this cat.
- **d** Give the row number in column 1 for a *reflection* of this cat.

3 Start with row 4 column 5 cat.

a Give the row numbers for any column 2 cats *congruent* to this one.

b Give the row and column numbers of any cats that are a *rotation* of this cat.

c Give the row and column numbers for any cats that are a *reduction* of this cat.



4 What is the transformation shown in the pair of cats?

5 Which *two* transformations are used to move the cats in

a row 1 column 1 to row 1 column 5

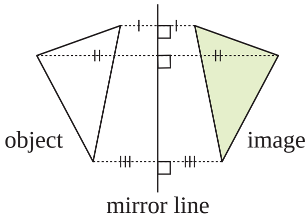
b row 2 column 4 to row 5 column 4

c row 2 column 5 to row 5 column 5

d row 3 column 6 to row 2 column 6?

A REFLECTIONS AND LINE SYMMETRY

REFLECTIONS

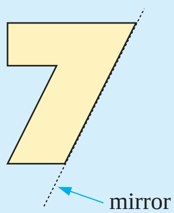


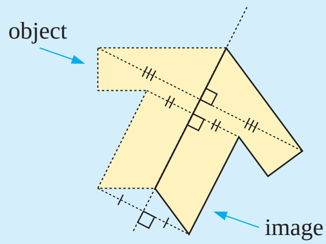
To reflect an object in a mirror line we draw lines at right angles to the mirror line which pass through key points on the object. The images of these points are the same distance away from the mirror line as the object points, but on the opposite side of the mirror line.

DEMO

Example 1 Self Tutor

Draw the mirror image of:

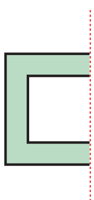




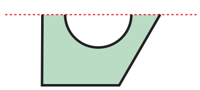
EXERCISE 22A.1

1 Place a mirror on the mirror line shown using dashes, and observe the mirror image. Draw the object and its mirror image in your work book.

a



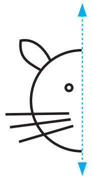
b



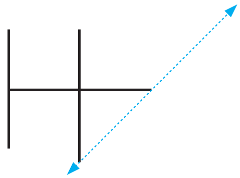
c

2 **a** Draw the image of the following if a mirror was placed on the mirror line shown:

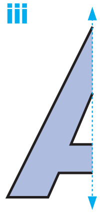
i



ii



iii

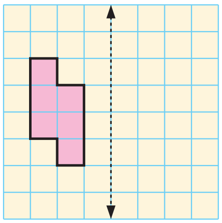


PRINTABLE DIAGRAMS

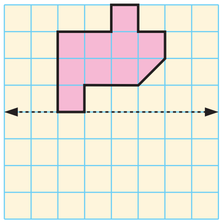
b Check your answers to **a** using a mirror.

3 On grid paper, reflect the geometrical shape in the mirror line shown:

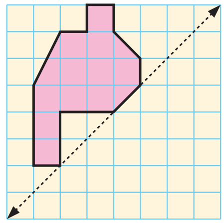
a



b



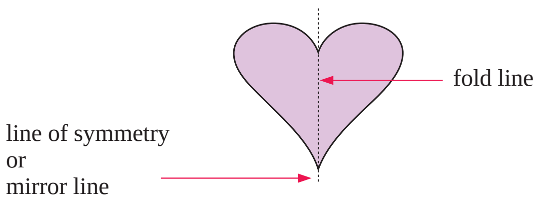
c



LINE SYMMETRY

A **line of symmetry** is a line along which a shape may be folded so that both parts of the shape will match.

For example:



DEMO

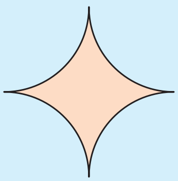
If a mirror is placed along the line of symmetry, the reflection in the mirror will be exactly the same as the half of the figure "behind" the mirror.

A shape has **line symmetry** if it has at least one line of symmetry.

Example 2 Self Tutor

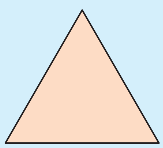
For each of the following figures, draw all lines of symmetry:

a

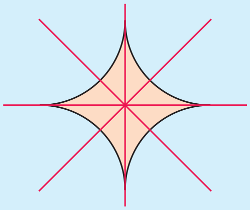


b

c



a

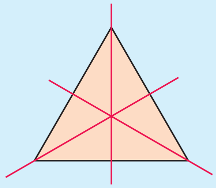


4 lines of symmetry

b

no lines of symmetry

c

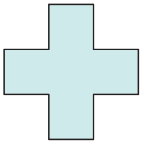


3 lines of symmetry

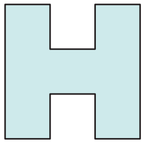
EXERCISE 22A.2

1 Copy the following figures and draw the lines of symmetry. Check your answers using a mirror.

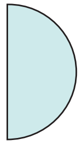
a



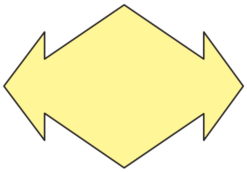
b



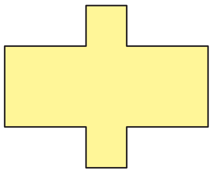
c



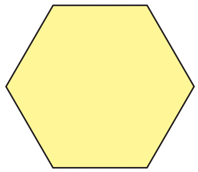
d



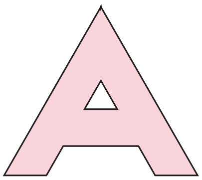
e



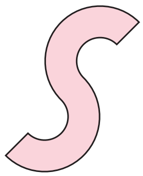
f



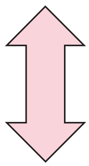
g



h

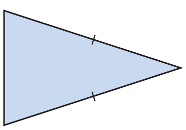


i

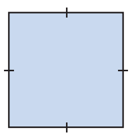


2 **a** Copy the following shapes and draw in all lines of symmetry.

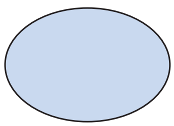
i



ii



iii



iv

b Which of these figures has the most lines of symmetry?

3 How many lines of symmetry do these patterns have?

a

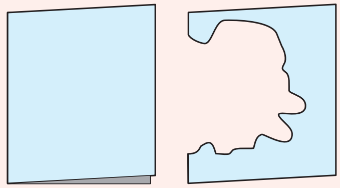
b

4 **a** How many lines of symmetry can a triangle have? Draw all possible cases.

b How many lines of symmetry can a quadrilateral have? Draw all possible cases.

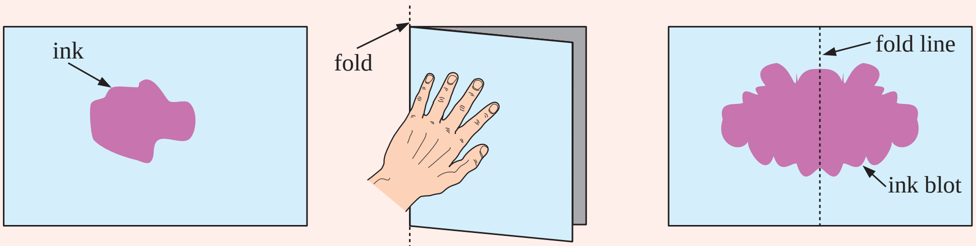
ACTIVITY 2 MAKING SYMMETRICAL SHAPES

You will need: paper, scissors, pencil, ink or paint

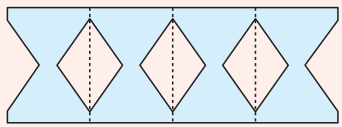


1. Take a piece of paper and fold it in half.
2. Cut out a shape along the fold line.
3. Open out the sheet of paper and observe the shapes revealed.
4. Record any observations about symmetry that you notice.
5. Try the following:
   a. Fold the paper twice before cutting out your shape.
   b. Fold the paper three times before cutting out your shape.

   In each case record your observations about the number of lines of symmetry.
6. Place a blob of ink or paint in the centre of a rectangular sheet of paper. Fold the paper in half and press the two pieces together. Open the paper and comment on the symmetry observed.



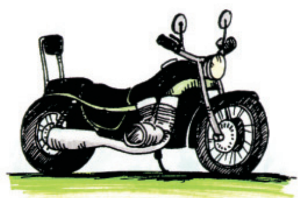
7. Make symmetrical patterns by folding a piece of paper a number of times and cutting out a shape. How many folds would you need and what shape would you need to cut out to get the result shown?



B ROTATIONS AND ROTATIONAL SYMMETRY

We are all familiar with things that rotate, such as the hands on a clock or the wheels of a motorbike.

DEMO


The point about which the hands of the clock, or the spokes of the wheel rotate, is called the **centre of rotation**.

The angle through which the hands or the spokes turn is called the **angle of rotation**.

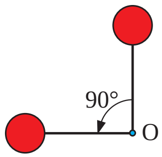
The globe of the world rotates about a line called the **axis of rotation**.

During a rotation, the distance of any point from the centre of rotation does not change.

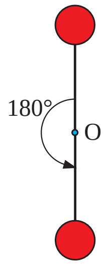
A **rotation** is the turning of a shape or figure about a point and through a given angle.

DEMO

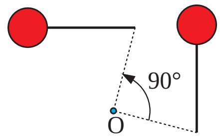
For example:



The figure is rotated anticlockwise about O through 90^o.



The figure is rotated anticlockwise about O through 180^o.



The figure is rotated anticlockwise about O through 90^o.

You will notice that under a rotation, the figure does not change in size or shape.

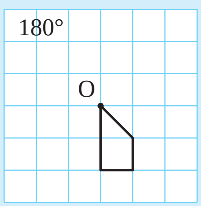
In mathematics we rotate in an anticlockwise direction unless we are told otherwise.

You should remember that 90^o is a $\frac{1}{4}$-turn, 180^o is a $\frac{1}{2}$-turn, 270^o is a $\frac{3}{4}$-turn, and 360^o is a full turn.

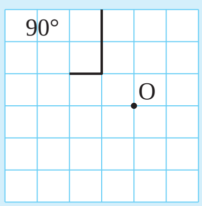
Example 3 Self Tutor

Rotate the given figures about O through the angle indicated:

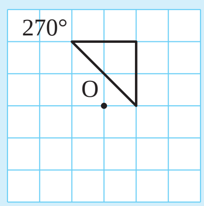
a



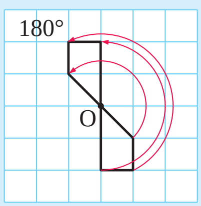
b



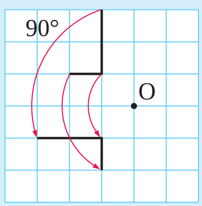
c



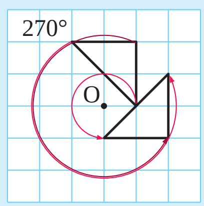
a



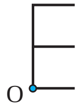
b

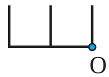


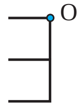
c

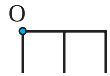


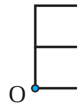
EXERCISE 22B.1

1 Consider the rotations of  which follow:

A 

B 

C 

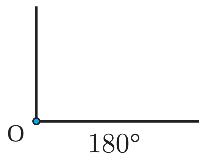
D 

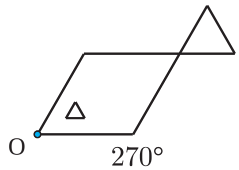
Which of **A**, **B**, **C**, or **D** is a rotation of the object through:

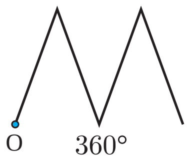
a 180^o **b** 360^o **c** 90^o **d** 270^o?

2 Copy and rotate each of the following shapes about the centre of rotation O, for the number of degrees shown. You could use tracing paper to help you.

a 
90°

b 
180°

c 
270°

d 
360°

e 
90°

f 
270°

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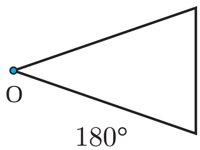
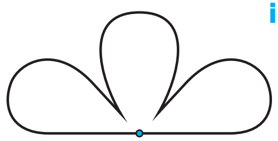
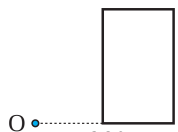
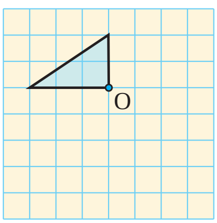
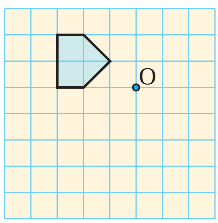
g 
180°

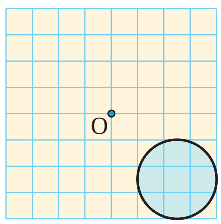
h 
90°

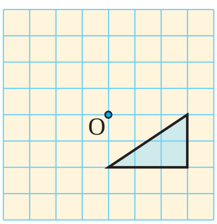
i 
90°

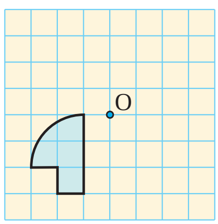
3 Rotate about O through the angle given:

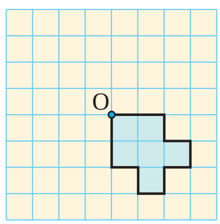
a 90^o 

b 180^o 

c 270^o 

d 180^o 

e 90^o 

f 270^o 

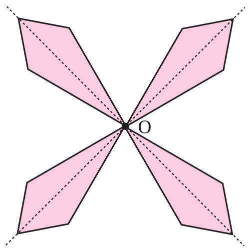
DEMO

ROTATIONAL SYMMETRY

A full rotation does not mean that a shape has rotational symmetry. Every shape fits exactly onto itself after a rotation of 360^o.

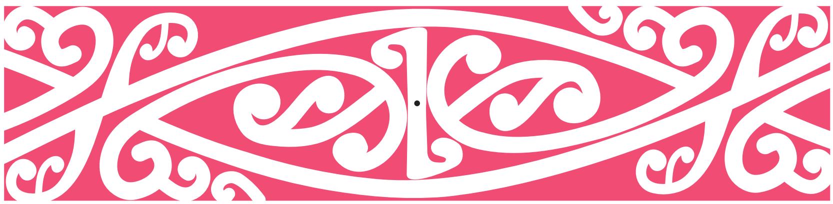
A shape has **rotational symmetry** if it can be fitted onto itself by turning it through an angle of **less than 360^o**, or one full turn.

The **centre of rotational symmetry** is the point about which a shape can be rotated onto itself.



The 'windmill' shown will fit onto itself every time it is turned about O through 90^o. O is the centre of rotational symmetry.

This fabric pattern also shows rotational symmetry.



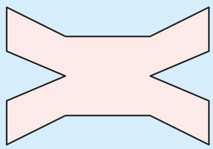
DEMO

If a figure has more than one line of symmetry then it will also have rotational symmetry. The centre of rotational symmetry will be the point where the lines of symmetry meet.

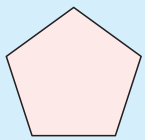
Example 4 Self Tutor

For the following figures, find the centre of rotational symmetry.

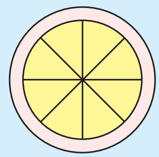
a



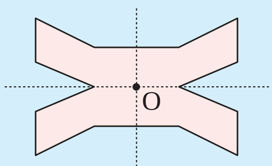
b



c

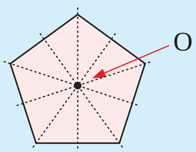


a



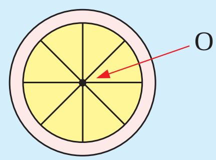
centre is O

b



centre is O

c



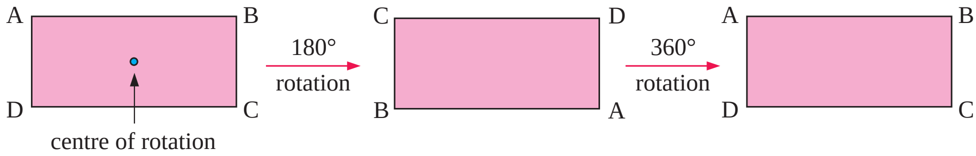
centre is O

THE ORDER OF ROTATIONAL SYMMETRY

DEMO

The **order of rotational symmetry** is the number of times a figure maps onto itself during one complete turn about the centre.

For example:

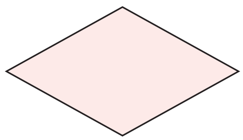
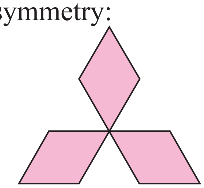


The rectangle has order of rotational symmetry of 2 since it moves back to its original position under rotations of 180^o and 360^o.

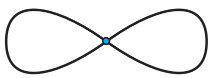
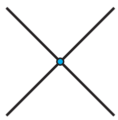
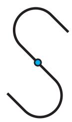
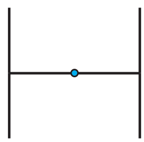
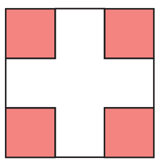
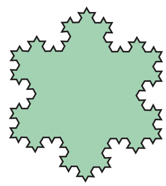
Click on the icon to find the order of rotational symmetry for an equilateral triangle.

EXERCISE 22B.2

1 For each of the following shapes, find the centre of rotational symmetry:

a 
b 
c 

2 For each of the following shapes find the *order* of rotational symmetry. You may use tracing paper to help you.

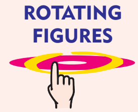
a 
b 
c 
d 
e 
f 
g 
h 

3 Design your own shape which has order of rotational symmetry of:

a 2 **b** 3 **c** 4 **d** 6

ACTIVITY 3 USING TECHNOLOGY TO ROTATE

In this activity we use a computer package to construct a shape that has rotational symmetry.



What to do:

1 Click on the icon to load the software.

2 From the menu, choose an angle to rotate through.

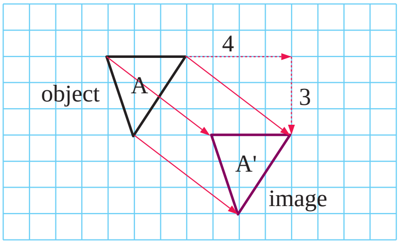
3 Make a simple design in the sector which appears, and colour it.

4 Press finish to see your creation.

C TRANSLATIONS

A **translation** of a figure occurs when every point on the figure is moved the same distance in the same direction.

Under a translation the original figure and its image are **congruent**.

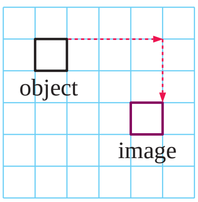


In the translation shown, the original figure has been translated 4 units right and 3 units down to give the image.

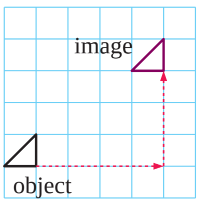
EXERCISE 22C

1 Describe each of the following translations:

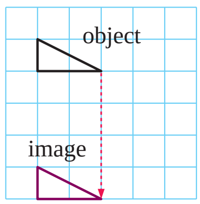
a



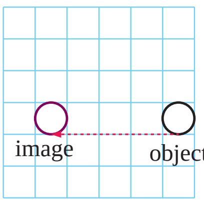
b



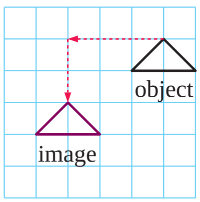
c



d

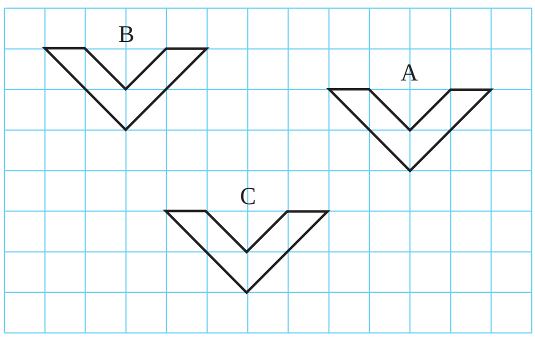


e



2 For the given figures, describe the translation from:

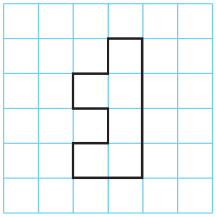
a A to B **b** B to A
c B to C **d** C to B
e A to C **f** C to A

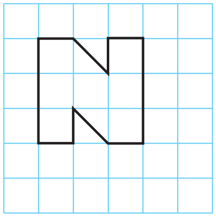


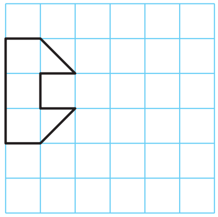
3 Copy onto grid paper and translate using the given directions:

a 3 right, 4 down **b** 6 left, 4 up **c** 2 right, 5 up

DEMO



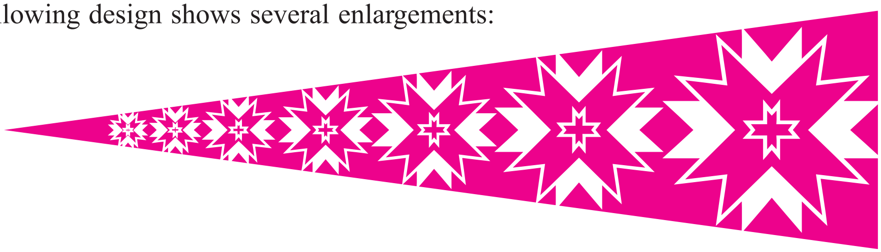




D ENLARGEMENTS AND REDUCTIONS

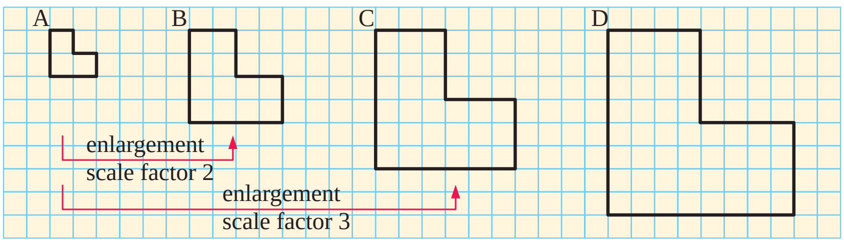
We are all familiar with enlargements in the form of photographs or looking through a microscope or telescope. Plans and maps are examples of **reductions**. The size of the image has been reduced but the proportions are the same as the original. Most photocopiers can perform enlargements and reductions.

The following design shows several enlargements:



In any enlargement or reduction, we multiply the lengths in the object by the **scale factor** to get the lengths in the image.

Look at the figures in the grid below:



DEMO

For the enlargement with scale factor 2, lengths have been *doubled*.

For the enlarement with scale factor 3, lengths have been *trebled*.

If shape B is reduced to shape A, the lengths are *halved* and the scale factor is $\frac{1}{2}$.

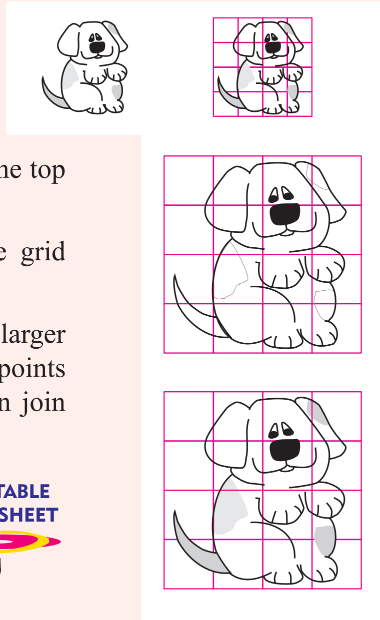
If shape D is reduced to shape A, the lengths are *quartered* and the scale factor is $\frac{1}{4}$.

A scale factor is:
- *greater* than 1 for an enlargement
- *less* than 1 for a reduction.

ACTIVITY 4 ENLARGEMENT BY GRIDS

You will need: Paper, pencil, ruler

What to do:



1. Copy the picture alongside.
2. Draw a grid 5 mm by 5 mm over the top of the dog as shown alongside:
3. Draw a grid 10 mm by 10 mm alongside the grid already drawn.
4. Copy the dog from the smaller grid onto the larger grid. To do this accurately, start by transferring points where the drawing crosses the grid lines. Then join these points and finish the picture.
5. Use this method to change the size of other pictures. You may like to try making the picture smaller as well as larger, by making your new grid smaller than the original.

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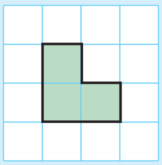
Example 5 Self Tutor

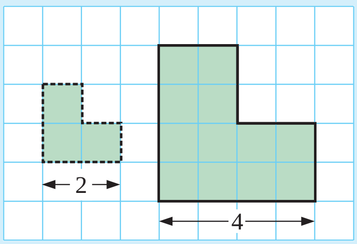
Enlarge



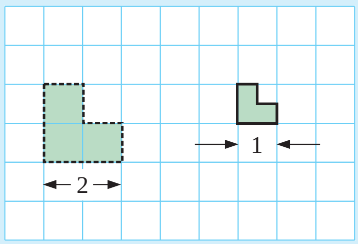
using a scale factor of:

a 2 **b** $\frac{1}{2}$

a



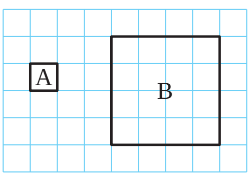
b



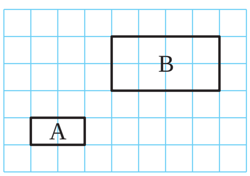
EXERCISE 22D

1 In the following diagrams, A has been enlarged to B. Find the scale factor.

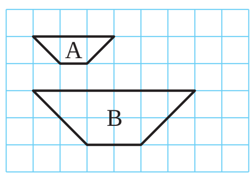
a



b

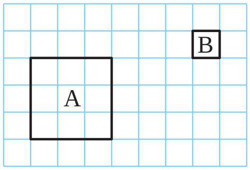


c

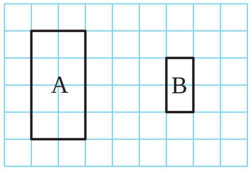


2 In the following diagrams, A has been reduced to B. Find the scale factor.

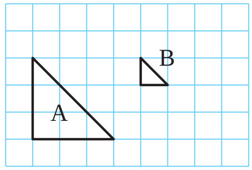
a



b



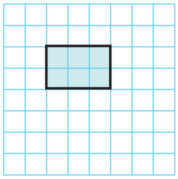
c



3 Enlarge or reduce the following objects by the scale factor given:

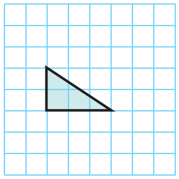
PRINTABLE WORKSHEET

a



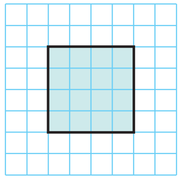
scale factor 2

b



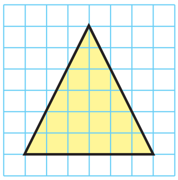
scale factor 3

c



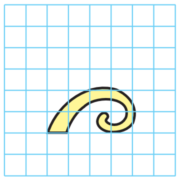
scale factor $\frac{1}{2}$

d



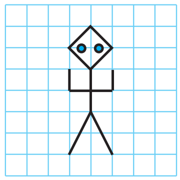
scale factor $\frac{1}{3}$

e



scale factor 4

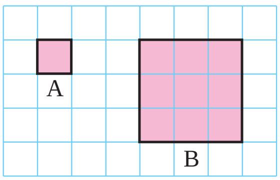
f



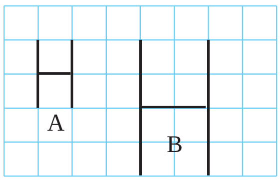
scale factor 2

4 Find the scale factor when A is transformed to B:

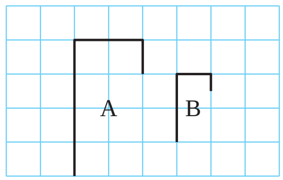
a



b



c

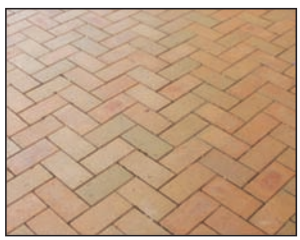


5 For each grid in **4**, write down the scale factor which transforms B into A.

E TESSELLATIONS

A **tessellation** is a pattern made using figures of the same shape and size. They must cover an area without leaving any gaps.

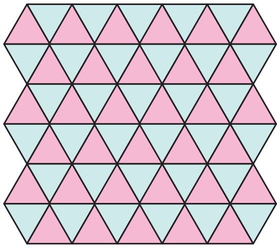
The photograph alongside shows a tessellation of bricks used to pave a footpath.

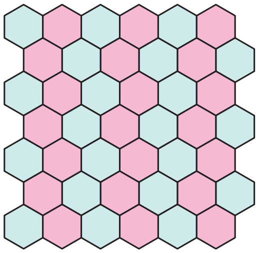


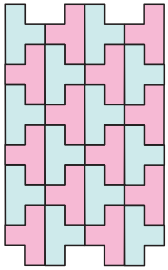
Tessellations are also found in carpets, wall tiles, floor tiles, weaving and wall paper.

This brick design is **not a tessellation** as it is constructed from two different brick sizes.

The following tile patterns are all tessellations:

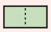


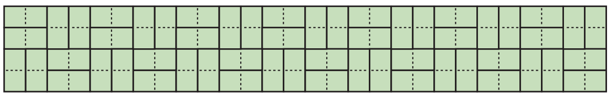


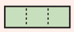


ACTIVITY 5 PAVING BRICKS

What to do:

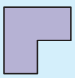
1 Using the "2 × 1" rectangle  , form at least two different tessellation patterns. One example is:

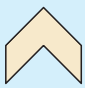


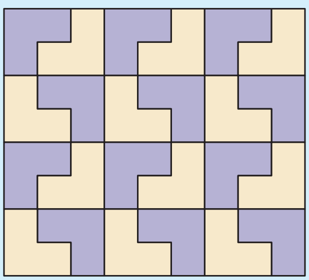
2 Repeat **1** using a "3 × 1" rectangle  .

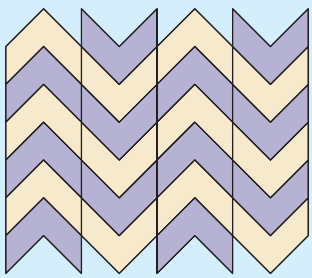
Example 6 — Self Tutor

Draw tessellations of the following shapes.

a 

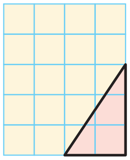
b 

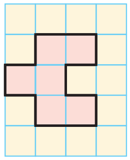
a 

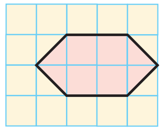
b 

EXERCISE 22E

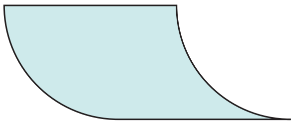
1 Draw tessellations using the following shapes:

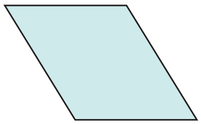
a 

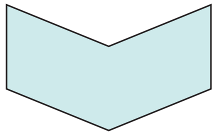
b 

c 

2 Draw tessellations using the following shapes:

a 

b 

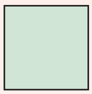
c 

ACTIVITY 6 — CREATING TESSELLATIONS

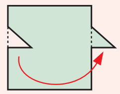
What to do:

Follow these steps to create your own tessellating pattern.

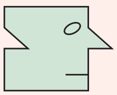
Step 1: Draw a square.



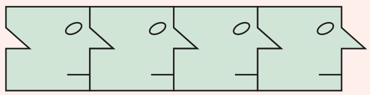
Step 2: Cut a piece from one side and 'glue' it onto the opposite side.



Step 3: Rub out any unwanted lines and add features.



Step 4: Photocopy this several times and cut out each face. Combine them to form a tessellation.

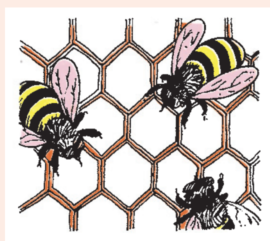


Make your own tessellation pattern and produce a full page pattern with 3 cm by 3 cm tiles. Be creative and colourful. You could use a computer drawing package to do this activity.

DISCUSSION — IN GOOD SHAPE

1 Research the shape of the cells in a beehive. Explain why they are that shape.

2 Look at the shapes of paving blocks. Explain what advantage some shapes have over others. When building walls, what are the advantages of rectangular bricks over square bricks?



ACTIVITY 7 COMPUTER TRANSFORMATIONS

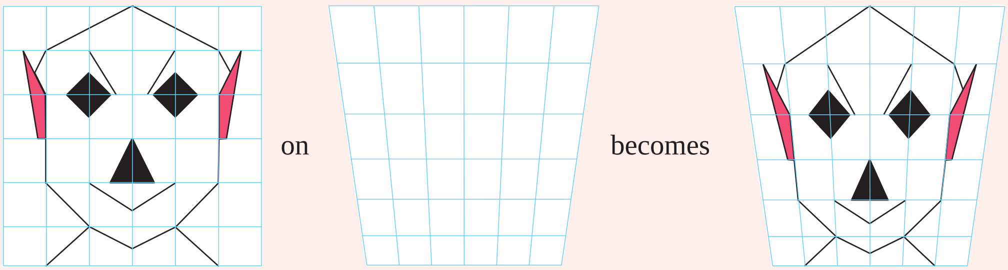
TESSELLATIONS BY COMPUTER

What to do:

1. Pick a shape and learn how to translate, reflect and rotate it.
2. Create a tessellation on your screen and colour it.
3. Print your final masterpiece.

ACTIVITY 8 DISTORTION TRANSFORMATIONS

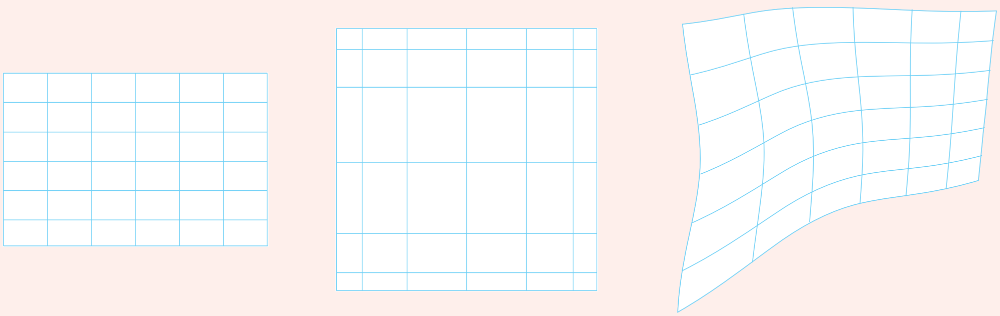
In this activity we copy pictures onto unusual graph paper to produce distortions of the original diagram. For example,



PRINTABLE GRIDS

What to do:

1. On ordinary squared paper draw a picture of your own choosing.
2. Redraw your picture on different shaped graph paper. For example:



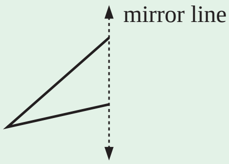
KEY WORDS USED IN THIS CHAPTER

- angle of rotation
- congruent
- line of symmetry
- reflection
- scale factor
- axis of rotation
- enlargement
- mirror line
- rotation
- tessellation
- centre of rotation
- image
- object
- rotational symmetry
- translation

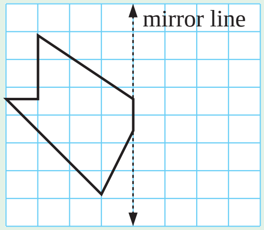
REVIEW SET 22A

1 Draw the mirror image of:

a

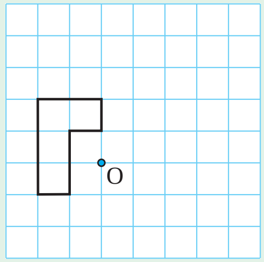


b



2 Draw the lines of symmetry for:

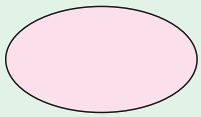
3 Rotate the given figure about O through 90^o anticlockwise.



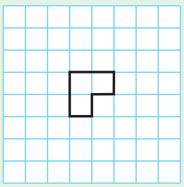
4 For the given shape:

a draw the lines of symmetry

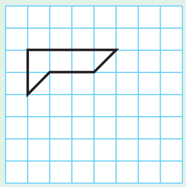
b state the order of rotational symmetry.



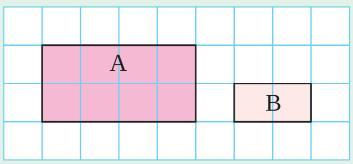
5 **a** Translate the figure three units left and one unit up.



b Enlarge the figure with scale factor 2.



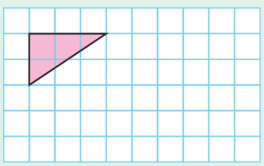
6



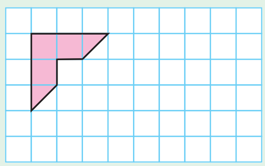
In the diagram A has been reduced to B. Find the scale factor.

7 Draw tessellations of the following shapes.

a



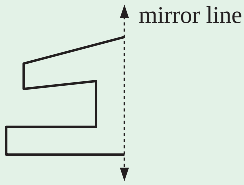
b



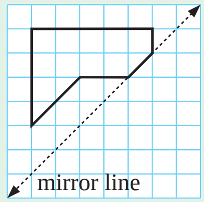
REVIEW SET 22B

1 Draw the mirror image of:

a

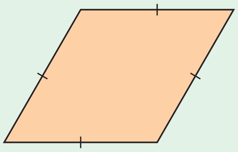


b

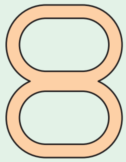


2 Draw the lines of symmetry for:

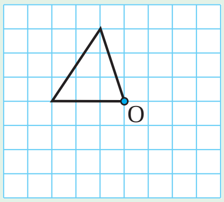
a



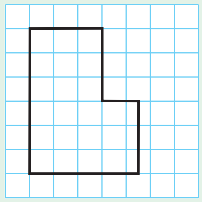
b



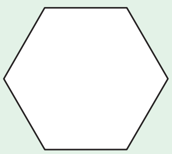
3 **a** Rotate the figure shown through 90^o anticlockwise about O.



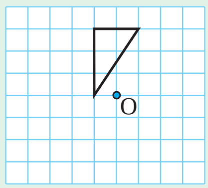
b Enlarge the figure with scale factor $\frac{1}{3}$.



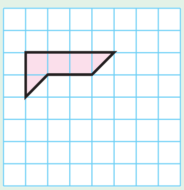
4 **a** Find the order of rotational symmetry for:



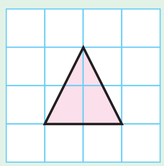
b Rotate the given figure 180^o about O.



5 Translate the given figure one unit to the right and 3 units down.



6 Draw a tessellation using the given shape.

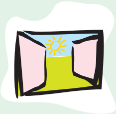


Chapter 23

Sets

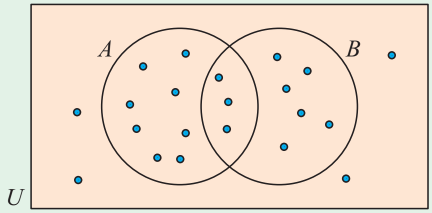
Contents:

OPENING PROBLEM



In the diagram alongside, how many dots are there in:

a circle A
b circle B
c circle A *and* circle B
d circle A *or* circle B
e neither circle A nor circle B
f exactly one of the two circles?



A SETS AND THEIR MEMBERS

A **set** is a group of objects or symbols.

For example, the prime numbers less than 13 are 2, 3, 5, 7 and 11.

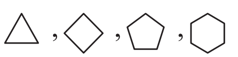
We could write them as the set $A = \{2, 3, 5, 7, 11\}$

A set is usually given a letter like A to identify it, especially when two or more sets are being considered.

We can also use words to help define a set. For example, the set of all multiples of three which are less than 13 could be written as

$M = \{\text{multiples of 3 which are} < 13\}$ or $M = \{3, 6, 9, 12\}$.

Other examples of sets are:

- {  } is the set of all polygons with less than 7 sides.
- {blue, grey, hazel, brown, green} is the set of all eye colours.
- $\{a, e, i, o, u\}$ is the set of all vowels in the English alphabet.

The objects or symbols in a set are called the **elements** or **members** of the set.

SET NOTATION

- $\in$ means: '*is a member of*' or '*is an element of*' or '*is in*' or '*belongs to*'
- $\notin$ means: '*is not a member of*' or '*is not an element of*' or '*is not in*' or '*does not belong to*'.
- $n(A)$ means: '*the number of elements in set A*'.

For example, if $A = \{2, 3, 5, 7, 11\}$ then $5 \in A$, $8 \notin A$, and $n(A) = 5$.

Example 1 — Self Tutor

a Use set notation to list the elements of:

  i P, the set of all prime numbers between 10 and 30

  ii F, the set of all factors of 21.

b True or false:

  i $21 \in P$ **ii** $23 \notin P$ **iii** $4 \notin F$ **iv** $21 \in F$?

c Find: **i** $n(P)$ **ii** $n(F)$.

---

a **i** $P = \{11, 13, 17, 19, 23, 29\}$ **ii** $F = \{1, 3, 7, 21\}$

b **i** False. 21 is not a prime number, so is not in P.

  ii False. 23 is in the set P.

  iii True. 4 is not in the set F.

  iv True. 21 is in the set F.

c **i** $n(P) = 6$ {as there are 6 elements in P}

  ii $n(F) = 4$ {as there are 4 elements in F}

EXERCISE 23A.1

1 List the members of the set of all:

  a positive even numbers less than 14

  b positive odd numbers between 15 and 30

  c prime numbers less than 23

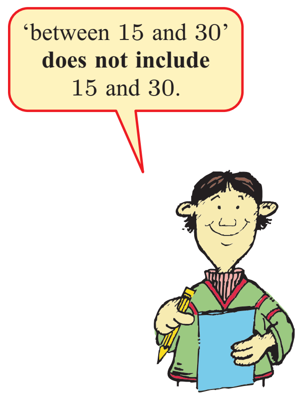
  d even prime numbers

  e prime factors of 12

  f multiples of 7 which are less than 50

  g common multiples of 2 and 5 which are less than 40

  h square numbers between 20 and 100.



2 Which of the following are true?

  a $17 \in$ {prime numbers less than 30} **b** $14 \notin$ {multiples of 4}

  c $1 \in$ {prime numbers} **d** $257 \in$ {odd numbers}

  e $91 \notin$ {prime numbers} **f** $\frac{9}{3} \notin$ {whole numbers}

  g $u \in$ {English vowels}

  h $3 \in$ {multiples of 3} and $3 \in$ {factors of 6}

3 List the elements of the following sets. State the number of elements in each set using $n(......)$ notation:

For example, if $M =$ {multiples of 11 which are less than 50}, then

$M = \{11, 22, 33, 44\}$ and $n(M) = 4$.

  a $P =$ {prime numbers less than 25}

  b $M =$ {multiples of 7 between 20 and 60}

  c $D =$ {numbers which divide exactly into 32}

4 Suppose P is the set of all prime numbers between 0 and 15, Q is the set of all factors of 27, and R is the set of all multiples of 3 which are less than 18.

a List the elements of the sets P, Q and R using set notation.

b Find x if:

i $x \in P$ and $x \in Q$ **ii** $x \in Q$ and $x \in R$ **iii** $x \in R$ but $x \notin Q$

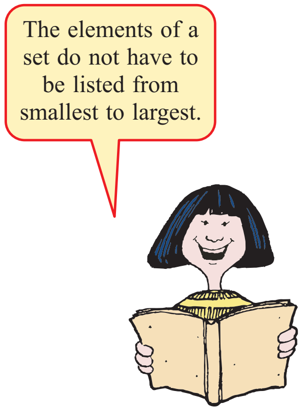
iv $x \in P$ but $x \notin Q$ **v** $x \in P$ and $x \in R$ **vi** $x \in R$ but $x \notin P$.

5 Suppose $A = \{2, 5, 8, 11, 14\}$. Find all values of x for which:

a $x \in A$ and $x + 3 \in A$ **b** $x \in A$ and $x + 6 \in A$

c $x \in A$ and $x - 3 \in A$.

EQUAL SETS

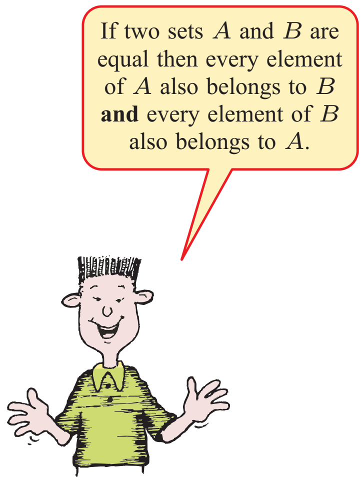


Two sets are **equal** if they have exactly the same elements.

For example,

if $A = \{2, 3, 8\}$ and $B = \{3, 8, 2\}$, then $A = B$.

EXERCISE 23A.2



1 If $A = \{3, 5, 1, 4\}$, $B = \{1, 3, 4, x\}$ and $A = B$, find x.

2 If $P = \{\text{even numbers between 10 and 20}\}$ and $Q = \{\text{multiples of 2 between 11 and 19}\}$, is $P = Q$?

3 If $K = \{1, 3, x, y\}$, $L = \{9, 5, 1, 3\}$ and $K = L$ what can be said about x and y?

4 True or false:

a If $A = B$, then $n(A) = n(B)$.

b If $n(A) = n(B)$, then $A = B$.

SUBSETS

If $M = \{2, 7, 8\}$ and $N = \{1, 2, 3, 5, 7, 8, 11\}$ we notice that every element of M is also an element of N. We say that M is a *subset* of N.

Set A is a **subset** of set B is every element of A is also in B.

If A is a subset of B, we write $A \subseteq B$.

EMPTY SETS

An **empty set** is a set which contains no elements.

The symbols $\{\ \}$ or $\varnothing$ are used to represent an empty set.

For example, the set of all whole numbers between 2 and 3 could be written as $\varnothing$.

Notice that the empty set is always a subset of any given set.

DISCUSSION — SUBSETS

Discuss the truth of these statements:

- The empty set $\varnothing$ is a subset of $\{*, \#\}$.
- Any set is a subset of itself. For example $\{1, 2, 3\} \subseteq \{1, 2, 3\}$.

Example 2 — Self Tutor

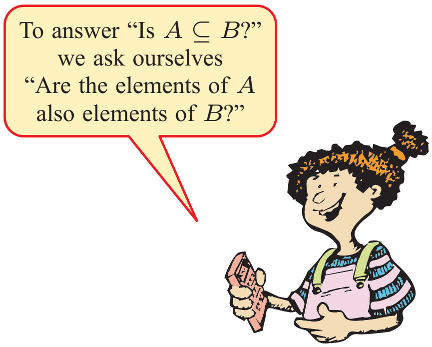
List all the subsets of $\{1, 2, 3\}$.

The subsets of $\{1, 2, 3\}$ are: $\varnothing$, the empty set

$\{1\}, \{2\}, \{3\}$	{the subsets containing one element}
$\{1, 2\}, \{2, 3\}, \{1, 3\}$	{the subsets containing two elements}
$\{1, 2, 3\}$	{any set is a subset of itself}

There are 8 subsets in all.

EXERCISE 23A.3



1 True or false?

- **a** $\{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4, 5, 6\}$
- **b** $\{1, 2, 3, 4\} \subseteq \{4, 3, 2, 1\}$
- **c** $\{1, 2, 3, 4\} \subseteq \{2, 3, 4\}$
- **d** $\{2, 4, 6\} \subseteq$ {even numbers}
- **e** $A \subseteq A$ for any set A

2 If $M = \{3, 4, 5\}$, list the subsets of M which contain exactly:

a one element **b** two elements **c** three elements.

3 If $N = \{1, 2, 3, 4\}$, list the subsets of N which contain exactly:

a two elements **b** three elements.

4 Suppose A and B are two sets. Copy and complete:
$A = B$ if $A \subseteq B$ and B...... .

B THE INTERSECTION OF SETS

In a class of students, $P = \{\text{Adam, Bert, Con, Dina, Eva}\}$ is the set of piano players
and, $V = \{\text{Con, Eva, Mandy, Quenda}\}$ is the set of violin players.

We notice that Con and Eva play both instruments, as they belong to both set P **and** set V.

The set {Con, Eva} is called the *intersection* of sets P and V.

The **intersection** of two sets A and B is the set of all elements which are common to both sets A **and** B.

The intersection of sets A and B is written $A \cap B$.

Example 3 Self Tutor

If $P = \{+, *, \blacktriangleright, \#, @\}$ and $Q = \{-, \times, *, \blacktriangleleft, @, \bullet\}$, list the set $P \cap Q$.

$*$ and @ are in both sets P and Q, so $P \cap Q = \{*, @\}$

Example 4 Self Tutor

If $A = \{\text{factors of } 12\}$ and $B = \{\text{factors of } 18\}$, find $A \cap B$.
What does $A \cap B$ represent?

$A = \{1, 2, 3, 4, 6, 12\}$ and $B = \{1, 2, 3, 6, 9, 18\}$

So, $A \cap B = \{1, 2, 3, 6\}$

This set represents all common factors of 12 and 18.

EXERCISE 23B

1. List the set $A \cap B$ for $A = \{a, e, i, o, u\}$ and $B = \{a, r, e, s, t\}$

2. Let M be the set of all letters used to write the word *apartment* and N be the set of all letters used to write the word *prospector*.
   a. List the sets M and N.
   b. Write down the set $M \cap N$.

3. If $M = \{\text{multiples of 4 less than } 16\}$ and $F = \{\text{factors of } 16\}$:
   a. list the sets M and F
   b. list the set $M \cap F$ and find $n(M \cap F)$.

4. If $P = \{\text{prime numbers less than } 18\}$ and $F = \{\text{factors of } 35\}$:
   a. list the sets P and F
   b. list the set $P \cap F$ and find $n(P \cap F)$.

5 List these intersections:

- **a** $\{1, 3, 5, 7\} \cap \{2, 4, 6, 8\}$
- **b** $\{\text{factors of } 12\} \cap \{\text{factors of } 8\}$
- **c** $\{\text{multiples of 5 that are} < 100\} \cap \{\text{multiples of 15 that are} < 100\}$

C THE UNION OF SETS

The **union of sets** A and B is the set of all elements which are in A **or** B.

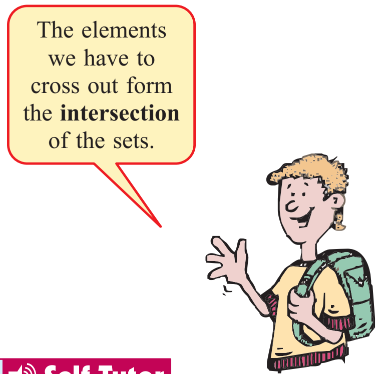
Elements in both A and B **are included** in the union of A and B.

The union of sets A and B is written $A \cup B$.

The union of two sets is made up by combining the elements of the two sets, then removing any elements which have been listed twice.

For example, suppose $A = \{3, 5, 6, 7, 9\}$ and $B = \{1, 2, 3, 4, 5\}$.

To find $A \cup B$, we first list the elements of both A and B, then cross out the elements listed twice:
$\{3, 5, 6, 7, 9, 1, 2, \cancel{3}, 4, \cancel{5}\}$



We can then list the numbers in order:

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9\}$

One advantage of this method is that the numbers we cross out are the elements of the **intersection** of A and B. So, $A \cap B = \{3, 5\}$.

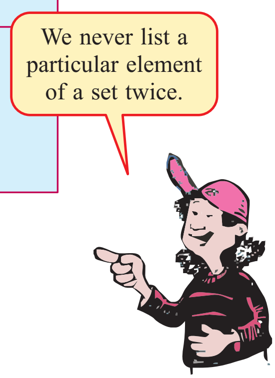
Example 5 **Self Tutor**

If $P = \{1, 3, 5, 7, 9\}$ and $Q = \{2, 4, 5, 6, 7, 8\}$, list the sets:

a $P \cup Q$ **b** $P \cap Q$

a $P \cup Q = \{1, 3, 5, 7, 9, 2, 4, \cancel{5}, 6, \cancel{7}, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

b $P \cap Q = \{5, 7\}$



EXERCISE 23C

1 Find $A \cup B$ for:

- **a** $A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6, 7, 8\}$
- **b** $A = \{a, c, d, f, m\}$ and $B = \{b, c, e, f, g\}$
- **c** $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7, 9\}$
- **d** $A = \{*, \#, !, \times\}$ and $B = \{\#, :, 5, \times, +\}$

2 If $A = \{1, 5, 6, 8\}$ and $B = \{1, 2, 4, 5, 9\}$, find:

a $A \cup B$ **b** $A \cap B$

3 If $P = \{2, 5, 7, 9\}$ and $Q = \{3, 6, 7, 11, 13\}$, find:

a $P \cup Q$ **b** $P \cap Q$.

4 If $R = \{6, 8, 10, 12\}$ and $S = \{5, 7, 9, 11, 13\}$, find:

a $R \cup S$ **b** $R \cap S$.

D VENN DIAGRAMS

In any problem dealing with sets:

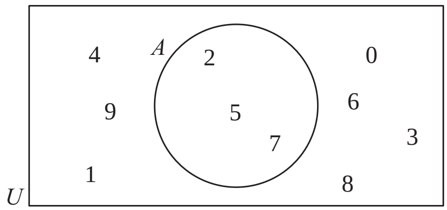
The **universal set** is the set which contains all of the elements we are considering.

For example, consider the set $A = \{2, 5, 7\}$, which is a subset of the set of all single digit numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

In this case the universal set is $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

A **Venn diagram** shows the relationship between sets. The universal set is represented by a rectangle and the other sets are represented by circles within it.

For $A = \{2, 5, 7\}$ and $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the Venn diagram is:

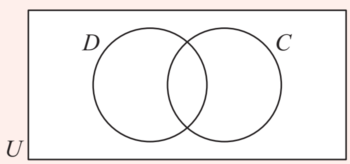


ACTIVITY VENN DIAGRAMS

Click on the icon to load a demonstration of a Venn diagram. 20 people are asked whether they own a cat or a dog or both. The information is sorted onto a **Venn diagram** which consists of two overlapping circles within a rectangle.

DEMO

Circle D represents the people who own a dog and circle C represents the people who own a cat. The circles overlap because some people own both a cat *and* a dog.



What to do:

1. Start the demonstration. Press continue to see how each person's response is added to the diagram.
2. Identify which region of the Venn diagram represents people who own:
   - a dog
   - a cat
   - a dog and a cat
   - neither a dog nor a cat
   - a dog but not a cat
   - a cat but not a dog
   - a dog or a cat

Example 6 **Self Tutor**

Consider the single digit numbers 0, 1, 2, 3, 4, ... up to 9.

Let $P = \{\text{prime numbers less than } 9\}$ and $F = \{\text{factors of } 6\}$.

a State the universal set.

b List the elements of P and F.

c Find **i** $P \cap F$ **ii** $P \cup F$

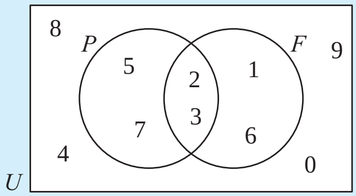
d Illustrate the sets on a Venn diagram.

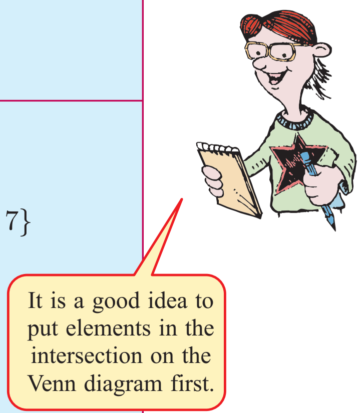
a $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

b $P = \{2, 3, 5, 7\}$ and $F = \{1, 2, 3, 6\}$

c **i** $P \cap F = \{2, 3\}$ **ii** $P \cup F = \{1, 2, 3, 5, 6, 7\}$

d

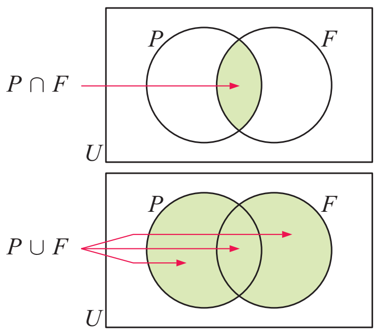




From **Example 6** we can see that the **intersection** of two sets is represented by the intersection or overlap of the two circles.

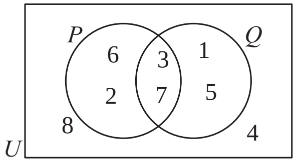
The **union** of the two sets is shown in the second Venn diagram alongside.

An element in the union could be in one set or the other or in both sets. It must lie within *at least one* of the circles.



EXERCISE 23D

1. Consider the sets $U = \{0, 1, 2, 3, ..., 9\}$, $A = \{3, 5, 6, 7, 9\}$, and $B = \{1, 4, 6, 8, 9\}$.
   a Find: **i** $A \cap B$ **ii** $A \cup B$
   b Illustrate the sets on a Venn diagram.

2 

From the Venn diagram shown, list the sets:

a P **b** Q **c** U

d $P \cap Q$ **e** $P \cup Q$.

3 $U = \{1, 2, 3, 4, 5, 6, ..., 17\}$, $M = \{$multiples of 3 which are less than 16$\}$, and $F = \{$factors of 15$\}$.

a List the elements of M and F.

b Find: **i** $M \cap F$ **ii** $M \cup F$ **iii** $n(M \cap F)$ **iv** $n(M \cup F)$

c Illustrate the sets on a Venn diagram.

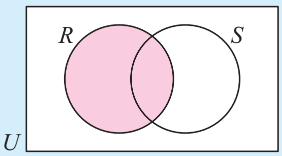
4 Suppose $U = \{a, b, c, d, e, f, g, h, i, j, k\}$, $A = \{b, c, e, g, i, k\}$ and $B = \{a, b, d, e, f, g, k\}$.

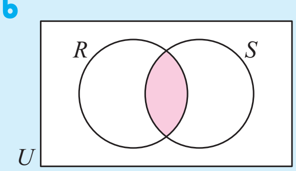
a List the sets: **i** $A \cap B$ **ii** $A \cup B$

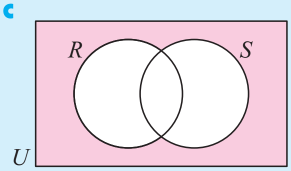
b Illustrate the sets on a Venn diagram.

Example 7 **Self Tutor**

Describe in words the shaded region:

a 

b 

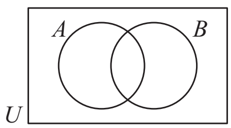
c 

a The elements in set R.

b The elements in both set R and set S.

c The elements in neither set R nor set S.

5 On separate Venn diagrams like the one illustrated, shade the region which indicates that you are:



a in U **b** in A

c in B **d** not in A

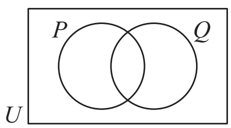
e not in B **f** in both A and B

g in A but not B **h** in B but not A

i in either A or B.

PRINTABLE PAGES OF VENN DIAGRAMS

6 On separate Venn diagrams, shade the regions representing those members:

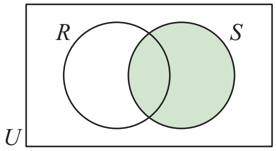


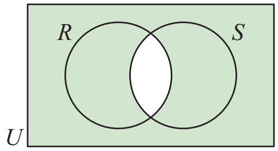
a in Q **b** not in P

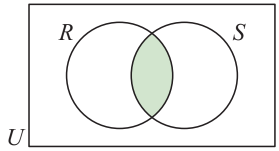
c in P but not Q **d** in both P and Q

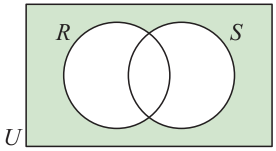
e neither in P nor in Q **f** in P or in Q.

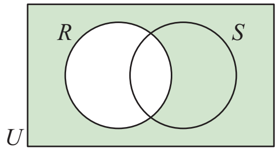
7 Describe in words the shaded region:

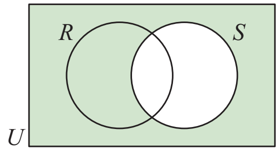
a


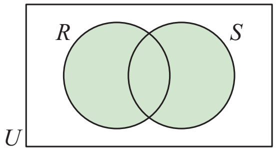
b


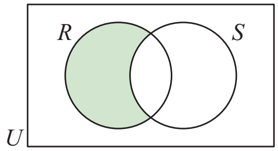
c


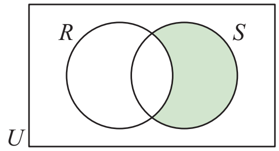
d


e


f


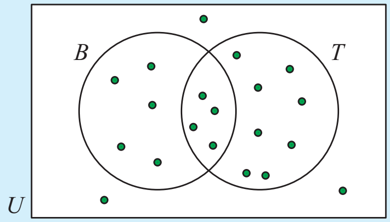
g


h


i


Example 8 **Self Tutor**

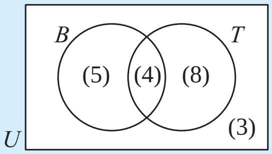
The Venn diagram shows the sports played by the students in a class. Each dot represents one person. B represents the students who play basketball and T represents the students who play tennis.



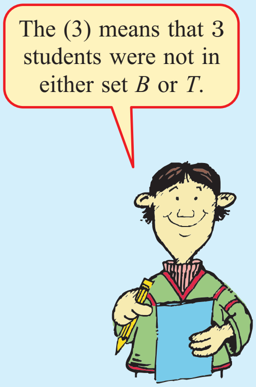
a How many students are there in the class?

b How many students play:

i basketball
ii tennis
iii basketball and tennis
iv neither basketball nor tennis
v basketball but not tennis
vi basketball or tennis?



The numbers in brackets are counts for each region.



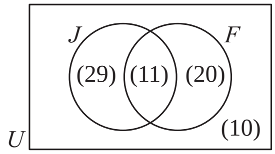
a There are $5 + 4 + 8 + 3 = 20$ students in the class.

b **i** There are $5 + 4 = 9$ students who play basketball.
ii There are $4 + 8 = 12$ students who play tennis.
iii There are 4 students who play basketball and tennis.
iv There are 3 students who play neither sport.
v 5 students play basketball but not tennis.
vi $5 + 4 + 8 = 17$ play basketball or tennis.

8 Suppose $U = \{$all letters of the English alphabet$\}$, $A = \{a, d, m, n, p, x, z\}$ and $B = \{b, c, d, f, h, n, q, y, z\}$.

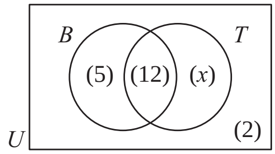
- **a** Display A, B and U on a Venn diagram.
- **b** Shade the region represented by A in blue.
- **c** Shade the region represented by B in yellow.
- **d** What are the elements of $A \cap B$?
- **e** In words, what is represented by the unshaded region?

9 The Venn diagram represents the people at a conference.
J represents those who understand Japanese.
F represents those who understand French.



- **a** How many people are at the conference?
- **b** How many people at the conference understand:
  - **i** Japanese
  - **ii** French
  - **iii** both Japanese and French
  - **iv** Japanese but not French
  - **v** neither Japanese nor French
  - **vi** Japanese or French?

10 A youth club in Xi'an has 32 members. The Venn diagram alongside shows those who play badminton (B) and tabletennis (T).



- **a** Find x.
- **b** How many of the club's members play:
  - **i** badminton
  - **ii** table tennis
  - **iii** badminton but not table tennis
  - **iv** both of the sports
  - **v** neither of the sports?

KEY WORDS USED IN THIS CHAPTER

- element
- empty set
- equal sets
- intersection
- member
- set
- subset
- union
- universal set
- Venn diagram

REVIEW SET 23A

1 Using set notation, list the elements of the set of all:

- **a** multiples of 8 which are less than 50
- **b** prime numbers between 25 and 40.

2 Let $A = \{$even numbers between 25 and 35$\}$.

- **a** List the elements of A.
- **b** Find $n(A)$.

3 Suppose $A = \{2, 4, 6, 11\}$ and $B = \{3, 6, 9, 15\}$.
Find the possible values of x for which:

- **a** $x \in A$ and $x \in B$
- **b** $x \in A$ and $x + 3 \in B$
- **c** $x \in B$ and $x - 2 \in A$

4 Suppose $C = \{1, 6, 7, x, 8\}$ and $D = \{7, y, 8, 1, 4\}$. If $C = D$, find x and y.

5 True or false?

a {multiples of 4 less than 30} $\subseteq$ {positive even numbers less than 30}

b {prime numbers} $\subseteq$ {odd numbers}

6 Suppose $U = \{1, 2, 3, ... , 20\}$, $E =$ {positive even numbers less than 20} and $F =$ {factors of 20}.

a List the elements of the sets: **i** E **ii** F **iii** $E \cap F$ **iv** $E \cup F$

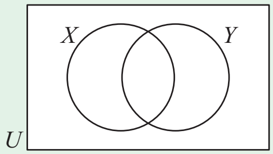
b Illustrate the sets on a Venn diagram.

7 Suppose $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 3, 6, 8, 9\}$ and $B = \{1, 3, 5, 7, 8\}$.

a Illustrate A, B and U on a Venn diagram.

b Find: **i** $A \cap B$ **ii** $A \cup B$ **iii** $n(A \cap B)$ **iv** $n(A \cup B)$.

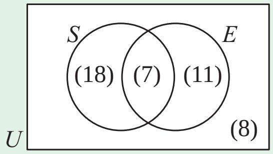
8 On separate Venn diagrams, shade the regions representing:



a in X **b** not in Y

c in both X and Y **d** in neither X nor Y.

9 The Venn diagram represents the people attending a convention on climate control.
S represents all the scientists and E represents all the environmentalists.



a How many people are at the convention?

b How many people at the convention are:

i scientists **ii** environmentalists

iii both scientists and environmentalists

iv neither scientists not environmentalists

v environmentalists but not scientists?

REVIEW SET 23B

1 Using set notation, list the elements of the set of all:

a square numbers between 10 and 60 **b** factors of 18.

2 Which of the following are true?

a $23 \in$ {prime numbers less than 20} **b** $36 \notin$ {square numbers}

c $63 \in$ {multiples of 7}

3 Let $M =$ {multiples of 3 less than 40} and $N =$ {multiples of 4 less than 40}.

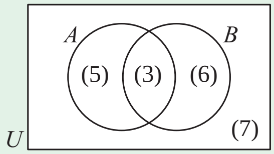
a List the sets M and N. **b** Find $M \cap N$.

c Copy and complete: {multiples of 3} $\cap$ {multiples of 4} $=$ {multiples of ......}

4 If $K = \{1, 5, 7, 9\}$, list all the subsets of K which contain:

a two elements **b** three elements.

5 For the Venn diagram shown, find:



a $n(A)$ **b** $n(B)$

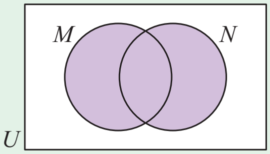
c $n(A \cap B)$ **d** $n(A \cup B)$

e $n(U)$

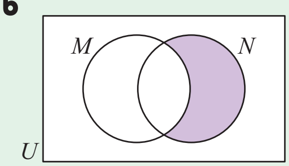
6 Consider the positive integers 1, 2, 3, ..., 12. Let $F = \{\text{factors of } 12\}$ and $P = \{\text{prime numbers less than } 12\}$. Represent these two sets on a Venn diagram.

7 What is represented by these shaded regions?

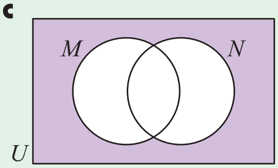
a



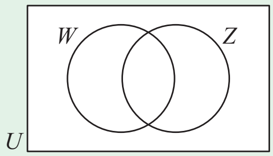
b



c



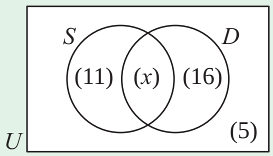
8 On separate Venn diagrams, shade the regions represented by:



a in Z **b** in W and in Z

c in W or in Z

9 A tennis club with 40 members holds a tournament. Some members are entered to play singles and some to play doubles.



a Find the value of x.

b How many members are entered to play:

i singles **ii** doubles

iii both singles and doubles **iv** singles but not doubles

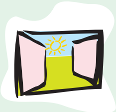
v at least one of the events **vi** exactly one of the events?

Chapter 24

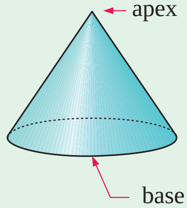
Solids and polyhedra

Contents:

OPENING PROBLEM

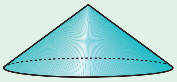
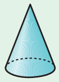


Alongside is a sketch of a **solid cone**. It has a circular base and a **curved surface** up to a point called its **apex**.



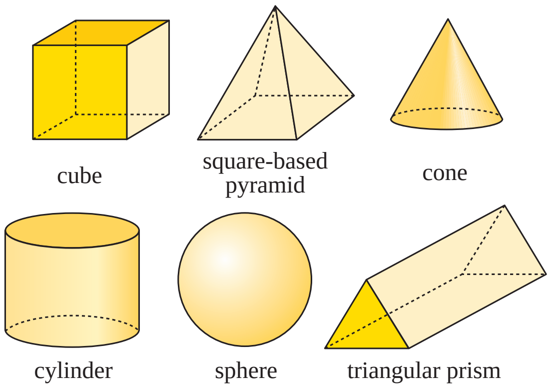
Things to think about:

- How can we make a cone from a piece of timber?
- How can we make a model of a cone from a sheet of paper?
- If we make a model from paper, how do we make the cone be flat like

 or pointy like  ?

A **solid** is a body which occupies space.

These diagrams show some special solids. You should learn their names and be able to draw neat freehand sketches of them.



The dashed lines show edges which are hidden at the back of the solid. The dashed lines remind us these edges are there, even if we cannot normally see them.

The boundaries of a solid are called **surfaces**. These surfaces may be flat surfaces, curved surfaces, or a mixture of both.

Which of the above solids have only flat surfaces, only curved surfaces or a combination of both types?

DEMO

Click on the icon to obtain models of the solids above. Rotate them to help you appreciate their 3-dimensional nature.

DID YOU KNOW?

Did you know that bronze statues are generally hollow inside? This is mainly because bronze is very expensive.

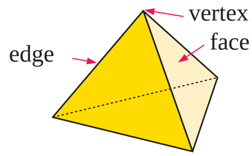
A TYPES OF SOLIDS

POLYHEDRA

A **polyhedron** is a solid which contains all flat surfaces.
The plural of polyhedron is **polyhedra**.

Cubes and pyramids are examples of polyhedra. Spheres and cylinders are not.

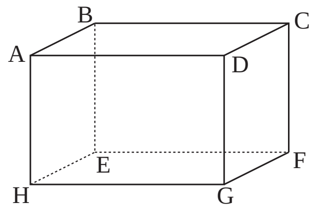
Each flat surface of a polyhedron is called a **face** and has the shape of a polygon.
Each corner point of a polyhedron is called a **vertex**.
Each intersection of two faces is called an **edge**.



The solid opposite is a triangular-based pyramid, often called a **tetrahedron**.

Labelling a figure helps describe its features.
For example:

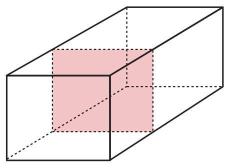
A, B, C, D, E, F, G and H are all vertices of this polyhedron.
ABCD is one face. There are five other faces.
[AB] is one edge. There are eleven other edges.



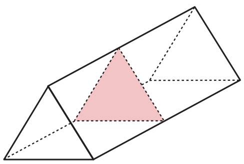
PRISMS

A **prism** is a polyhedron with a uniform cross-section.

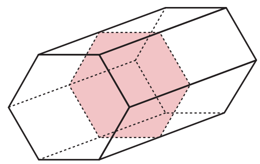
Examples of prisms:



rectangular prism

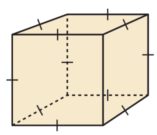


triangular prism



hexagonal prism

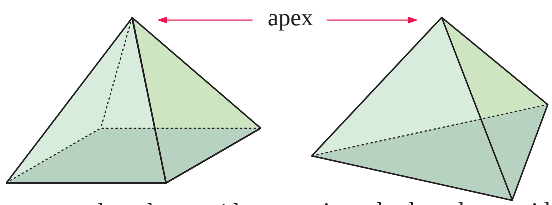
A **cube** is a rectangular prism with 6 square faces.
All of its edges are the same length.



PYRAMIDS

A **pyramid** is a solid with a polygon for a base, and triangular faces which come from the base to meet at a point called the **apex**.

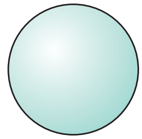
Examples of pyramids:



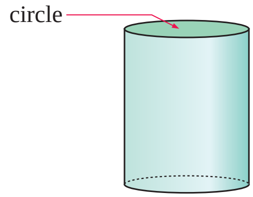
square-based pyramid triangular-based pyramid

SOLIDS WITH CURVED SURFACES

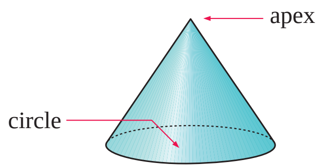
A **sphere** is a perfectly round ball.



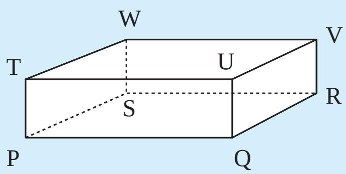
A **cylinder** is a solid with a uniform circular cross-section.



A **cone** has a circular base and a curved surface which rises up to a point called its **apex**.



Example 1 — Self Tutor



a Classify the given solid.
b Name all of its vertices.
c List all of its edges.
d What face is closest to us?

a A rectangular prism {all faces are rectangles}

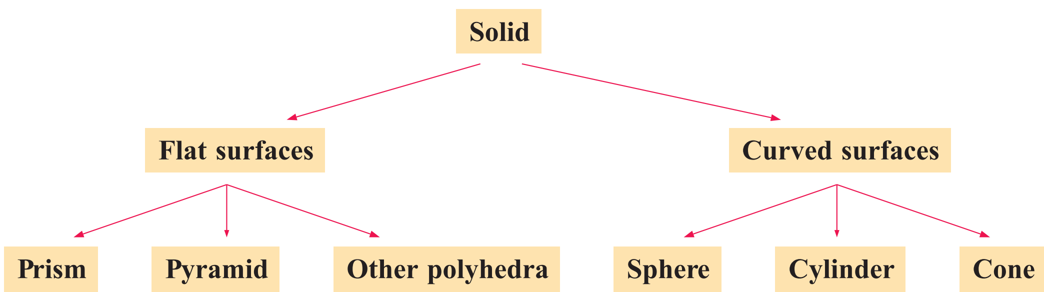
b P, Q, R, S, T, U, V and W are its vertices.

c Its edges are: [PQ], [QR], [RS] and [SP] {on the base}
[PT], [QU], [RV] and [SW] {verticals}
[TU], [UV], [VW] and [TW] {on the top}.

d The face PQUT is closest to us.

CLASSIFYING SOLIDS

The following flowchart gives us a way of classifying solids:



EXERCISE 24A

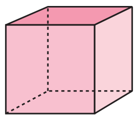
1 Draw a neat diagram to represent a:

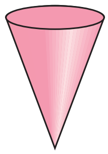
a cube **b** cone **c** cylinder
d sphere **e** rectangular prism **f** triangular-based pyramid

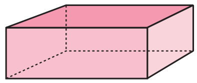
2 Name the shape which best resembles:

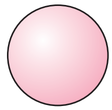
a a basketball **b** the top part of a funnel **c** a tennis ball container
d a six-faced die **e** a cereal box **f** a broom handle

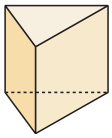
3 Classify these solids:

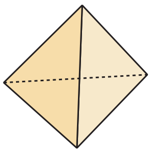
a


b


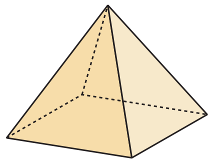
c


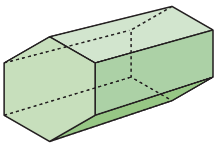
d


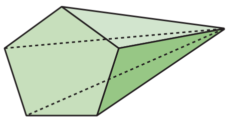
e


f


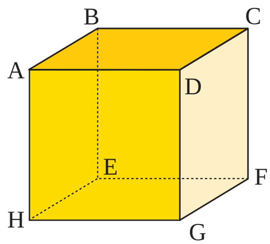
g


h


i


j


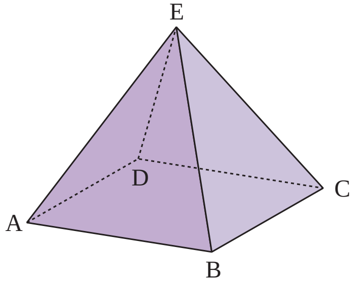
4 **a** Name all the vertices of this cube.
b Name all of its faces.
c Name all of its edges.



5 What shapes are the side faces of:

a a prism **b** a pyramid?

6



For the given pyramid, name and count the:

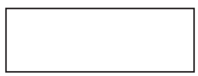
a faces **b** vertices
c edges.

B FREEHAND DRAWINGS OF SOLIDS

Making freehand sketches of special solids is not easy. Following are step by step instructions on how to do them accurately.

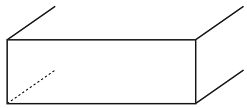
RECTANGULAR PRISM

Step 1:



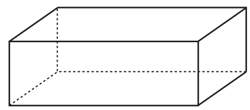
Draw a rectangle for the **front face**.

Step 2:



From each of the vertices draw lines back to create the edges. Their lengths are drawn slightly shorter than their real length to give perspective.

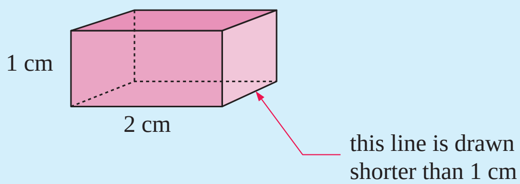
Step 3:

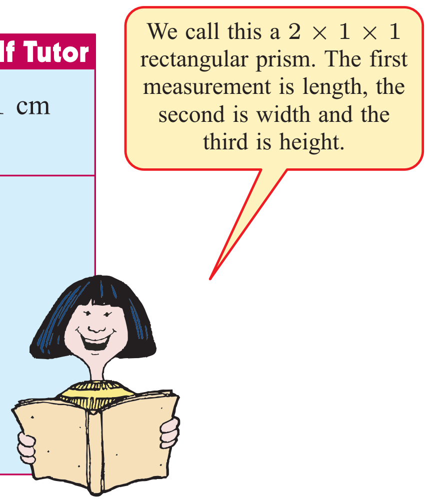


Complete the drawing by joining the appropriate vertices. Use dotted lines for the hidden edges.

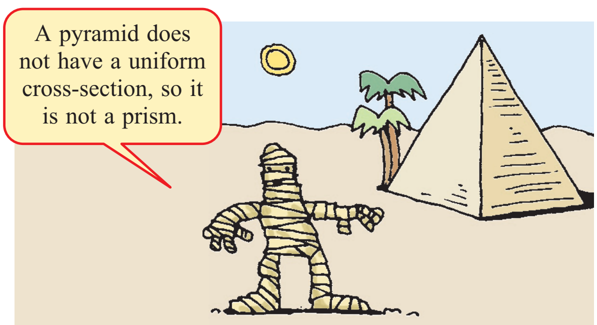
Example 2 Self Tutor

Draw a regular prism 2 cm long by 1 cm wide by 1 cm high.





PYRAMIDS



In the picture of the pyramid alongside, only five edges, four vertices and two faces can be seen.

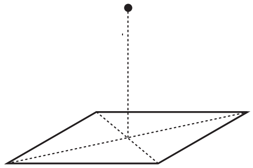
In fact, this pyramid has a square base and four triangular faces.

To draw a pyramid we use the following steps:

Step 1:

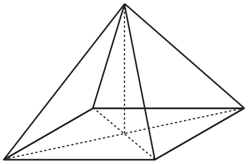
Draw a parallelogram to represent the base.

Step 2:



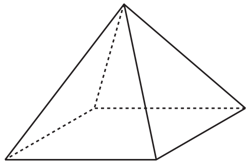
To find the centre of the parallelogram, draw its diagonals and find their point of intersection. Draw a point above the centre to represent the **apex** of the pyramid.

Step 3:



Join each vertex of the base to the apex to complete the pyramid.

Step 4:



Looking at the picture of the pyramid above, not all edges can be seen at the one time. We show hidden edges as dotted lines.

CYLINDERS

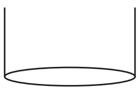
You are probably familiar with cylinders such as tin cans. You would be aware that their top and bottom is a circle, but when we look at it on an angle it will *appear* as an **ellipse** or oval.

To draw a cylinder we use the following steps.

Step 1:

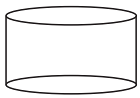
Draw an ellipse to represent the base.

Step 2:



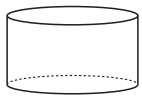
Draw the sides of the cylinder from the "ends" of the ellipse.

Step 3:



Complete the cylinder by drawing another ellipse on the top.

Step 4:



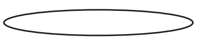
We use a dashed curve to show the part of the base that is hidden.

CONES

Just like a cylinder, we represent the circular base of a cone using an ellipse.

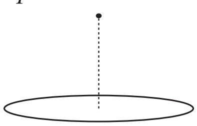
To draw a cone we use the following steps:

Step 1:



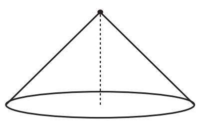
Draw an ellipse to represent the base.

Step 2:



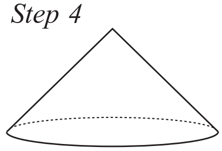
Mark a point directly above the centre of the ellipse. This will be the apex of the cone.

Step 3:



Join the "ends" of the ellipse to the apex to complete the cone.

Step 4



We use a dashed curve to show the part of the base that is hidden.

ACTIVITY 1 VIEWING SOLIDS

In this activity we revisit the demonstration of 3-dimensional solids.

What to do:

1 Click on the icon to run the software.

2 Rotate each of the objects so you can view them from:
   a directly above **b** directly below **c** directly alongside.

In each case, sketch your results.

EXERCISE 24B

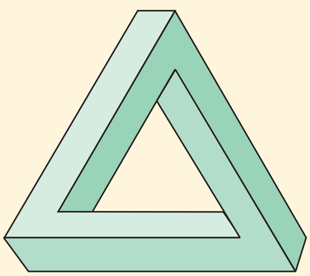
1 Draw a rectangular prism that is:
   a 1 cm × 1 cm × 2 cm **b** 1 cm × 2 cm × 1 cm **c** 2 cm × 2 cm × 1 cm

2 Draw freehand sketches of:
   a a square-based pyramid
   b a tetrahedron
   c a hexagonal prism
   d a triangular-based prism
   e a hexagonal-based pyramid.

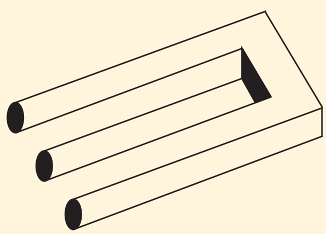
3 Draw freehand sketches of:
   a a cylinder which is 3 cm high and has a base that is 2 cm wide
   b a cone which is 4 cm high and has a base that is 3 cm wide.

4 Sketch a sphere. Use shading to show how it curves.

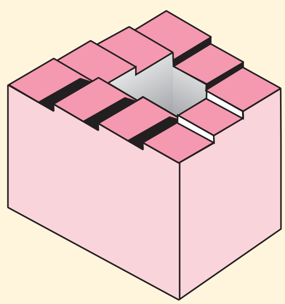
VISUAL ILLUSIONS



Trigon



Eat from this fork!

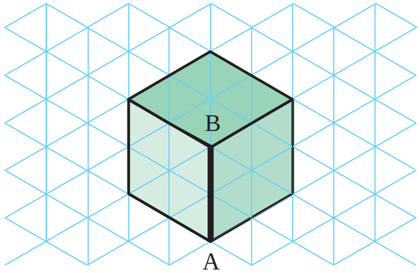


Ever-ascending steps

C ISOMETRIC PROJECTIONS

When drawing a rectangular object, we can also use an **isometric projection**. This uses special graph paper made up of equilateral triangles.

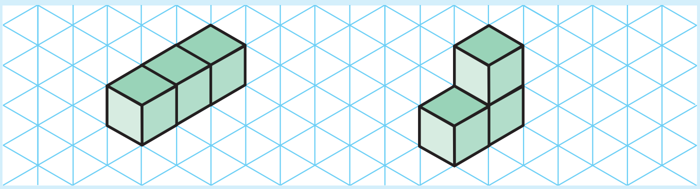
The diagram alongside shows the isometric projection of a cube. The edge [AB] appears closest to us, and this is often the **starting edge** of the figure, or the first edge drawn.



Example 3 — Self Tutor

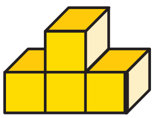
On isometric graph paper, draw the only two different shapes which can be made from three cubes of the same size and which have at least one face in full contact with one of the other cubes.

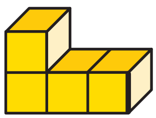
PRINTABLE ISOMETRIC PAPER

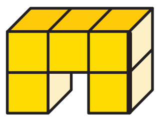


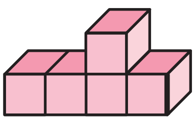
EXERCISE 24C

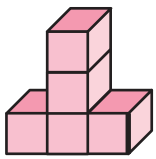
1 Redraw the following figures on isometric graph paper:

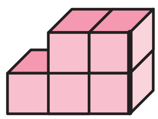
a


b


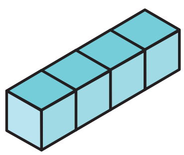
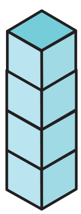
c


d


e


f


2 On isometric paper draw all possible different shapes which can be made from four cubes of the same size and which have at least one face in full contact with one of the other cubes.

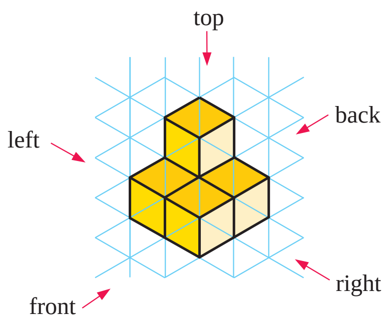
Note that  and  are the same.

D CONSTRUCTING BLOCK SOLIDS

When an architect draws plans of a building, separate drawings are made from several viewing directions.

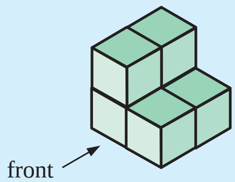
Given a drawing on isometric graph paper, there are 5 directions we consider:

The top view is also called the **plan**. We use numbers on the plan to indicate the height of each pile.



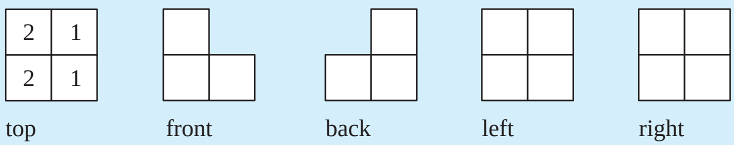
Example 4 Self Tutor

Draw top, front, back, left and right views of:



BLOCK BUSTER

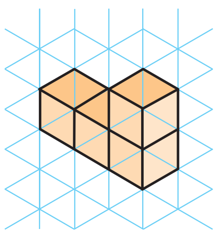
The views are:



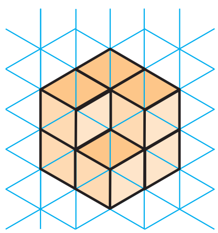
EXERCISE 24D

1 Draw top, front, back, left and right views of:

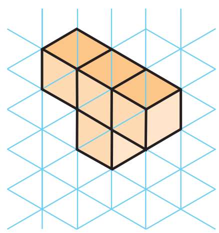
a



b

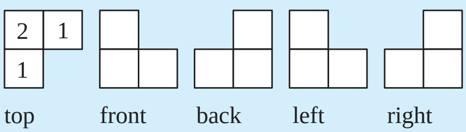


c

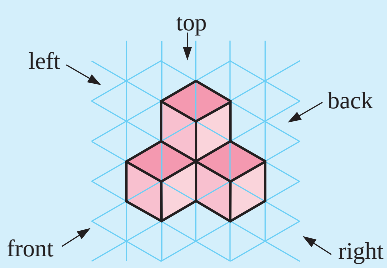


Example 5 Self Tutor

The given diagrams show different views of the same shape:

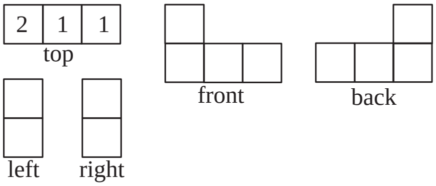


Draw the object on isometric paper.

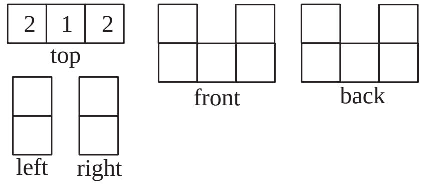


2 Draw the 3-dimensional object whose views are:

a



b

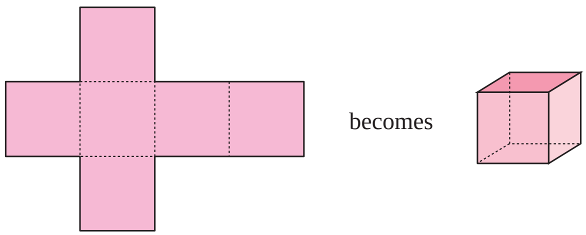


3 Draw four different objects made from five cubes whose view from the top is .
They must be free standing and not glued together.

E NETS OF SOLIDS

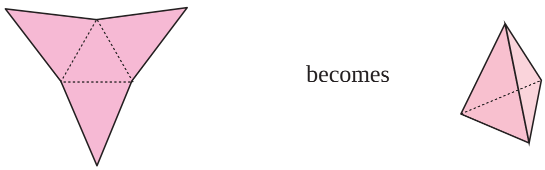
A **net** is a two-dimensional shape which may be folded to form a solid.

For example, a **cube** is formed when the "net" shown is cut out and folded along the dotted lines.

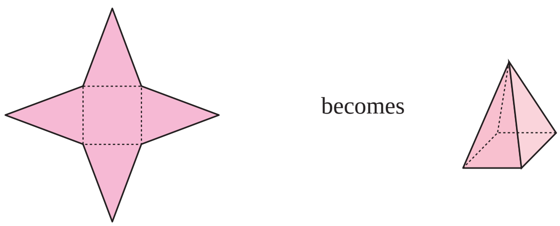


DEMO

A **triangular-based pyramid** is formed when this "net" is cut out and folded along the dotted lines.

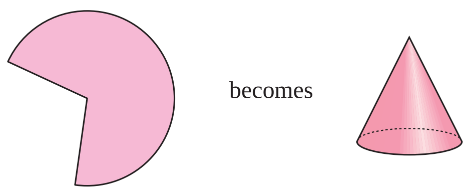


A **square-based pyramid** is formed when this "net" is cut out and folded along the dotted lines.



All the examples above are polyhedra and have **flat surfaces**. However, some solids with **curved surfaces** can also be made from nets.

For example, the curved surface of a **cone** is formed from a "net" which is part of a circle. A separate circle can then be cut out to be the base of the cone.

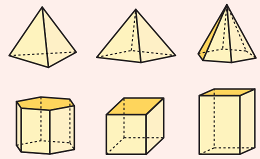


ACTIVITY 2 MAKING MODELS OF SOLIDS

Click on the solid for which you want a printable net. If possible print it on light card rather than ordinary paper.

PRINTABLE TEMPLATE

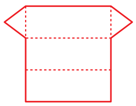
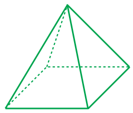
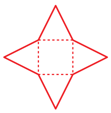
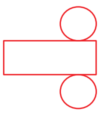
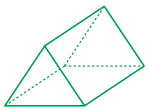
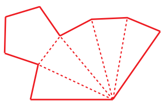
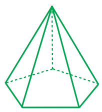
What to do:



1. Construct the solids from the nets provided.
2. Make a mobile from the solids to hang in your classroom.

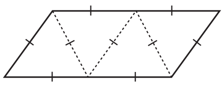
EXERCISE 24E

1. Match the net given in the first column with the correct solid and the correct name:

Net	Solid	Name
a 	**A** 	**(1)** Pentagonal-based pyramid
b 	**B** 	**(2)** Triangular prism
c 	**C** 	**(3)** Square-based pyramid
d 	**D** 	**(4)** Cylinder

2. Click on the icon to obtain nets for the solids in **1**. They have extra tabs to help you glue the solid together.

PRINTABLE NETS

3. Is  a possible net for a triangular-based pyramid?

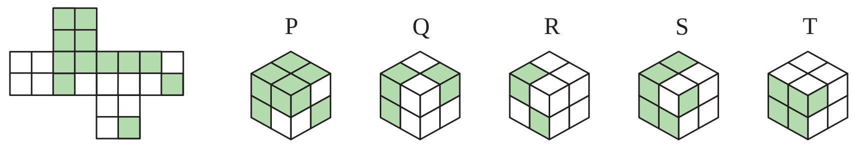
ACTIVITY 3 WHICH CUBE IS IT?

What to do:

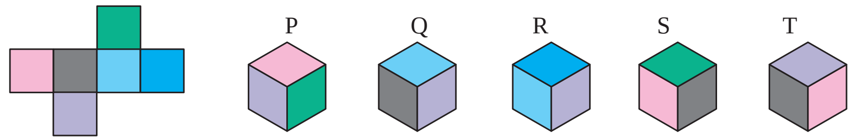
For each of the cases following:

1. Carefully study the nets and the sets of cubes given.
2. Determine which cube can be made from the net and write down your answer.
3. Construct an actual net showing the **exact** same patterns on the faces. Make the cube and hence check your answer to **2**.

Case 1:



Case 2:



KEY WORDS USED IN THIS CHAPTER

- apex
- cylinder
- isometric projection
- prism
- sphere
- triangular prism
- cone
- edge
- net
- pyramid
- square-based pyramid
- vertex
- cube
- face
- polyhedron
- solid
- tetrahedron

LINKS
click here

PLATONIC SOLIDS

Areas of interaction:
Human ingenuity, Approaches to learning

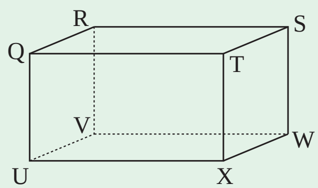
REVIEW SET 24A

1 Draw the following solids:

a a cube **b** a cone.

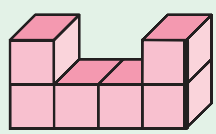
2 For the rectangular prism shown, name all of the:

a vertices **b** faces **c** edges.

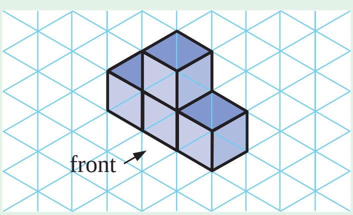


3 Draw a freehand sketch of a cylinder which is 45 mm high and has a base 12 mm wide.

4 Draw the following object as an isometric projection:



5



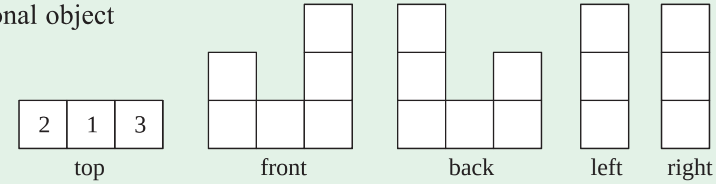
For the solid shown, draw views from the:

a top **b** left

c right **d** front

e back.

6 Draw the 3-dimensional object whose views are:



7 Draw a net for:

a a rectangular prism **b** a cube **c** a triangular-based pyramid

8 Draw and name the solid corresponding to the net:

REVIEW SET 24B

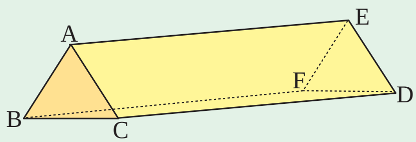
1 Draw the following solids:

a a triangular prism **b** a cylinder

2 For the given figure, name:

a all vertices **b** all edges

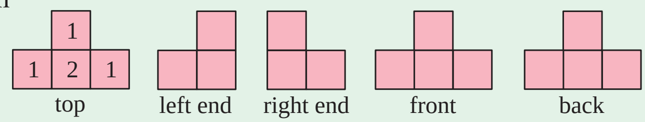
c all faces.



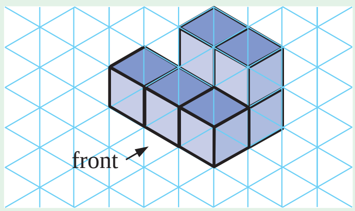
3 Draw a freehand sketch of:

a a cube **b** a 2 cm × 1 cm × 2 cm rectangular prism.

4 Draw an isometric diagram of the solid with views:



5 Draw top, front, back, left and right views of:

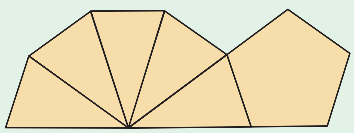


6 On isometric graph paper, draw 5 of the possible arrangements of 4 identical blocks where every block is in full contact with at least one full face of another block.

7 Draw a net for:

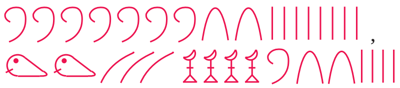
a a cone **b** a square-based pyramid.

8 Draw and name the solid which corresponds to the following net:



ANSWERS

EXERCISE 1A.1

1 **a** 27 **b** 

2 **a** 31 427 **b** 21 123 423

EXERCISE 1A.2

1 **a** 13 **b** 26 **c** 204 **d** 3240 **e** 723 **f** 5259

2 **a**  **b**  **c** 
d  **e**  **f** 

EXERCISE 1A.3

1 **a** 8 **b** 14 **c** 16 **d** 31 **e** 110 **f** 81 **g** 125
h 216 **i** 62 **j** 1156 **k** 550 605 **l** 720 **m** 419
n 555 501 **o** 2 300 000

2 **a** XVIII **b** XXXIV **c** CCLXXIX
d CMII **e** MXLVI **f** MMDLI

3 **a** 88 = LXXXVIII **b** 1500 = MD **c** MCMXCIX

4 **a** DCCVIII swords **b** MCCXCIV denarii

EXERCISE 1A.4

1 **a**  **b**  **c** 
d **e**  **f** 

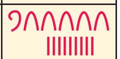
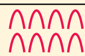
2 **a** 14 **b** 120 **c** 218 **d** 168 **e** 313 **f** 380

EXERCISE 1A.5

1 **a** 765 **b** 3248 **c** 9999

2 **a** 四百九十七 **b** 八千四百 **c** 一千一百一十一

3

	words	Hind.-Arab.	Roman	Egypt.
a	thirty seven	37	XXXVII	
b	one hundred and four	104	CIV	
c	one hundred and fifty nine	159	CLIX	
d	eighty	80	LXXX	

	words	Mayan	Chinese Japanese
a	thirty seven		三十七
b	one hundred and four		一百四
c	one hundred and fifty nine		一百五十九
d	eighty		八十

EXERCISE 1B

1 **a** 8 **b** 80 **c** 8 **d** 800 **e** 80 **f** 8000 **g** 800
h 8000 **i** 8 **j** 80 000 **k** 8000 **l** 80 000

2 **a** 3 thousands, 5 ten thousands, 8 tens
b 3 thousands, 5 hundreds, 8 tens
c 3 units, 5 ten thousands, 8 tens
d 3 hundreds, 5 thousands, 8 hundred thousands

3 **a** 864 **b** 974 210 **c** 997 722
d 345, 354, 435, 453, 534, 543 (6 numbers)

4 **a** 8, 16, 19, 54, 57, 75 **b** 6, 60, 600, 606, 660
c 1008, 1080, 1800, 1808, 1880
d 40 561, 45 061, 46 051, 46 501, 46 510
e 207 653, 227 635, 236 705, 265 703
f 545 922, 554 922, 594 522, 595 242

5 **a** 631, 613, 361, 316, 163, 136
b 9877, 9787, 8977, 8779, 7987, 7897, 7789
c 498 321, 498 231, 492 813, 428 931, 428 391
d 675 034, 673 540, 607 543, 576 304, 563 074

6 **a** 86 **b** 674 **c** 9638 **d** 50 240 **e** 27 003
f 73 298 **g** 500 375 **h** 809 302

7 **a** $9 \times 100 + 7 \times 10 + 5 \times 1$ **b** $6 \times 100 + 8 \times 10$
c $3 \times 1000 + 8 \times 100 + 7 \times 10 + 4 \times 1$
d $9 \times 1000 + 8 \times 10 + 3 \times 1$
e $5 \times 10\,000 + 6 \times 1000 + 7 \times 100 + 4 \times 10 + 2 \times 1$
f $7 \times 10\,000 + 5 \times 1000 + 7 \times 1$
g $6 \times 100\,000 + 8 \times 100 + 2 \times 10 + 9 \times 1$
h $3 \times 100\,000 + 5 \times 10\,000 + 4 \times 1000 + 7 \times 100 + 1 \times 10 + 8 \times 1$

8 **a** 27 **b** 80 **c** 608 **d** 1016 **e** 8200 **f** 19 538
g 75 403 **h** 602 818

9 **a** 7 **b** 13 **c** 21 **d** 299 **e** 4007 **f** 9997
g 400 004 **h** 209 026

10 **a** 1000 times larger **b** 1000 times smaller
c The 4 which is left of the second 7. 100 times larger.

EXERCISE 1C

1 **a** 80 **b** 50 000 000 **c** 600 **d** 400 000 **e** 70 000
f 2

2 **a** 3 000 000, 600 000, 40 000, 8000, 500, 90, 7
b 30 000 000, 4 000 000, 800 000, 60 000, 5000, 200, 70, 1

3 **a** 37 000 000 **b** 200 000 000, 17 000 000
c 150 000 000 **d** $111 240 463.10
e 21 240 657 **f** 415 000 000 **g** 1 048 576 bytes

4 Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto

5 **a** **i** Asia **ii** Africa, Asia, North America
b Antarctica, Australia

REVIEW SET 1A

1 **a** 165 **b** 2634

2 **a**  **b** 

3 **a** 18 **b** 79 **4** MMXII **5** **a**  **b** 

6 **a** 476 **b** 359

7 **a** 400 **b** 40 000

8 **a** 800 **b** 800 000 **9** 87 410

10 79 562, 96 572, 569 207, 652 097, 795 602 **11** 2497

12 17 304 **13** 4 000 000, 500 000, 30 000, 2000, 600, 80, 1

14 9 984 700 square kilometres

REVIEW SET 1B

1 **a**  **b** 

2 **a** 253 **b** 122 305 **3** CLXXXVIII

4 **a** 三百八十六 **b** 三千一百一十三

5 **a** 70 **b** 7000

6 885 500

7 6 080 699, 968 099, 698 096, 680 969, 608 699

8 **a** $2 \times 1000 + 1 \times 100 + 5 \times 10 + 9 \times 1$
b $3 \times 100\,000 + 6 \times 1000 + 4 \times 100 + 2 \times 10 + 8 \times 1$

9 **a** 23 **b** 991

10 30 000, 7 000 000, 400 000, 5000, 900, 20, 2

11 5 890 000 km **12** **a** 2 billion **b** 2000

13 **a** 1000 times **b** 10 times **c** the first 6; 10 times

EXERCISE 2A.1

1 **a** 807 **b** 1330 **c** 3995 **d** 1644 **e** 1597 **f** 13 059

2 **a** 79 **b** 107 **c** 748 **d** 696 **e** 2155 **f** 6565 **g** 814 **h** 4955 **i** 4619

3 **a** 82 **b** 44 **c** 109 **d** 453 **e** 665 **f** 3656

4 **a** 34 **b** 48 **c** 6 **d** 22 **e** 182 **f** 476 **g** 376 **h** 3767

EXERCISE 2A.2

1 22 m **2** $432 **3** 6 kg **4** €41 **5** 22
6 3923 km **7** 1178 cm

EXERCISE 2B.1

1 **a** 500 **b** 5000 **c** 50 000 **d** 6900 **e** 69 000 **f** 690 000 **g** 12 300 **h** 246 000 **i** 96 000 **j** 490 000 **k** 49 000 **l** 490 000

2 **a** 120 **b** 148 **c** 496 **d** 1272 **e** 405 **f** 2744 **g** 14 580 **h** 23 112 **i** 5754 **j** 45 026 **k** 10 413 **l** 26 864

EXERCISE 2B.2

1 **a** 200 **b** 20 **c** 2 **d** 5700 **e** 570 **f** 57 **g** 24 300 **h** 2430 **i** 243 **j** 4500 **k** 450 **l** 45 **m** 72 000 **n** 7200 **o** 720 **p** 600 000 **q** 60 000 **r** 6000

2 **a** 14 **b** 54 **c** 21 **d** 75 **e** 901 **f** 619

3 **a** 6 **b** 25 **c** 52 **d** 48 **e** 208 **f** 817

EXERCISE 2B.3

1 1000 hours **2** 10 000 hours **3** 90 **4** $80 **5** 82 min
6 2450 mm **7** 672 km **8** 81 min **9** 20 000 hours
10 20 of them **11** 240 bags

12 **a** 20 000 **b** 4000 **c** 2500 **d** 500 **e** 10 000 **f** 12 500 **g** 8000 **h** 25 000

EXERCISE 2B.4

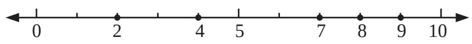
1 **a** $375 + 836 \approx 1200$ **b** $79 \times 8 \approx 640$
c $978 - 463 = 515$ **d** $7980 \div 20 \approx 400$
e $455 + 544 = 999$ **f** $50 \times 400 = 20\,000$
g $2000 - 1010 = 990$ **h** $3000 \div 300 = 10$

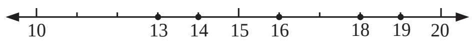
2 **a** $5268 - 3179 < 4169$ **b** $29 \times 30 < 900$
c $672 + 762 < 1444$ **d** $720 \div 80 > 8$
e $20 \times 80 > 160$ **f** $700 \times 80 > 54\,000$
g $5649 + 7205 > 12\,844$ **h** $6060 - 606 > 5444$

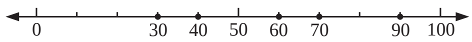
EXERCISE 2C

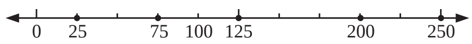
1 $26 **2** RM 201 **3** 11 oranges **4** 54 **5** £11
6 €30 **7** $1.44 **8** $1860 **9** €743
10 No, he was €10 short. **11** $550 **12** 60 km **13** 600 g

EXERCISE 2D

1 **a** 

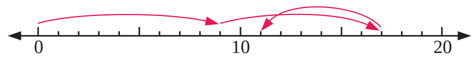
b 

c 

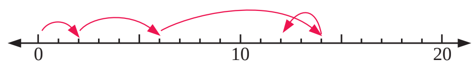
d 

e 

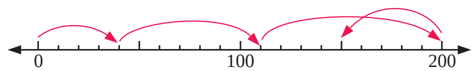
2 **a** $3 + 6 + 9 = 18$ **b** $11 + 9 - 13 = 7$
c $20 + 30 - 10 - 10 - 10 - 10 - 10 = 0$
d $250 + 400 - 350 = 300$ **e** $70 - 40 + 20 = 50$
f $17 - 4 - 4 - 4 - 5 + 4 = 4$

3 **a** 

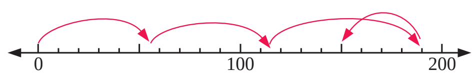
$9 + 8 - 6 = 11$

b 

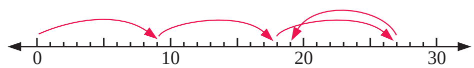
$2 + 4 + 8 - 2 = 12$

c 

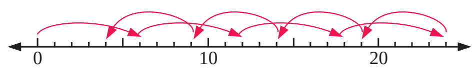
$40 + 70 + 90 - 50 = 150$

d 

$55 + 60 + 75 - 40 = 150$

e 

$3 \times 9 - 8 = 19$

f 

$4 \times 6 \div 5 = 4$ remainder 4

EXERCISE 2E.1

1 **a** 20 **b** 50 **c** 70 **d** 80 **e** 90 **f** 200 **g** 460 **h** 790 **i** 1730 **j** 2800 **k** 3950 **l** 6980

2 **a** 40 **b** 70 **c** 90 or 100 **d** 130 **e** 460 **f** 730 or 740 **g** 820 **h** 1220 or 1230 **i** 6740

3 **a** 20 **b** 40 **c** 50 **d** 70 **e** 100 **f** 210 **g** 310 **h** 500 **i** 890 **j** 3660 **k** 7440 **l** 8710 **m** 9610 **n** 14 080 **o** 30120 **p** 47 780 **q** 69 570 **r** 70 100

4 **a** 100 **b** 300 **c** 800 **d** 1700 **e** 3000 or 3100 **f** 6200

5 **a** 500 **b** 7600 **c** 3000

6 **a** 100 **b** 200 **c** 600 **d** 800 **e** 1100 **f** 2700 **g** 7000 **h** 13 200 **i** 27 700 **j** 38 500 **k** 55 400 **l** 85 100

7 **a** 1000 **b** 0 **c** 1000 **d** 5000 **e** 8000 **f** 7000 **g** 10 000 **h** 9000 **i** 13 000 **j** 8000 **k** 246 000 **l** 500 000

8 **a** 20 000 **b** 50 000 **c** 50 000 **d** 80 000
e 90 000 **f** 50 000 **g** 90 000 **h** 100 000

9 **a** 200 000 **b** 300 000 **c** 700 000 **d** 700 000
e 100 000 **f** 500 000 **g** 300 000 **h** 100 000

10 **a** 40 musicians **b** 60 singers **c** €580 **d** \$4100
e 700 kg **f** \$25 000 **g** 35 600 km **h** 40 000 km
i £460 000 **j** 1 500 000 people

EXERCISE 2E.2

1 **a** 80 **b** 300 **c** 400 **d** 400 **e** 1000 **f** 3000
g 7000 **h** 60 000

2 **a** 780 **b** 270 **c** 750 **d** 340 **e** 1600 **f** 6600
g 8800 **h** 35 000

3 **a** €50 000 **b** 8200 km **c** £500
d \$308 000 **e** 6700 people **f** 33 000 people

EXERCISE 2F.1

1 **a** 270 **b** 540 **c** 360 **d** 240 **e** 700 **f** 42 000
g 64 000 **h** 270 000

2 **a** \$240 **b** \$640 **c** \$1400 **d** \$36 000

3 **a** 2000 **b** 2400 **c** 4900 **d** 6400 **e** 4000
f 8000 **g** 8000 **h** 40 000

4 **a** \$1000 **b** €2400 **c** £3500 **d** \$5400

5 **a** 60 000 **b** 200 000 **c** 300 000 **d** 900 000
e 24 000 **f** 180 000 **g** 200 000 **h** 32 000 000

6 **a** \$80 000 **b** €140 000 **c** \$240 000 **d** £240 000

7 **a** 100 **b** 300 **c** 20 000 **d** 200 **e** 10 **f** 200
g 30 **h** 200 **i** 60 **j** 150 **k** 25 **l** 2

8 **a** €25 000 **b** 100 min **9** **a** **B** **b** **C** **c** **A** **d** **B**

10 **a** 2400 words **b** **i** 2800 trees **ii** 840 000 apples
c 30 barrels **d** 10 hours
e **i** 5400 seats **ii** \$162 000

EXERCISE 2F.2

1 **a** 432 **b** 768

REVIEW SET 2A

1 **a** 224 **b** 272 **2** \$29 **3** No, €1 extra is needed.
4 **a** 3400 **b** 59 **5** **a** 522 **b** 128

6 **a** True **b** False **c** True **d** False

7 **a** $60 \times 1000 > 59\,000$ **b** $499\,994 > 499\,949$ **8** £2688
9 €475 each **10** \$728 **11** **a** 16 000 000 **b** 5 times larger
12 166 days 16 hours **13** **a** 40 **b** 4000 **c** 500 000
14 £600 **15** 40 000 **16** 2 kg

REVIEW SET 2B

1 **a** 448 **b** 2843 **2** €192 **3** 1200

4 **a** 53 200 **b** 46

5 **a** False **b** False **c** True **d** True

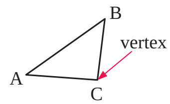
6 **a** $237 + 384 \approx 620$ **b** $8020 \div 20 \approx 400$

7 **a** 897 **b** 51 **8** 10 sections **9** \$184.80
10 20 000 fills **11** **a** £39 800 **b** 60 000
12 **a** \$80 **b** 16 000 **13** \$360 **14** 140 000
15 About 3 times larger.

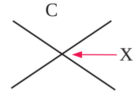
EXERCISE 3A

1 **a** (1) a speck of dust
(2) corner where two walls and the floor meet
b (1) where two walls meet
(2) bottom of blackboard

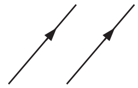
2 **a** A vertex is a corner point of figure A, B and C are all vertices.



b A point of intersection is the point where two lines meet.



c Two lines which are always the same distance apart.

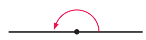


3 **a** (LM) or (ML) **b** (CD), (CE), (DE), (DC), (EC), (ED)
4 **a** B **b** C **5** **a** B **b** [AB]
6 **a** Q **b** [QR] **c** [QR]

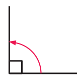
EXERCISE 3B

1 **a** 60^o **b** 75^o **c** 128^o **d** 103^o **e** 27^o
f 23^o **g** 135^o **h** 155^o **i** 87^o **j** 96^o

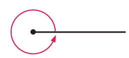
2 **a**



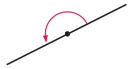
b



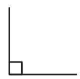
c



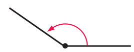
3 **a**



b

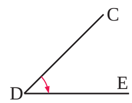


c

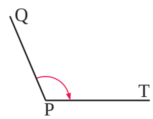


4 **a** $\widehat{ABC}$, acute angle **b** $\widehat{PQR}$, obtuse angle
c $\widehat{KLM}$, reflex angle

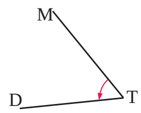
5 **a**



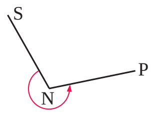
b



c



d



6 **a** $B\widehat{A}C = 83^o$, $A\widehat{C}B = 31^o$, $A\widehat{B}C = 66^o$
b $F\widehat{D}E = 119^o$, $D\widehat{E}F = 32^o$, $D\widehat{F}E = 29^o$
c $A\widehat{B}C = 94^o$, $B\widehat{C}D = 78^o$, $C\widehat{D}A = 78^o$, $D\widehat{A}B = 110^o$
d $P\widehat{Q}R = 54^o$, $Q\widehat{R}S = 127^o$, $R\widehat{S}T = 127^o$, $S\widehat{T}P = 91^o$, $T\widehat{P}Q = 141^o$

7 Your answer might look like this:

	Estimate	*Actual*
a	50^o	53^o
b	90^o	92^o
c	20^o	17^o

9 $A\widehat{B}C$ is larger as its degree measure is larger.

EXERCISE 3C

1 **a** $x = 25$ **b** $y = 45$ **c** $a = 30$ **d** $n = 107$
e $p = 50$ **f** $x = 60$ **g** $x = 90$ **h** $b = 45$
i $m = 36$

2 **a** $a = 270$ **b** $b = 120$ **c** $c = 318$ **d** $d = 89$
e $e = 120$ **f** $f = 81$ **g** $x = 90$ **h** $y = 135$
i $x = 148$ **j** $g = 112$ **k** $h = 95$ **l** $i = 67$

EXERCISE 3D

1 **a** $a=85$ **b** $b=25$ **c** $c=46$ **d** $d=53$ **e** $e=37$
f $f=46$ **g** $g=23$ **h** $h=117$ **i** $x=60$
j $x=70$ **k** $y=45$ **l** $x=30,\ y=42$

2 **a** $p+q+r=180$ **b** $a+b=90$ **c** $a+b=140$
d $r+s=90$ **e** $m+n=75$ **f** $p+m=90$

EXERCISE 3E

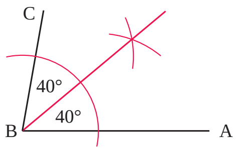
1 **a** $x=110$ **b** $x=90$ **c** $x=40$ **d** $x=80$
e $x=90$ **f** $x=90$

2 **a** $a+b+c+d=360$ **b** $m+n=170$ **c** $x+y=180$

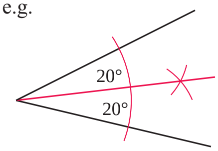
3 **a** 180^o, 180^o, 360^o
b The angle sum at the vertices is exactly 360^o.
c This argument 'works' for **all** quadrilaterals regardless of their size and shape. No measurement is needed and no guessing.

EXERCISE 3F

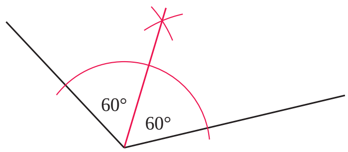
1 a/b



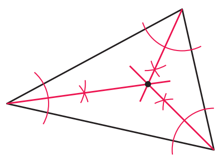
2 a/b e.g.



3 a/b e.g.



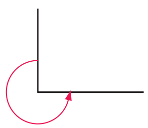
4 a/b (Reduced)



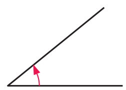
c All three angle bisectors meet at the same point.

REVIEW SET 3A

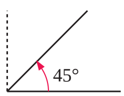
1 **a** A **b** (BC) **2** **a**



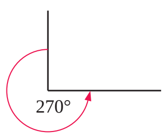
b



3 **a**



b



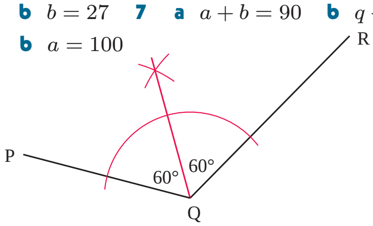
4 **a** 90^o **b** 180^o

5 **a** $x=112^o$ {angles at a point}
b $a=145^o$ {angles at a point}

6 **a** $a=60$ **b** $b=27$ **7** **a** $a+b=90$ **b** $q+r=72$

8 **a** $x=95$ **b** $a=100$

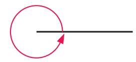
9



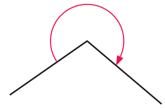
REVIEW SET 3B

1 **a** [BC] **b** [BC]

2 **a**



b



3 **a** $A\widehat{B}C$ (or $C\widehat{B}A$), acute **b** $D\widehat{E}F$ (or $F\widehat{E}D$), reflex

4 **a** estimate 15^o, actual 15^o **b** estimate 30^o, actual 32^o

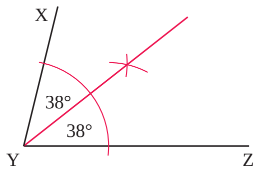
5 **a** $a=90$ {angles on a line} **b** $a=35$ {angles on a line}

6 **a** $x=33$ {angles of a triangle}
b $x=16$ {angles of a triangle}

7 **a** $a=94$ **b** $c=112$

8 **a** $a+b+c=260$ **b** $x+y=190$

9



EXERCISE 4A

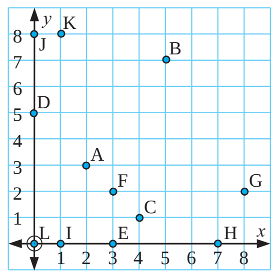
1 **a** **i** A5 **ii** D5 **iii** A2
b **i** Gloucester Park Trotting Ground
ii East Perth Cemetery **iii** Queen's Gardens
iv Police Traffic Branch **v** Police Headquarters

2 **a** **i** G5 **ii** A6 **iii** I2
b **i** Eiffel Tower **ii** Au Jardin du Luxembourg
iii Centre National D Art Et Culture Georges Pompidou

3 **a** **i** C2 **ii** C4 **iii** C3 **iv** E1 **v** B4
b **i** Buckingham Palace **ii** Royal Courts of Justice
iii Covent Garden **iv** Piccadilly Circus
v Waterloo Station

EXERCISE 4B

1



2 **a** 0
b 0

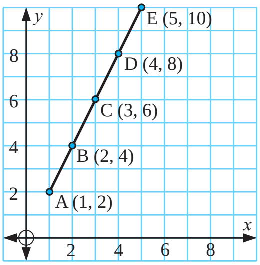
3 **a** 5, 1, 6, 0 **b** 3, 8, 7, 0
a A(5, 4), B(8, 5), C(1, 3), D(2, 7), E(0, 5), F(6, 0), G(4, 8), H(0, 7), I(2, 0)
b O(0, 0)

4 **a** C, D. Lie on same vertical line.
b B, E. Lie on same horizontal line. **c** D(4, 4)

5 D(5, 2)

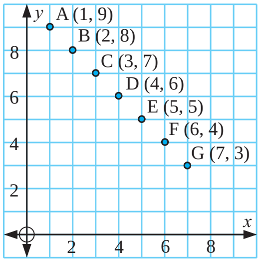
6 **a** **i** (7, 5) **ii** (9, 2) **iii** (7, 7) **iv** (2, 1) and (6, 6)
b **i** Treasure Trove **ii** Lion's Den
iii Mt Ogre **iv** Oasis

7



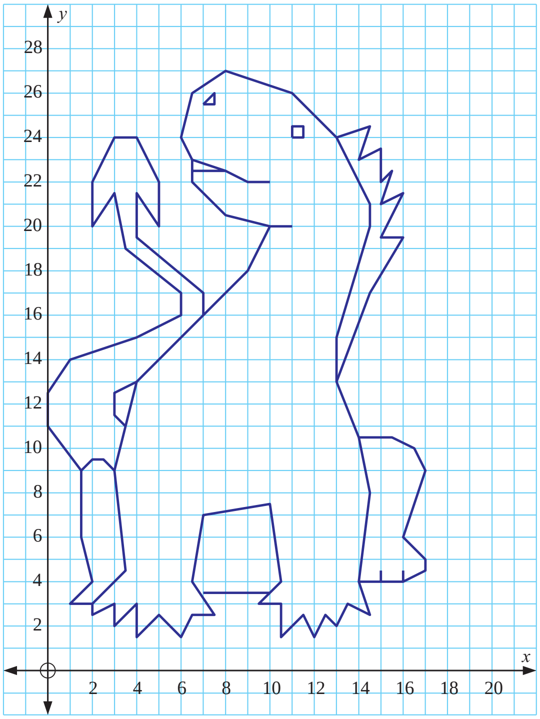
a lie on a straight line
b E(5, 10)

8

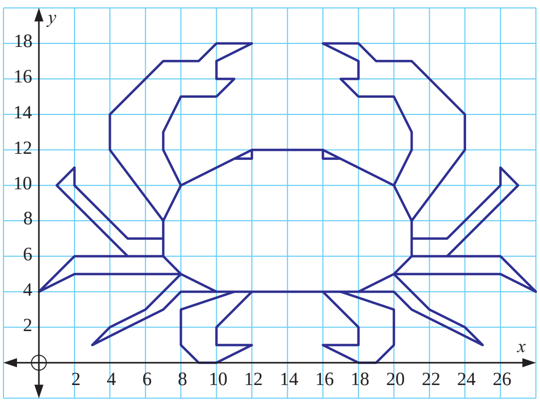


(5, 5), (6, 4), (7, 3)

9



10

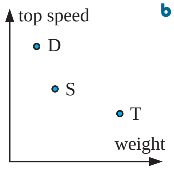


EXERCISE 4C

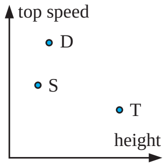
1 **a** **i** weight, age **ii** age **iii** weight
b **i** Christopher **ii** Christopher **iii** Dragan **iv** Emilio
c **i** False **ii** True **iii** True **iv** False

2 **a** Sarah **b** Voula **c** 8 years, size 7
d 16 years, size 9 **e** 12 years, size 6

3 **a**



b



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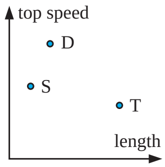
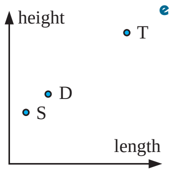
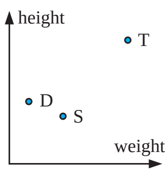


d



e



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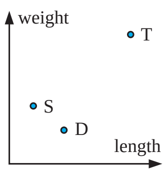
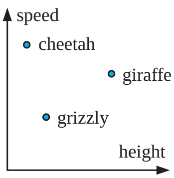
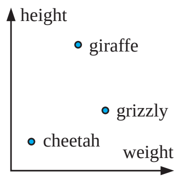


4 **a** P cheetah, Q giraffe, R grizzly bear, X giraffe, Y cheetah, Z grizzly bear
b giraffe
c



d



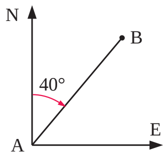
5 **a** 200 km/h, 5 hrs **b** 500 km/h, 2 hrs
c 1000 km/h, 1 hr **d** increase speed, decrease flight time

6 **a** Californian Redwood **b** Red Gum
c diameter 1 m, height 32 m **d** 70 m **e** 10 m **f** 7 m

EXERCISE 4D

1 **a** N70°E **b** S30°W **c** N50°W **d** S60°W
e S65°E **f** N75°E

2 **a**



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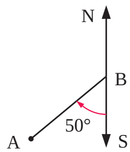

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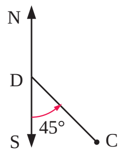

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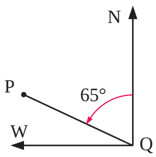

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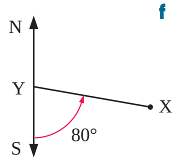
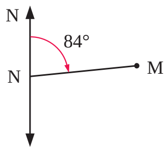
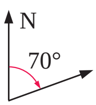


f



3 **a** 070° **b** 210° **c** 310° **d** 240° **e** 115°
f 075°

4 **a**



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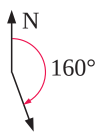
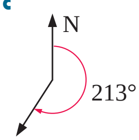


c



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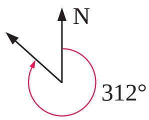

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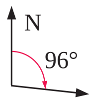


5 **a** West **b** True North

6 **a** 000° **b** 045° **c** 090° **d** 135° **e** 180°
f 225° **g** 270° **h** 315°

7 **a** **i** H9 **ii** G5 **iii** K2 **iv** D2
b **i** 068° **ii** 127° **iii** 307° **iv** 349°

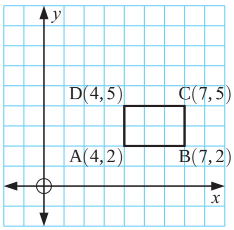
8 **a** N **b** SE **c** **i** 041° **ii** 333° **iii** 096°

REVIEW SET 4A

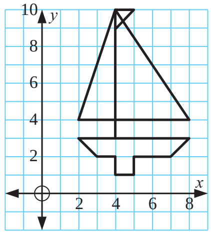
1 **a** **i** Kiwi Park **ii** Medical Centre **b** **i** E3 **ii** F11
c C7

2 **a** (1, 1) **b** (2, 5) **c** (5, 2) **d** (4, 4) **e** (3, 0)

3



4



5 **a** Claire **b** Anna **c** 130 cm, size 7

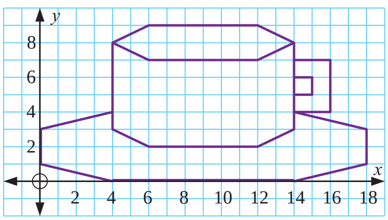
6 **a** point X **b** (11, 1) **c** **i** 243° **ii** 299°

REVIEW SET 4B

1 **a** **i** Museum **ii** Art School **b** **i** E6 **ii** B2

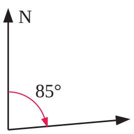
2 **a** (6, 3) **b** (0, 2) **c** (5, 0) **d** (2, 6) **e** (3, 2)

3

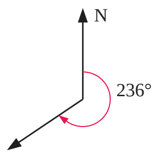


4 **a** Z **b** Y **c** 15 million **d** 300 000 km^2 **e** X

5 **a**



b



6 **a** **i** Q(8, 1) **ii** R(2, 4) **iii** S(6, 9)
b **i** 039° **ii** 346° **iii** 297°

EXERCISE 5A

1 **a** 19 **b** 32 **c** 41 **d** 98

2 **a** 4 **b** 9 **c** 19 **d** 157

3 **a** 23 **b** 19 **c** 17 **d** 20

4 **a** 16 **b** 29 **c** 20 **d** 138 **e** 85 **f** 257
g 144 **h** 90

5 **a** 29 **b** 55 **6** €66 500 **7** 125 kg **8** 926 kg

9 **a** £2586 **b** £1691

EXERCISE 5B

1 **a** 54 **b** 143 **c** 70 **d** 264 **e** 120

2 **a** 4 **b** 4 **c** 11 **d** 15

3 **a** 130 **b** 1900 **c** 2100 **d** 19 000 **e** 420
f 97 000 **g** 5400 **h** 6000

4 **a** 221 **b** 20 **c** 42

5 **a** 6 **b** 60 **c** 600 **d** 6000 **e** 35 **f** 350
g 3500 **h** 35 000 **i** 33 **j** 330 **k** 3300
l 330 000

6 **a** 3 **b** 30 **c** 300 **d** 30 **e** 5 **f** 50
g 500 **h** 5 **i** 3 **j** 30 **k** 3 **l** 300

7 $1400 **8** 27 **9** 12 000 trees

10 **a** 210 rooms **b** €31 500 **c** €441 000

11 **a** 6000 m **b** 8400 m **c** 14 000 m

12 **a** 159 mowers
b $31 025 from Standard, $22 310 from Advanced, $18 200 from Deluxe
c $71 535 **d** $34 955 profit

EXERCISE 5C

1 **a** 7 **b** 7 **c** 0 **d** undefined **e** 18 **f** 7
g undefined **h** 0 **i** 15 **j** 23

2 **a** 73 **b** 0 **c** undefined **d** 0 **e** undefined
f 0 **g** 3 **h** 125 **i** 0 **j** 45 **k** 0 **l** 0
m 0 **n** 0 **o** 235

EXERCISE 5D

1 **a** 7^2 **b** 8^3 **c** 7^4 **d** $2^2 \times 5^2$ **e** $3^3 \times 11$ **f** $4^2 \times 5^3$
g $2 + 3^3$ **h** $2^2 + 3^2$ **i** $7^2 + 2^3$ **j** $2^3 - 2$ **k** $3^2 - 2^3$
l $5 + 2^3 + 7^2$

2 **a** 10^2 **b** 10^3 **c** 10^4 **d** 10^6 **e** 10^9 **f** 10^{12}

3 **a** 16 **b** 625 **c** 2401 **d** 5000 **e** 633
f 11 **g** 44 **h** 32

4 **a** 3^2 **b** equal **c** 2^5 **d** 3^7

EXERCISE 5E

1 **a** 14 **b** 10 **c** 10 **d** 8 **e** 4 **f** 11 **g** 6
h 1 **i** 18 **j** 7 **k** 2 **l** 3 **m** 42 **n** 20
o 28 **p** 2 **q** 6 **r** 12 **s** 6 **t** 9 **u** 5

2 **a** 10 **b** 26 **c** 36 **d** 55 **e** 4 **f** 38 **g** 0
h 21 **i** 11 **j** 31 **k** 7 **l** 41 **m** 15 **n** 5
o 19 **p** 21

3 **a** 2 **b** 4 **c** 1 **d** 30 **e** 0 **f** 32 **g** 8
h 17 **i** 26

4 **a** 29 **b** 49 **c** 18 **d** 36 **e** 1 **f** 10 **g** 86
h 45

5 **a** $4 + 18 \div 3 = 10$ **b** $6 \times 7 - 12 = 30$
c $(17 + 3) \div 5 = 4$ **d** $(18 - 2) \div 8 = 2$
e $12 \div 4 + 10 \times 2 = 23$ **f** $12 \times 4 - 10 \div 2 = 43$

EXERCISE 5F

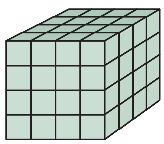
1 **a** 862 953 **b** 3 050 709 **c** 20 369 068 **d** 1 011 190
e 9836 **f** 890 637 **g** 50 875 000

2 **a** $(9 \times 1000) + (7 \times 100) + (3 \times 10) + (8 \times 1)$
b $(2 \times 10\,000) + (9 \times 1000) + (7 \times 100) + (8 \times 10) + (2 \times 1)$
c $(4 \times 10\,000) + (4 \times 100) + (4 \times 1)$
d $(6 \times 100\,000) + (5 \times 10\,000) + (7 \times 1000) + (9 \times 100) + (3 \times 10) + (1 \times 1)$
e $(8 \times 100\,000) + (8 \times 100) + (8 \times 10) + (8 \times 1)$
f $(1 \times 1\,000\,000) + (2 \times 100\,000) + (4 \times 10\,000) + (7 \times 1000) + (9 \times 10) + (1 \times 1)$
g $(4 \times 10\,000\,000) + (9 \times 1\,000\,000) + (7 \times 100\,000) + (5 \times 10\,000) + (5 \times 1000) + (4 \times 100)$
h $(6 \times 1\,000\,000) + (7 \times 100\,000) + (7 \times 10\,000) + (7 \times 1000) + (7 \times 100) + (7 \times 10) + (7 \times 1)$

3 **a** $(6\times10^2)+(5\times10^1)+(8\times1)$
b $(3\times10^3)+(8\times10^2)+(7\times10^1)+(4\times1)$
c $(9\times10^4)+(5\times10^3)+(6\times10^2)+(3\times10^1)+(6\times1)$
d $(1\times10^5)+(1\times10^2)$
e $(5\times10^5)+(5\times10^3)+(7\times10^2)+(5\times10^1)$
f $(1\times10^6)+(2\times10^5)+(7\times10^4)+(4\times10^3)$ $+(9\times10^2)+(4\times10^1)+(7\times1)$
g $(3\times10^7)+(6\times10^6)+(6\times10^5)$
h $(4\times10^6)+(2\times10^5)+(9\times10^4)+(3\times10^3)$ $+(3\times10^2)+(7\times10^1)+(5\times1)$
i $(4\times10^5)+(6\times10^2)+(8\times10^1)+(7\times1)$
j $(2\times10^7)+(3\times10^6)+(6\times10^5)+(9\times10^4)$ $+(7\times10^3)+(5\times10^2)$

EXERCISE 5G

1 **a** 16 **b** 25 **c** 49 **d** 100 **e** 20 **f** 36 **g** 21 **h** 9
2 **a** 0, 1, 4, 5, 6, 9 **b** no
3 **a** 1 **b** 4 **c** 6 **d** 9 **e** 12
4 **a** 7 **b** 8 **c** 10 **d** 0 **e** 20
5 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
6 **a** 4 **b** 120 **c** 72 **d** 294
7 **a**



4^3 blocks

b Each block is
n units long $\times$ n units wide $\times$ n units high
$= n\times n\times n$
$= n^3$

8 **a** 3 **b** 4 **c** 5 **d** 10

EXERCISE 5H

1 **a** **i** 1, 2 **ii** 1, 3 **iii** 1, 2, 4 **iv** 1, 5 **v** 1, 7 **vi** 1, 2, 4, 8 **vii** 1, 3, 9 **viii** 1, 2, 5, 10 **ix** 1, 11 **x** 1, 13 **xi** 1, 2, 7, 14 **xii** 1, 3, 5, 15 **xiii** 1, 2, 4, 8, 16 **xiv** 1, 17 **xv** 1, 2, 3, 6, 9, 18 **xvi** 1, 19 **xvii** 1, 2, 4, 5, 10, 20 **xviii** 1, 3, 7, 21
b 2, 3, 5, 7, 11, 13, 17, 19
c **i** 6, 8, 10, 14, 15, 21 **ii** 12, 16, 18, 20
2 **a** 1, 23 **b** 1, 2, 3, 4, 6, 8, 12, 24
c 1, 2, 4, 5, 10, 20, 25, 50, 100
d 1, 3, 5, 9, 15, 45 **e** 1, 2, 4, 8, 16, 32, 64
f 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
3 **a** 8, 10, 12 **b** 17, 19, 21, 23, 25
4 **a** 2 and 8 **b** 1 and 19, 3 and 17, 5 and 15, 7 and 13
c 2, 4, 14 and 2, 6, 12 and 2, 8, 10 and 4, 6, 10
5 **a** even **b** even **c** odd **d** odd **e** even **f** odd **g** even

EXERCISE 5I

1 **a** yes **b** yes **c** no **d** yes **e** yes
2 **a** yes **b** yes **c** no **d** yes **e** yes
3 **a** yes **b** no **c** yes **d** no **e** yes
4 **a** no **b** yes **c** no **d** yes **e** no
5 **a** yes **b** no **c** yes **d** no **e** yes
6 **a** 0 to 9 **b** 1, 4 or 7 **c** 0, 2, 4, 6 or 8 **d** 1, 4 or 7
7 **a** $2^3-1^3-1=6$ $3^3-2^3-1=18$
$4^3-3^3-1=36$ $5^3-4^3-1=60$
b $10^3-9^3-1=270$
All of the answers in **a** and **b** are divisible by 6.
8 **a** 1, 4 or 7 **b** 2, 5 or 8 **c** 1, 4 or 7 **d** 2, 5 or 8

EXERCISE 5J.1

1 **a** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
b 1 has only one factor and so cannot be a prime.
c One, only the number 2.
d **i** 31, 37 **ii** 61, 67 **iii** 97, 101, 103, 107, 109
2 **a** 7 **b** 2, 3 **c** 2, 5 **d** 2, 3, 7 **e** 2, 3 **f** 2, 3, 5, 7
3 **a** Is even. **b** Is divisible by 5. **c** Is even.
d Is divisible by 3. **e** Is divisible by 3.
f Is divisible by 3.
4 **a** 2^2 **b** 3^2 **c** 5^2 **d** 2^3 **e** 3^3 **f** 2^5 **g** 3^4 **h** 2^6 **i** 5^3 **j** 3^5 **k** 2^7 **l** 7^3
5 **a** $2^3\times3^2$ **b** $2^5\times5^1$ **c** $2^2\times3^2\times5^1$
d $2^3\times11^2$ **e** $2^4\times5^1\times7^2$ **f** $2^2\times3^3\times5^3$

EXERCISE 5J.2

1 **a** 1 **b** 2 **c** 6 **d** 4 **e** 18 **f** 9 **g** 14 **h** 8 **i** 12 **j** 24 **k** 11 **l** 26
2 **a** 1 **b** 4 **c** 6 **d** 6

EXERCISE 5K

1 **a** 6, 12, 18, 24, 30, ... **b** 11, 22, 33, 44, 55, ...
c 12, 24, 36, 48, 60, ... **d** 15, 30, 45, 60, 75, ...
e 20, 40, 60, 80, 100, ... **f** 35, 70, 105, 140, 175, ...
2 91
3 **a** 12 **b** 8 **c** 24 **d** 35 **e** 45 **f** 12 **g** 24 **h** 12
4 198 **5** 495, 585, 675, 765, 855 or 945

REVIEW SET 5A

1 **a** 32 **b** 0 **c** 338 **d** 0 **2** 297 **3** 14
4 **a** 7 **b** 4 **5** **a** 3 **b** 8 **c** 10
6 **a** 0 **b** 1700 **c** 4600 **7** 345 books
8 $32\div8+4+4=12$ **9** 1, 2, 3, 6, 9, 18, 27, 54
10 25 **11** 23, 29 **12** 42 **13** 8 **14** 24
15 **a** 2, 5, 8 **b** 0, 2, 4, 6, 8 **16** 72
17 **a** $3^2\times5^1$ **b** $2^4\times3^2$

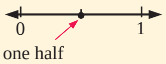
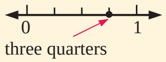
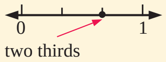
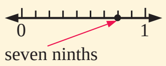
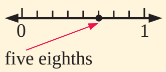
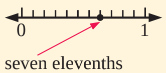
REVIEW SET 5B

1 **a** 18 **b** 78 **c** 104 **d** 23 **2** 236
3 **a** 0 **b** 0 **c** undefined **d** 417 **4** $50
5 **a** 36 **b** 27 **c** 27 **6** $2\times8\div4+2=6$
7 **a** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 **b** 53, 59 **c** 14, 21
8 $2^2\times23^1$ **9** 288
10 **a** an even number **b** an odd number **11** 25 students
12 **a** 1, 4 or 7 **b** 1 to 9 **c** 1, 4, 7
13 **a** 16 **b** 27 **c** 9 **d** 5
14 503806 **15** 2 and 7, $392=2^3\times7^2$

EXERCISE 6A

1

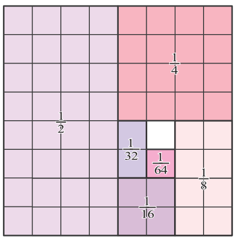
	Symbol	*Words*	*Num.*	*Denom.*
a	$\frac{1}{2}$	one half	1	2
b	$\frac{3}{4}$	three quarters	3	4
c	$\frac{2}{3}$	two thirds	2	3
d	$\frac{2}{7}$	two sevenths	2	7
e	$\frac{7}{9}$	seven ninths	7	9
f	$\frac{5}{8}$	five eighths	5	8
g	$\frac{7}{11}$	seven elevenths	7	11

	Meaning	*Number line*
a	One whole divided into two equal parts and one is being considered.	
b	One whole divided into four equal parts and three are being considered.	
c	One whole divided into three equal parts and two are being considered.	
d	One whole divided into seven equal parts and two are being considered.	
e	One whole divided into nine equal parts and seven are being considered.	
f	One whole divided into eight equal parts and five are being considered.	
g	One whole divided into eleven equal parts and seven are being considered.	

EXERCISE 6B

1 **C**, **D**, **E**, **H**

2



a $\frac{63}{64}$ **b** shaded square is one square out of 64 **c** 64
d $\frac{1}{64}$ **e** $\frac{63}{64}$

3 **a** 4 **b** $\frac{1}{4}$ **c** $\frac{1}{4}$
d piece 1 is $\frac{1}{4}$, piece 2 is $\frac{1}{4}$, piece 3 is $\frac{1}{16}$, piece 4 is $\frac{1}{8}$, piece 5 is $\frac{1}{16}$, piece 6 is $\frac{1}{8}$, piece 7 is $\frac{1}{8}$

EXERCISE 6C

1 **a** $\frac{2}{8}$ **b** $\frac{4}{8}$ **c** $\frac{6}{8}$ **d** $\frac{8}{8}$

2 **a** $\frac{15}{30}$ **b** $\frac{24}{30}$ **c** $\frac{25}{30}$ **d** $\frac{9}{30}$ **e** $\frac{6}{30}$ **f** $\frac{20}{30}$ **g** $\frac{30}{30}$
h $\frac{18}{30}$

3 **a** $\frac{2}{16}$ **b** $\frac{4}{16}$ **c** $\frac{16}{16}$ **d** $\frac{0}{16}$ **e** $\frac{14}{16}$ **f** $\frac{12}{16}$ **g** $\frac{10}{16}$
h $\frac{32}{16}$

4 **a** $\frac{50}{100}$ **b** $\frac{25}{100}$ **c** $\frac{80}{100}$ **d** $\frac{90}{100}$ **e** $\frac{28}{100}$ **f** $\frac{26}{100}$
g $\frac{100}{100}$ **h** $\frac{85}{100}$

5 **a** $\frac{5\times2}{6\times2}=\frac{10}{12}$ **b** $\frac{8\times3}{9\times3}=\frac{24}{27}$ **c** $\frac{5\times5}{7\times5}=\frac{25}{35}$
d $\frac{3\times8}{4\times8}=\frac{24}{32}$ **e** $\frac{4\times10}{5\times10}=\frac{40}{50}$ **f** $\frac{7\times4}{8\times4}=\frac{28}{32}$

6 **a** $\frac{6\div2}{8\div2}=\frac{3}{4}$ **b** $\frac{8\div2}{10\div2}=\frac{4}{5}$ **c** $\frac{10\div5}{15\div5}=\frac{2}{3}$
d $\frac{18\div3}{21\div3}=\frac{6}{7}$ **e** $\frac{15\div5}{25\div5}=\frac{3}{5}$ **f** $\frac{18\div2}{20\div2}=\frac{9}{10}$

7 **a** $\square=1$ **b** $\square=4$ **c** $\square=8$ **d** $\square=3$
e $\square=3$ **f** $\square=1$ **g** $\square=3$ **h** $\square=8$

8 **a** $\Delta=20$ **b** $\Delta=120$ **c** $\Delta=8$ **d** $\Delta=25$
e $\Delta=40$ **f** $\Delta=81$ **g** $\Delta=69$ **h** $\Delta=66$

EXERCISE 6D.1

1 **a** 9 **b** 10 **c** 3 **d** 2 **e** 3 **f** 2 **g** 2
h 1 **i** 4

EXERCISE 6D.2

1 **a** $\frac{4}{5}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{3}{7}$ **e** $\frac{4}{7}$ **f** $\frac{5}{7}$
g $\frac{4}{7}$ **h** $\frac{1}{5}$ **i** $\frac{41}{100}$ **j** $\frac{5}{8}$

2 **a** $\frac{4}{5}$ **b** $\frac{9}{10}$ **c** $\frac{3}{4}$ **d** $\frac{5}{7}$ **e** $\frac{7}{13}$ **f** $\frac{3}{4}$
g $\frac{3}{4}$ **h** $\frac{3}{11}$ **i** $\frac{41}{100}$ **j** $\frac{7}{8}$

3 **a** $\frac{8}{11}$ **b** $\frac{9}{16}$ **c** $\frac{3}{5}$ **d** $\frac{1}{3}$ **e** $\frac{1}{4}$ **f** $\frac{1}{17}$
g $\frac{8}{27}$ **h** $\frac{1}{4}$ **i** $\frac{1}{15}$ **j** $\frac{3}{8}$

4 **b**, **c**, **h**, **j**, **k**

EXERCISE 6E

1 **a** $\frac{9}{20}$ **b** $\frac{8}{15}$ **c** $\frac{5}{12}$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{13}$ **d** $\frac{3}{13}$ **e** $\frac{4}{13}$ **f** $\frac{5}{26}$

3 **a** $\frac{1}{5}$ **b** $\frac{39}{100}$ **c** $\frac{1}{2}$ **d** $\frac{2}{7}$ **e** $\frac{5}{24}$ **f** $\frac{1}{15}$
g $\frac{1}{10}$ **h** $\frac{27}{100}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{3}{4}$ **d** $\frac{1}{5}$

5 **a** $\frac{1}{24}$ **b** $\frac{1}{6}$ **c** $\frac{1}{48}$ **d** $\frac{1}{1440}$

6 $\frac{7}{10}$ **7** $\frac{13}{40}$ **8** $\frac{4}{9}$ **9** $\frac{1}{3}$

10 **a** 5 **b** 18 **c** 4 **d** 15 **e** 5 **f** 11 **g** 6
h 24 **i** 5 **j** 21 **k** 6 **l** 50

11 $\frac{53}{60}$ **12** $\frac{1}{19}$

13 **a** 4 people **b** 5 lollies **c** 7 drinks **d** 65 grams
e €19 **f** 15 min

14 5 games **15** 49 students **16** 37 cars

17 312 RMB **18** 84 plants

19 **a** **i** 90^o **ii** 180^o **iii** 270^o **b** **i** $\frac{1}{12}$ **ii** $\frac{1}{6}$ **iii** $\frac{2}{3}$

20 18 children **21** 2 h **22** 14 goals

23 **a** 1875 kg **b** 50 boxes **24** **a** $\frac{1}{7}$ **b** $\frac{3}{7}$ **c** €200 000

EXERCISE 6F

1 **a** 21 **b** 15 **c** 6 **d** 36 **e** 72 **f** 30
g 330 **h** 36

2 **a** $\frac{1}{4}$, $\frac{1}{2}=\frac{2}{4}$ **b** $\frac{2}{3}=\frac{8}{12}$, $\frac{3}{4}=\frac{9}{12}$
c $\frac{1}{2}=\frac{7}{14}$, $\frac{4}{7}=\frac{8}{14}$ **d** $\frac{5}{8}$, $\frac{3}{4}=\frac{6}{8}$
e $\frac{7}{10}=\frac{21}{30}$, $\frac{5}{6}=\frac{25}{30}$ **f** $\frac{3}{4}=\frac{27}{36}$, $\frac{7}{9}=\frac{28}{36}$

g $\frac{5}{8}=\frac{25}{40},\ \frac{8}{10}=\frac{32}{40}$ **h** $\frac{5}{8}=\frac{55}{88},\ \frac{8}{11}=\frac{64}{88}$

i $\frac{1}{4}=\frac{25}{100},\ \frac{7}{20}=\frac{35}{100},\ \frac{9}{25}=\frac{36}{100}$

3 **a** $\frac{7}{10},\ \frac{1}{2}=\frac{5}{10},\ \frac{2}{5}=\frac{4}{10}$ **b** $\frac{3}{4}=\frac{6}{8},\ \frac{5}{8},\ \frac{1}{2}=\frac{4}{8}$

c $\frac{4}{6}=\frac{8}{12},\ \frac{7}{12},\ \frac{1}{2}=\frac{6}{12}$

EXERCISE 6G

1 **a** 4 **b** 4 **c** 3 **d** 5 **e** 5 **f** 10 **g** 3
h 30 **i** 1 **j** 8 **k** 5 **l** 9

2 **a** $1\frac{1}{4}$ **b** $1\frac{1}{6}$ **c** $4\frac{1}{2}$ **d** $3\frac{1}{6}$ **e** $7\frac{1}{2}$ **f** $5\frac{2}{3}$
g $2\frac{2}{7}$ **h** $2\frac{7}{8}$ **i** $3\frac{1}{7}$ **j** $3\frac{8}{9}$ **k** $10\frac{1}{4}$ **l** $9\frac{1}{12}$

3 **a** $\frac{7}{2}$ **b** $\frac{14}{3}$ **c** $\frac{11}{4}$ **d** $\frac{5}{3}$ **e** $\frac{3}{2}$ **f** $\frac{15}{4}$
g $\frac{9}{5}$ **h** $\frac{13}{2}$ **i** $\frac{41}{9}$ **j** $\frac{47}{8}$ **k** $\frac{48}{7}$ **l** $\frac{23}{12}$

4 **a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{5}{2}$ **d** 36

e $\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=\frac{6}{6},\ \frac{2}{1}=\frac{4}{2}=\frac{6}{3},\ \frac{3}{1}=\frac{6}{2},$
$\frac{4}{1},\ \frac{5}{1},\ \frac{6}{1}$

REVIEW SET 6A

1 **a** $\frac{3}{5}$ **b** $\frac{5}{12}$ **c** $1\frac{8}{9}$ or $\frac{17}{9}$ **2** **a** $\frac{10}{12}$ **b** $\frac{9}{12}$

3 **a** 24 **b** 30 **4** **a** False **b** False **c** True **d** True

5 **a** €64 **b** 40 g **6** **a** $\frac{3}{7}$ **b** $\frac{7}{20}$

7 **a** $\frac{1}{4}$ **b** $\frac{3}{10}$ **c** 5 days

8 **a** $\frac{2}{9}=\frac{8}{36},\ \frac{1}{4}=\frac{9}{36}\ \therefore\ \frac{2}{9},\ \frac{1}{4}$
b $\frac{5}{8}=\frac{55}{88},\ \frac{7}{11}=\frac{56}{88}\ \therefore\ \frac{5}{8},\ \frac{7}{11}$

9 $\frac{4}{9}$ **10** **a** $5\frac{1}{3}$ **b** 9

REVIEW SET 6B

1 **a** $\frac{5}{8}$ **b** $\frac{2}{4}$ or $\frac{1}{2}$ **c** $\frac{5}{6}$ **2** **a** $\frac{3}{24}$ **b** $\frac{10}{24}$ **c** $\frac{18}{24}$

3 **a** $\square=3$ **b** $\square=8$ **4** **a** $4\frac{7}{8}$ **b** $\frac{1}{5}$ **c** $\frac{1}{4}$

5 $\frac{2}{5}=\frac{8}{20},\ \frac{3}{4}=\frac{15}{20}$ So, order is $\frac{3}{4},\ \frac{13}{20},\ \frac{2}{5}$.

6 **a** 21 **b** $\square=15,\ \Delta=36$

7 **a** True **b** False **c** False

8 **a** $\frac{1}{9}$ **b** 1200 sheep **c** 360 students

9 **a** $\frac{3}{4}=\frac{30}{40},\ \frac{7}{10}=\frac{28}{40}$ **b** $\frac{7}{9}=\frac{77}{99},\ \frac{8}{11}=\frac{72}{99}$

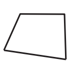
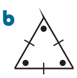
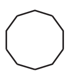
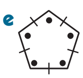
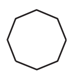
10 **a** $\frac{17}{6}$ **b** $\frac{31}{7}$

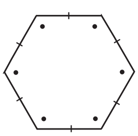
EXERCISE 7A

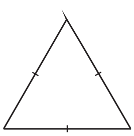
1 **a** not closed **b** not all straight sides
c some sides cross each other

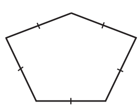
2 **a** no, as all angles are not equal **b** yes, is regular
c no, angles not equal

3 **a** regular quadrilateral **b** quadrilateral **c** triangle
d quadrilateral **e** heptagon **f** decagon
g dodecagon **h** quadrilateral **i** dodecagon
j decagon **k** quadrilateral **l** quadrilateral

4 **a**  **b**  **c**  **d**  **e**  **f** 

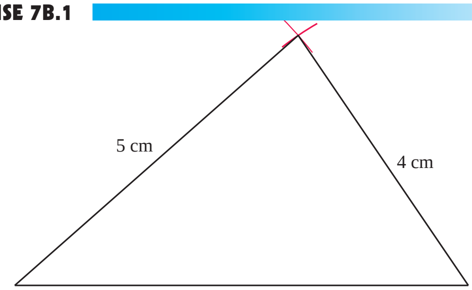
5 **a** 
regular hexagon

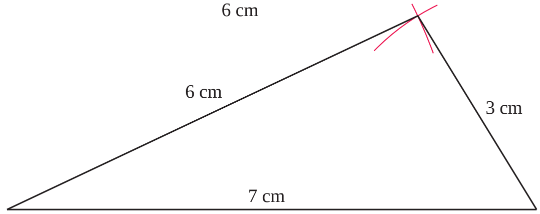
b 
equilateral triangle

c 
equilateral non-regular pentagon

6 **a** irregular **b** irregular **c** regular **d** regular
e regular **f** irregular

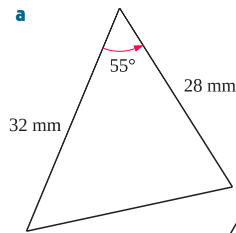
EXERCISE 7B.1

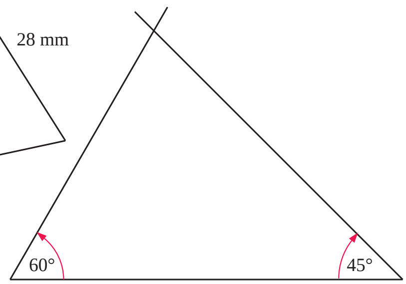
1 **a** 

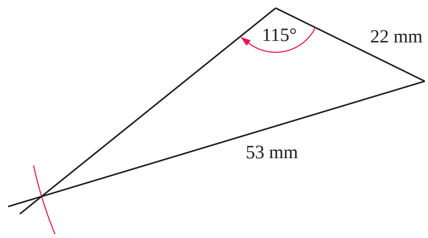
b 

2 **a** Equilateral as all sides have the same length of 5 cm.
b All three angles measure 60^o.
c "*All angles of an equilateral triangle measure* 60^o."

3 **a** The cut line is both [AB] and [AC].
So, AB = AC and $\therefore$ the triangle is isosceles.
b Angles ABC and ACB are formed from the same cut.

4 **a** 

b 

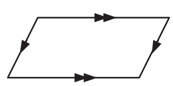
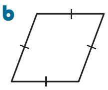
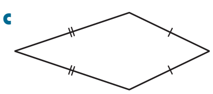
c 

EXERCISE 7B.2

1 **a** scalene **b** isosceles **c** scalene **d** equilateral

2 **a** $x=38$ **b** $x=45$ **c** $x=4,\ y=70$
d $x=46$ **e** $x=66$ **f** $x=4$ **g** $x=40$
h $x=70,\ y=4$ **i** $p=90$ **j** $x=56,\ y=68$
k $x=70,\ y=110$ **l** $x=44$

EXERCISE 7C.1

1 **a**  **b**  **c** 

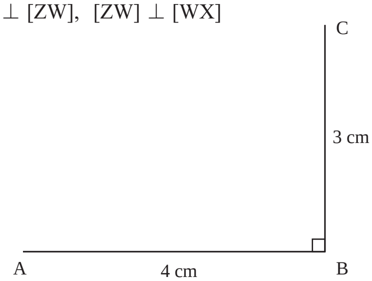
2 rectangle, rhombus, square

3 **a** square **b** rectangle **c** parallelogram
d rhombus **e** kite **f** trapezium

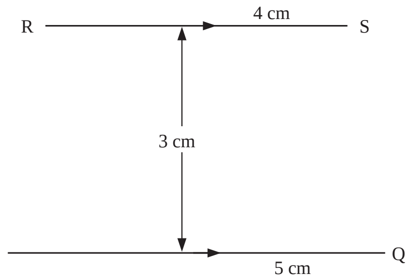
4 **a** True **b** True **c** True **d** True **e** True
f True **g** True **h** True

5 **a** [RQ] ⊥ [QP] **b** [AB] || [DC]
c [HI] || [KJ], [HI] ⊥ [IJ], [IJ] ⊥ [KJ] **d** [KM] ⊥ [LN]
e [PQ] || [SR], [PQ] ⊥ [SP], [SP] ⊥ [RS]
f [WX] || [ZY], [WZ] || [XY], [WX] ⊥ [XY], [XY] ⊥ [YZ], [YZ] ⊥ [ZW], [ZW] ⊥ [WX]

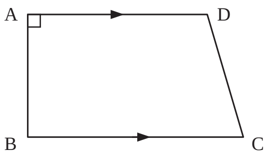
6 **a**



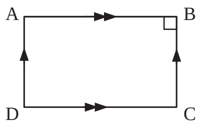
b



c



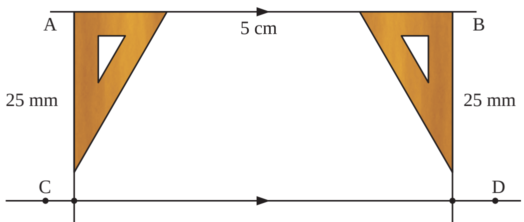
d



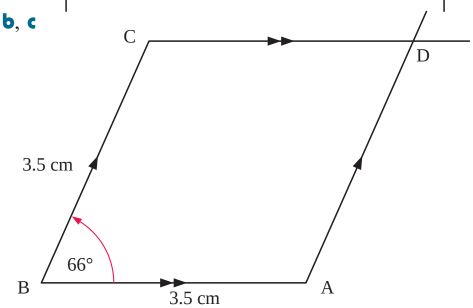
7 **a** $x = 3,\ y = 90$ **b** $x = 25,\ y = 90,\ z = 1$ **c** $x = 120$
d $x = 45,\ y = 90$ **e** $x = 7.2,\ y = 5$ **f** $x = 57$

EXERCISE 7C.2

1

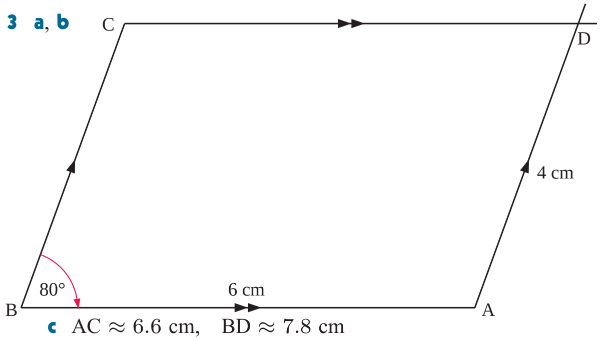


2 **a**, **b**, **c**



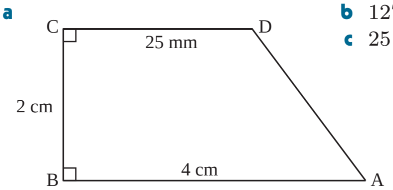
d a rhombus **e** ≈ 3.8 cm

3 **a**, **b**



c AC ≈ 6.6 cm, BD ≈ 7.8 cm

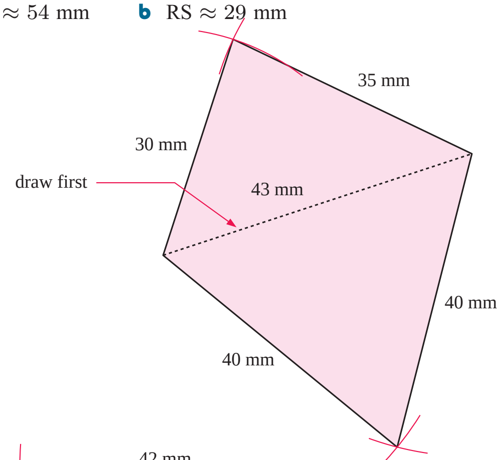
4 **a**



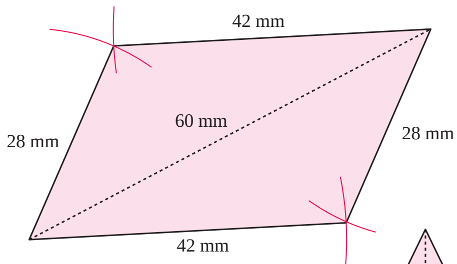
b 127° **c** 25 mm

5 **a** DB ≈ 54 mm **b** RS ≈ 29 mm

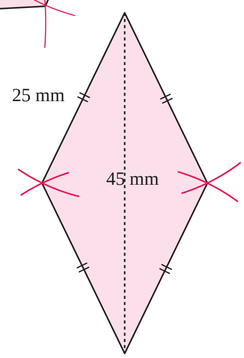
6 **a**



b



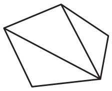
c



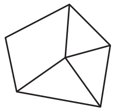
EXERCISE 7D

1 **a** 7 **b** 9 **c** 6 **d** 7 **e** 4 **f** 7

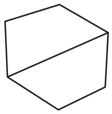
2 **a**



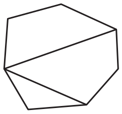
b



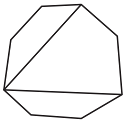
c



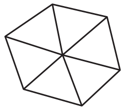
d



e

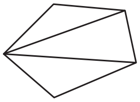


f

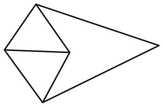


3 $E = 7, \ \ V = 5, \ \ R = 4$

(1)



(2)



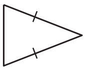
4 The figure is **not possible** as $V + R - 2 = 9 + 6 - 2$
$= 13$
$\neq E$
as $E = 12$

Conclusion: Salvi did not count correctly.

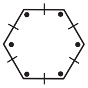
REVIEW SET 7A

1 **a** pentagon **b** quadrilateral **c** octagon

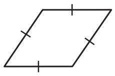
2 **a**



b

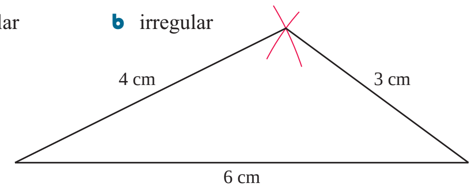


c



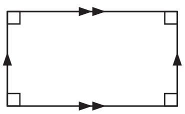
3 **a** irregular **b** irregular

4



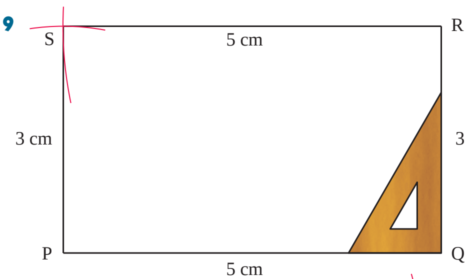
5 AC $\approx$ 63 mm **6** **a** scalene **b** equilateral

7 a rectangle

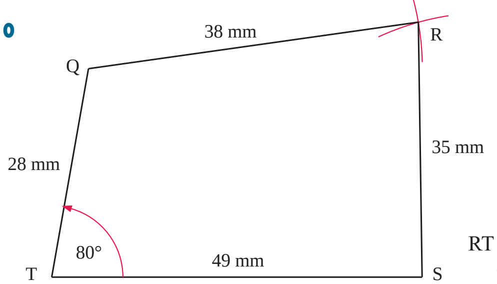


8 **a** $x = 4,$
$a = 130,$
$b = 50$
b $c = 108,$
$d = 3$

9



10



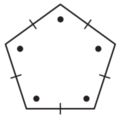
RT $\approx$ 54 mm or 55 mm

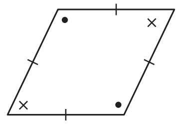
11 **a** 14 **b** 10

REVIEW SET 7B

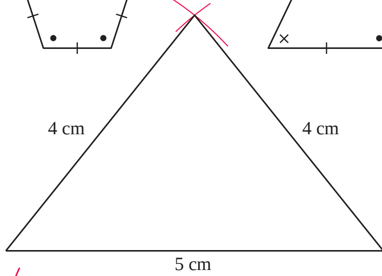
1 **a** a hexagon **b** a quadrilateral **c** a 12-gon (dodecagon)

2 **a** a regular pentagon **b** a rhombus

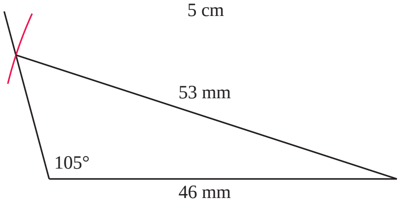




3



4



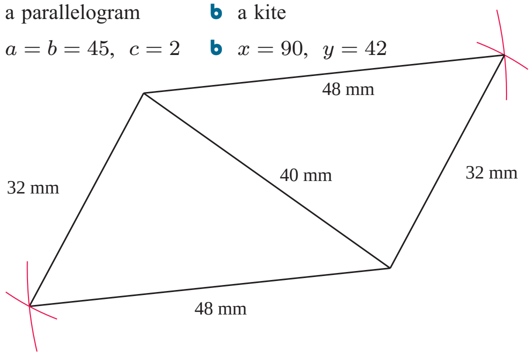
5 **a** isosceles **b** scalene

6 **a** $x = 3, \ \ y = 61$ **b** $x = 50$

7 **a** a parallelogram **b** a kite

8 **a** $a = b = 45, \ \ c = 2$ **b** $x = 90, \ \ y = 42$

9



10 **a** 9 **b** 6

EXERCISE 8A.1

1 **a** 1 **b** 1 **c** $\frac{5}{6}$ **d** $\frac{10}{11}$ **e** $1\frac{5}{8}$ **f** 1 **g** $3\frac{3}{5}$ **h** 5
i 2 **j** $\frac{6}{7}$ **k** 4 **l** 2 **m** $1\frac{2}{5}$ **n** $7\frac{1}{3}$ **o** $1\frac{1}{4}$

2 **a** $\frac{3}{4}$ **b** $\frac{8}{15}$ **c** $\frac{11}{12}$ **d** $1\frac{1}{12}$ **e** $\frac{3}{10}$ **f** $\frac{7}{10}$
g $1\frac{3}{20}$ **h** $\frac{11}{12}$ **i** $1\frac{1}{6}$ **j** $\frac{19}{20}$ **k** $1\frac{1}{21}$ **l** $\frac{59}{72}$

3 $\frac{11}{15}$ **4** **a** $\frac{13}{15}$ **b** $1\frac{3}{20}$ **c** $1\frac{1}{12}$ **d** $1\frac{1}{3}$ **e** $1\frac{1}{5}$ **f** $1\frac{5}{6}$

5 $\frac{31}{40}$

EXERCISE 8A.2

1 **a** $3\frac{1}{2}$ **b** $2\frac{11}{12}$ **c** $5\frac{1}{6}$ **d** $2\frac{27}{40}$ **e** $4\frac{17}{20}$ **f** $4\frac{11}{12}$
g $6\frac{1}{6}$ **h** $6\frac{13}{15}$ **i** $8\frac{1}{8}$

2 $5\frac{5}{6}$ hours

3 **a** $\frac{41}{56}$ **b** $1\frac{13}{144}$ **c** $7\frac{13}{60}$ **d** $7\frac{173}{198}$ **e** $1\frac{335}{476}$
f $2\frac{128}{273}$ **g** $5\frac{24}{253}$ **h** $25\frac{29}{35}$ **i** $4\frac{1}{360}$

EXERCISE 8B

1 **a** $\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$ **b** $\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$

2 **a** $\frac{6}{8}-\frac{4}{8}=\frac{2}{8}$ **b** $\frac{3}{4}-\frac{1}{2}=\frac{1}{4}$

3 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** $\frac{1}{5}$ **f** $\frac{5}{8}$
g $\frac{4}{11}$ **h** $1\frac{3}{5}$ **i** $2\frac{4}{7}$ **j** $3\frac{11}{13}$ **k** $1\frac{2}{5}$ **l** $2\frac{1}{8}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{8}$ **d** $\frac{3}{8}$ **e** $\frac{1}{8}$ **f** $\frac{1}{12}$
g $\frac{7}{15}$ **h** $\frac{1}{12}$ **i** $\frac{11}{20}$

5 **a** $\frac{1}{5}$ **b** 0 **c** $\frac{1}{8}$ **d** $\frac{5}{12}$ **e** $\frac{7}{24}$ **f** $\frac{11}{12}$

6 $\frac{4}{15}$ **7** $\frac{1}{6}$ **8** $\frac{1}{12}$

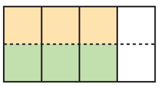
9 **a** 1 **b** $\frac{5}{6}$ **c** $1\frac{1}{3}$ **d** $1\frac{9}{10}$ **e** $1\frac{5}{8}$ **f** $\frac{7}{12}$
g $2\frac{7}{12}$ **h** $1\frac{11}{12}$ **i** $\frac{13}{20}$ **j** $2\frac{1}{3}$ **k** $1\frac{11}{14}$ **l** $4\frac{5}{12}$

10 $1\frac{1}{12}$ hours

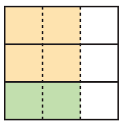
11 **a** $\frac{39}{88}$ **b** $\frac{81}{247}$ **c** $\frac{23}{40}$ **d** $3\frac{38}{221}$ **e** $2\frac{73}{84}$ **f** $1\frac{265}{396}$

EXERCISE 8C.1

1 **a**



b $\frac{3}{8}$ **2** **a**



b $\frac{4}{9}$

3 **a** $\frac{1}{2}\times\frac{3}{7}=\frac{3}{14}$ **b** $\frac{1}{3}\times\frac{4}{5}=\frac{4}{15}$ **c** $\frac{3}{4}\times\frac{4}{7}=\frac{12}{28}$ $(=\frac{3}{7})$

4 **a** $\frac{3}{20}$ **b** $\frac{10}{21}$ **c** $\frac{9}{16}$ **d** $\frac{16}{25}$ **e** $\frac{1}{4}$ **f** $\frac{4}{9}$
g $\frac{16}{105}$ **h** $\frac{48}{385}$

5 **a** $\frac{4}{5}$ **b** $\frac{5}{6}$ **c** $1\frac{1}{5}$ **d** $1\frac{1}{2}$ **e** $\frac{5}{6}$ **f** $3\frac{3}{8}$
g $1\frac{9}{16}$ **h** $8\frac{1}{3}$

EXERCISE 8C.2

1 **a** $\frac{2}{7}$ **b** $\frac{1}{8}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** 18 **f** 12 **g** 2
h 8 **i** $3\frac{1}{3}$ **j** 9 **k** 15 **l** $\frac{1}{2}$ **m** 11 **n** 10
o 20 **p** 15 **q** 15 **r** 24 **s** 21 **t** 250

2 150 mL **3** $10\frac{2}{5}$ m **4** $\frac{3}{8}$

5 **a** $\frac{15}{224}$ **b** $\frac{1}{51}$ **c** $\frac{33}{200}$ **d** $\frac{9}{23}$ **e** $\frac{5}{12}$ **f** $5\frac{4}{9}$
g $3\frac{3}{8}$ **h** $7\frac{7}{40}$

EXERCISE 8D

1 **a** $\square=\frac{3}{2}$ **b** $\square=\frac{1}{3}$ **c** $\square=\frac{3}{4}$ **d** $\square=\frac{1}{5}$

2 **a** $\frac{4}{3}$ **b** $\frac{4}{5}$ **c** 7 **d** $\frac{1}{5}$ **e** $\frac{3}{7}$

3 **a** 1 **b** 3 **c** 100 **d** 87 **e** 913 **f** 400

EXERCISE 8E

1 **a** $1\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $1\frac{1}{2}$ **e** $\frac{1}{4}$ **f** $\frac{1}{6}$
g $\frac{1}{6}$ **h** $\frac{1}{10}$ **i** 9 **j** 4 **k** 70 **l** $\frac{1}{70}$
m 30 **n** $\frac{1}{30}$ **o** $\frac{1}{500}$ **p** 500

2 **a** $\frac{1}{10}$ **b** $\frac{2}{3}$ **c** $1\frac{7}{8}$ **d** $2\frac{2}{15}$ **e** $\frac{15}{32}$ **f** $6\frac{3}{7}$ **g** $3\frac{4}{9}$
h $\frac{3}{35}$

3 $7\frac{1}{2}$ laps **4** 18 strides

EXERCISE 8F

1 $1\frac{5}{12}$ **2** €105 000 **3** 20 **4** $\frac{13}{60}$ **5** $\frac{5}{9}$ **6** 17 out of 20

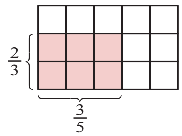
7 £98 **8** 9 lengths **9** $18\frac{3}{4}$ m **10** **a** $\frac{47}{60}$ **b** $\frac{13}{60}$

11 $\frac{1}{35}$ **12** $\frac{1}{10}$ **13** $26\frac{2}{3}$ laps **14** $\frac{7}{24}$ **15** $\frac{6}{35}$

16 $9\frac{9}{10}$ litres **17** $160 **18** **a** $\frac{1}{12}$ **b** $\frac{5}{24}$ **19** $\frac{2}{5}$ m

REVIEW SET 8A

1 **a** $2\frac{8}{9}$ **b** $7\frac{1}{8}$ **c** $1\frac{67}{209}$ **d** $\frac{3}{2}$ **e** $1\frac{7}{9}$ **f** $\square=\frac{11}{7}$
g



$\frac{6}{15}$ or $\frac{2}{5}$ **h** $\frac{2}{15}$

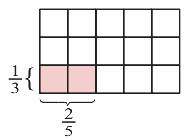
2 **a** $1\frac{1}{24}$ **b** $2\frac{7}{12}$ **c** $\frac{19}{24}$ **3** **a** $3\frac{15}{16}$ **b** $2\frac{1}{5}$ **c** 49

4 **a** 1 **b** 126 **5** **a** 56 m **b** $\frac{4}{15}$ **c** 36 lengths

6 $\frac{1}{6}$ of them **7** **a** $\frac{7}{8}$ **b** $9\frac{1}{4}$ **c** 9

REVIEW SET 8B

1 **a** $3\frac{6}{7}$ **b** $1\frac{29}{112}$ **c** $3\frac{2}{3}$
d



$\frac{1}{3}$ of $\frac{2}{5}=\frac{2}{15}$

e $\square=\frac{9}{8}$ **f** 37 **g** $6\frac{3}{5}$ **h** $\frac{22}{63}$

2 **a** $1\frac{11}{12}$ **b** $4\frac{17}{24}$ **c** $\frac{7}{20}$ **3** **a** 5 **b** 4 **c** 4

4 **a** 4900 **b** $2\frac{1}{3}$ **5** **a** 1 **b** 250 **c** $12\frac{1}{4}$

6 **a** 10 lengths **b** $\frac{5}{48}$ of it **c** $\frac{6}{35}$ remains

7 **a** $\frac{3}{8}$ **b** $2\frac{7}{16}$ **c** 12 **8** **a** 40 laps **b** $18 000

EXERCISE 9A

1 **a** zero point six **b** zero point four five
c zero point nine zero eight **d** eight point three
e six point zero eight **f** ninety six point zero two
g five point eight six four
h thirty four point zero zero three
i seven point five eight one **j** sixty point two six four

2 **a** 8.37 **b** 21.05 **c** 9.004 **d** 38.206

3 Final written numeral is
a 0.83 **b** 4.128 **c** 9.004 **d** 0.056 **e** 28.699
f 139.077

4 Final written numeral is
a 0.8 **b** 0.003 **c** 70.8 **d** 9000.002 **e** 209.04
f 8000.402 **g** 5020.3 **h** 980.034 **i** 60.89 **j** 36.42

5 **a** $5+\frac{4}{10}$ **b** $10+4+\frac{9}{10}$ **c** $2+\frac{3}{100}$
d $30+2+\frac{8}{10}+\frac{6}{100}$ **e** $2+\frac{2}{10}+\frac{6}{100}+\frac{4}{1000}$
f $1+\frac{3}{10}+\frac{8}{1000}$ **g** $3+\frac{2}{1000}$ **h** $\frac{9}{10}+\frac{5}{100}+\frac{2}{1000}$
i $4+\frac{2}{100}+\frac{4}{1000}$ **j** $2+\frac{9}{10}+\frac{7}{100}+\frac{3}{1000}$
k $20+\frac{8}{10}+\frac{1}{100}+\frac{6}{1000}$ **l** $7+\frac{7}{10}+\frac{7}{100}+\frac{7}{1000}$
m $9+\frac{8}{1000}$ **n** $100+50+4+\frac{4}{10}+\frac{5}{100}+\frac{1}{1000}$
o $800+8+\frac{8}{10}+\frac{8}{1000}$ **p** $\frac{6}{100}+\frac{4}{1000}$

6 **a** 0.6 **b** 0.09 **c** 0.43 **d** 0.809 **e** 0.007
f 0.052 **g** 0.568 **h** 0.0023 **i** 0.094 **j** 0.101
k 4.387 **l** 0.0308 **m** 0.3033 **n** 0.200 05 **o** 5.555

7 **a** 300 **b** $\frac{3}{10}$ **c** 30 **d** $\frac{3}{1000}$ **e** 3
f $\frac{3}{100}$ **g** $\frac{3}{10\,000}$ **h** 3000

8 **a** $\frac{5}{1000}$ **b** 500 **c** $\frac{5}{10}$ **d** $\frac{5}{100}$ **e** 5000
f 5 **g** 50 000 **h** $\frac{5}{10\,000}$

9 **a** 0.23 **b** 0.79 **c** 0.3 **d** 0.117 **e** 4.69
f 0.703 **g** 0.6 **h** 0.54 **i** 0.4672 **j** 0.36

10 **a** 17.465 **b** 12.096 **c** 3.694 **d** 4.22
e 980.034 **f** 36.42

11 **a** $\frac{2}{100}$ **b** 2 **c** $\frac{2}{100}$ **d** 20 **e** $\frac{2}{10\,000}$
f 200 **g** $\frac{2}{10}$ **h** $\frac{2}{1000}$

EXERCISE 9B

1 **a** 0.7 **b** 0.2 **c** 0.33 **d** 0.46

2 **a** 3.243 **b** 2.071 **c** 0.752 **d** 1.056 **e** 4.009

EXERCISE 9C

1 **a** \$7.25 **b** \$24.50 **c** \$61.10 **d** \$205.05
e \$12.70 **f** \$120.65

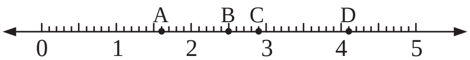
2 **a** \$4.47 **b** \$15.97 **c** \$7.55 **d** \$36.00
e \$150.00 **f** \$32.80 **g** \$85.05 **h** \$30.03

3 **a** **i** €0.35 **ii** €0.05 **iii** €4.05 **iv** €30.00
v €4.87 **vi** €2.95 **vii** €38.75 **viii** €6384.75
b €0.40, €34.05, €7.82, €6423.50 **c** €48.02, €6417.75

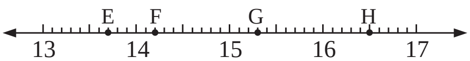
EXERCISE 9D

1 **a** N is 0.7 **b** N is 2.3 **c** N is 6.8 **d** N is 21.4
e N is 11.1 **f** N is 8.5

2 **a**



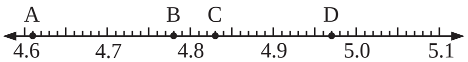
b



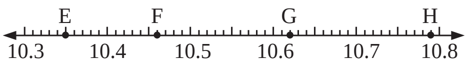
3 38.4°C **4** 75.5 cm **5** **a** 4.5 kg **b** 0.3 L

6 **a** N is 0.15 **b** N is 0.24 **c** N is 1.77 **d** N is 3.59
e N is 7.83 **f** N is 11.75

7 **a**



b



EXERCISE 9E

1 **a** A is 6.7, B is 6.2 and $A > B$
b A is 47.8, B is 47.3 and $A > B$
c A is 3.77, B is 3.78 and $A < B$
d A is 1.953, B is 1.956 and $A < B$
e A is 0.042, B is 0.047 and $A < B$
f A is 0.404, B is 0.402 and $A > B$

2 **a** $0.7 < 0.8$ **b** $0.06 > 0.05$ **c** $0.2 > 0.19$
d $4.01 < 4.1$ **e** $0.81 > 0.803$ **f** $2.5 = 2.50$
g $0.304 < 0.34$ **h** $0.03 < 0.2$ **i** $6.05 < 60.50$
j $0.29 = 0.290$ **k** $5.01 < 5.016$ **l** $1.15 > 1.035$
m $21.021 < 21.210$ **n** $8.09 = 8.090$ **o** $0.904 < 0.94$

3 **a** 0.4, 0.6, 0.8 **b** 0.1, 0.4, 0.9 **c** 0.06, 0.09, 0.14
d 0.46, 0.5, 0.51 **e** 1.06, 1.59, 1.61 **f** 0.206, 2.06, 2.6
g 0.0905, 0.095, 0.905 **h** 15.05, 15.5, 15.55

4 **a** 0.9, 0.8, 0.4, 0.3 **b** 0.51, 0.5, 0.49, 0.47
c 0.61, 0.609, 0.6, 0.596 **d** 0.42, 0.24, 0.04, 0.02
e 6.277, 6.271, 6.27, 6.027 **f** 0.311, 0.31, 0.301, 0.031
g 8.880, 8.088, 8.080, 8.008 **h** 7.61, 7.061, 7.06, 7.01

5 **a** 0.4, 0.5, 0.6 **b** 0.6, 0.5, 0.4 **c** 0.8, 1.0, 1.2
d 0.11, 0.13, 0.15 **e** 0.55, 0.5, 0.45 **f** 2.05, 2.01, 1.97
g 4.8, 4.0, 3.2 **h** 1.00, 1.25, 1.50
i 1.147, 1.159, 1.171 **j** 0.375, 0.500, 0.625

EXERCISE 9F

1 **a** 2.4 **b** 3.6 **c** 4.9 **d** 6.4 **e** 4.3

2 **a** 4.24 **b** 2.73 **c** 5.63 **d** 4.38 **e** 6.52

3 **a** 0.5 **b** 0.49 **4** **a** 3.8 **b** 3.79

5 **a** 0.2 **b** 0.18 **c** 0.184 **d** 0.1838

6 **a** 3.9 **b** 4 **c** 6.1 **d** 0.462 **e** 2.95 **f** 0.1756

EXERCISE 9G

1 **a** $\frac{1}{10}$ **b** $\frac{7}{10}$ **c** $1\frac{1}{2}$ **d** $2\frac{1}{5}$ **e** $3\frac{9}{10}$ **f** $4\frac{3}{5}$
g $\frac{19}{100}$ **h** $1\frac{1}{4}$ **i** $\frac{9}{50}$ **j** $\frac{13}{20}$ **k** $\frac{1}{20}$ **l** $\frac{7}{100}$
m $2\frac{3}{4}$ **n** $1\frac{1}{40}$ **o** $\frac{1}{25}$ **p** $2\frac{3}{8}$

2 **a** $\frac{4}{5}$ **b** $\frac{22}{25}$ **c** $\frac{111}{125}$ **d** $3\frac{1}{2}$ **e** $\frac{49}{100}$ **f** $\frac{1}{4}$
g $5\frac{3}{50}$ **h** $3\frac{8}{25}$ **i** $\frac{17}{200}$ **j** $3\frac{18}{25}$ **k** $1\frac{12}{125}$ **l** $4\frac{14}{25}$
m $\frac{8}{125}$ **n** $\frac{5}{8}$ **o** $\frac{23}{200}$ **p** $2\frac{11}{50}$

3 **a** $\frac{1}{5}$ kg **b** $\frac{1}{4}$ hour **c** $\frac{17}{20}$ kg **d** $1\frac{1}{2}$ km **e** $1\frac{3}{4}$ g
f $2\frac{37}{50}$ m **g** $4\frac{22}{25}$ tonnes **h** $6\frac{7}{25}$ L **i** €$1\frac{1}{4}$ **j** €$1\frac{19}{25}$
k €$3\frac{13}{20}$ **l** €$4\frac{21}{100}$ **m** €$8\frac{2}{5}$ **n** €$5\frac{1}{8}$ **o** £$3\frac{2}{25}$
p £$4\frac{11}{100}$ **q** £$18\frac{22}{25}$ **r** £$52\frac{1}{4}$

EXERCISE 9H

1 **a** 5 **b** 2 **c** 25 **d** 125 **e** 5 **f** 4
g 2 **h** 8 **i** 25 **j** 4 **k** 2 **l** 25

2 **a** 0.15 **b** 0.85 **c** 0.36 **d** 0.84 **e** 1.5
f 2.2 **g** 0.26 **h** 0.276 **i** 0.024 **j** 0.364
k 0.25 **l** 0.072 **m** 0.544 **n** 0.936 **o** 0.022
p 0.204 **q** 0.375 **r** 0.1775

3 **a** $\frac{1}{2} = 0.5$ **b** $\frac{1}{5} = 0.2,\ \frac{2}{5} = 0.4,\ \frac{3}{5} = 0.6,\ \frac{4}{5} = 0.8$
c $\frac{1}{4} = 0.25,\ \frac{2}{4} = 0.50,\ \frac{3}{4} = 0.75$
d $\frac{1}{8} = 0.125,\ \frac{2}{8} = 0.250,\ \frac{3}{8} = 0.375,\ \frac{4}{8} = 0.500,$
$\frac{5}{8} = 0.625,\ \frac{6}{8} = 0.750,\ \frac{7}{8} = 0.875$

REVIEW SET 9A

1 **a** \$16.95 **b** \$302.35 **2** **a** 1.8 **b** 0.54

3 **a** 0.003 **b** 0.027 **c** 0.308 **d** 1.315

4 **a** 0.73 **b** 0.107 **c** 5.069

5 **a** €25.35 **b** \$107.85 **c** 5.029
d $4 + \frac{3}{10} + \frac{6}{100}$ or $4\frac{36}{100}$ **e** $2 + \frac{4}{100} + \frac{9}{1000}$ **f** $\frac{2}{1000}$

6 **a** B is 0.87 **b** B is 2.374 **7** 3.2 **8** **a** 3.9 **b** 3.86

9 **a** $\frac{4}{5}$ **b** $\frac{3}{4}$ **c** $\frac{3}{8}$ **d** $\frac{17}{25}$

10 **a** 0.8 **b** 0.15 **c** 0.625 **d** 0.275

11 0.026, 0.062, 0.206, 0.216, 0.621 **12** 0.69, 0.66, 0.63

REVIEW SET 9B

1 **a** 1.493 **b** 2.058 **2** **a** 0.3 **b** 1.23

3 **a** 0.44 **b** 0.033 **c** 1.002 **d** 4.105

4 **a** 16.574 **b** $\frac{9}{10}+\frac{2}{100}+\frac{1}{1000}$ or $\frac{921}{1000}$ **c** $\frac{3}{100}$
d £12.35

5 **a** A is 2.46 **b** A is 0.063

6 4.44, 4.404, 4.044, 4.04, 0.444 **7** 1.5, 1.9, 2.3 **8** 2.3199

9 **a** 4.0 **b** 4.00 **10** **a** $\frac{31}{50}$ **b** $\frac{9}{20}$ **c** $\frac{7}{8}$ **d** $10\frac{2}{5}$

11 **a** 0.06 **b** 1.2 **c** 0.68 **d** 0.125

12 $\frac{1}{8}=0.125$, $\frac{2}{8}=0.250$, $\frac{3}{8}=0.375$, $\frac{4}{8}=0.500$, $\frac{5}{8}=0.625$

EXERCISE 10A

1 25, 26 and 16, 17, 18 and 6, 7, 8, 9, 10, 11

2 4 spiders and 9 beetles **3** $2\times(3+4)-5$ **4** 23 €1 coins

5
```
   386
 + 135
   1
 -----
   521
```
is one solution

6 $a=6$, $b=4$, $c=3$

7 12 cards **8** 10 **9** 6 boys and 3 girls

10 Kristina is 13, Fredrik is 11 and Frida is 17

EXERCISE 10B

1 34 **2** 12 **3** 6 **4** 20 **5** 12 ways **6** 15 **7** 36
8 10 **9** 20 **10** **a** 24 **b** 12 **c** 4

EXERCISE 10C

1 6 : (Seating arrangements deal with different right hand and left hand neighbours.)

2 9:42 am **3** 43 pages **4** 11 pieces **5** Row 9
6 Phil **7** 30 squares

8 C was 1st, E 2nd, A 3rd, D 4th and B 5th

9 Martin uses blue, Patel white, Lisa yellow, Owen green, Natalie red

10 Fill the 5 L bowl with water. Tip 3 L of this into the 3 L bowl. 2 L remains in the large bowl. Add this to the pasta. Empty the bowls, repeat the process and add another 2 L to the pasta.
There is another way. Can you find it?

EXERCISE 10D

1 128 **2** 20 **3** 91 lengths **4** 108 rails **5** 27
6 128 sections **7** 54 diagonals **8** 56 pieces

9 **a** **i** 1 **ii** 3 **iii** 6 **iv** 10
b $1=1$, $3=1+2$, $6=1+2+3$, $10=1+2+3+4$
c $H=\dfrac{P(P-1)}{2}$ **d** 18 336 handshakes

10 **a** 435 lines
b Each vertex is like a person. Each line is like a handshake.

EXERCISE 10E

1 4 months ago **2** 12 **3** 15 apples **4** \$70
5 Nima had 36, Kelly had 15 **6** **a** 88 kg **b** 89 kg
7 37 **8** 9:25 am

REVIEW SET 10A

1 7 £10 notes and 4 £20 notes **2** 8 different ways

3 **a** 15 $(8+4+2+1)$
b There are 15 losers, so 15 games. **c** 255 games

4 January 31 **5** 8

REVIEW SET 10B

1 10 **2** 8 m **3** 77 {as $77\to49\to36\to18\to9$}
4 24 ways **5** 155 cards

EXERCISE 11A

1 **a** 0.9 **b** 3.3 **c** 1.13 **d** 1.13 **e** 27.82
f 18.43 **g** 5.2 **h** 0.444 **i** 10.92 **j** 32.955
k 0.7006 **l** 4.748

2 **a** 0.8 **b** 1.5 **c** 0.4 **d** 1.4 **e** 2.3 **f** 2.26
g 2.67 **h** 0.01 **i** 9.02 **j** 5.593 **k** 0.001 **l** 0.001

3 **a** **i** 39.012 **ii** 2.134 **iii** 3.076 **iv** 8
b **i** 1.101 **ii** 0.099 **iii** 11.754 **iv** 22.694

4 **a** 64.892 **b** 27.493 **c** 12.214 **d** 21.2919
e 408.488 **f** 209.7442

5 **a** 5.981 **b** 1.011 **c** 4.481 **d** 167.348
e 58.626 **f** 3.1004 **g** 18.867 **h** 7.782
i 4.258 **j** \$5.30 **k** €5.97 **l** £4.60

6 **a** 55.1183 **b** 42.266 **c** 1.197 **d** \$118.10

7 **a** 15.867 **b** 2.731 **c** 0.681 **d** £6.85

8 €17.10 **9** 0.37 m **10** 69.4 kg **11** 237.4 m

12 No, he has only \$59.05 and needs another \$3.45.

13 27.95 kg **14** 3.38 kg **15** 13.079 m **16** €8.10

EXERCISE 11B.1

1

	Number	$\times10$	$\times100$
a	0.0943	0.943	9.43
b	4.0837	40.837	408.37
c	0.0008	0.008	0.08
d	24.6801	246.801	2468.01
e	\$57.85	\$578.50	\$5785
	$\times1000$	$\times10^4$	$\times10^6$
a	94.3	943	94 300
b	4083.7	40 837	4 083 700
c	0.8	8	800
d	24 680.1	246 801	24 680 100
e	\$57 850	\$578 500	\$57 850 000

2 **a** 430 **b** 8000 **c** 5 000 000 **d** 6
e 46 **f** 58 **g** 309 **h** 250 **i** 80
j 324 **k** 900 **l** 845 **m** 240 **n** 208.5
o 8940 **p** 53 **q** 0.094 **r** 71 800

3 **a** 100 **b** 1000 **c** 10 000 **d** 100 **e** 10 **f** 10
g 100 **h** 1 000 000 **i** 1000

EXERCISE 11B.2

1

	a	**b**	**c**	**d**
Num.	647.352	93 082.6	42 870	10.94
$\div10$	64.7352	9308.26	4287	1.094
$\div100$	6.473 52	930.826	428.7	0.1094
$\div1000$	0.647 352	93.0826	42.87	0.010 94
$\div10^5$	0.006 473 52	0.930 826	0.4287	0.000 109 4

2 **a** 0.23 **b** 0.036 **c** 0.426 **d** 0.3 **e** 5.8
f 0.58 **g** 39.4 **h** 0.07 **i** 0.458 **j** 0.8007
k 0.024 05 **l** 0.0632 **m** 5.79 **n** 0.579
o 0.0579 **p** 0.003 **q** 0.0003 **r** 0.000 046

3 **a** 10 **b** 100 **c** 100 **d** 1000 **e** 10
f 10 000 **g** 100 **h** 1000

EXERCISE 11C

1 **a** \$56.3K - \$61.8K **b** \$32.5K - \$34.9K
c \$23.2K - \$24.4K **d** \$70.8K - \$73.2K
e \$158.7K - \$165.7K **f** \$327.9K - \$348.4K

2 **a** Salary between \$38 700 and \$39 900
b Salary between \$43 200 and \$44 500
c Salary between \$95 500 and \$98 900

3 **a** 3.18 million **b** 91.73 million **c** 23.46 million
d 1.49 million **e** 30.08 million **f** 9.48 million

4 **a** 21 650 000 **b** 1 930 000 **c** 16 030 000

5 **a** 3 860 000 000 **b** 375 090 000 000
c 21 950 000 000 **d** 4 130 000 000

6 **a** 3.87 bn **b** 2.71 bn **c** 97.06 bn **d** 2.02 bn

EXERCISE 11D

1 **a** 0.8 **b** 2.4 **c** 3.5 **d** 4.8 **e** 0.28
f 0.2 **g** 0.018 **h** 0.018 **i** 0.33 **j** 0.6
k 0.0063 **l** 0.0003

2 **a** 7.2 **b** 26 **c** 13.5 **d** 0.035 **e** 0.144
f 0.0812 **g** 0.36 **h** 0.0016 **i** 0.024

3 **a** 95.2 **b** 9.52 **c** 0.952 **d** 0.952 **e** 0.952
f 0.0952 **g** 0.0952 **h** 0.000 952 **i** 0.952

4 **a** 1339.5 **b** 133.95 **c** 13.395 **d** 1.3395 **e** 13.395
f 1.3395 **g** 0.133 95 **h** 0.001 339 5 **i** 133.95

5 **a** 2.4 **b** 0.88 **c** 2.5 **d** 0.27 **e** 2.7
f 15.2 **g** 0.72 **h** 0.0063 **i** 0.0016 **j** 0.08
k 0.04 **l** 0.0009 **m** 0.072 **n** 1.21 **o** 0.01

6 **a** £39.51 **b** \$36.96 **c** 90 L

7 \$3.30 **8** 44.8 kg **9** £29 **10** \$15.30

11 $6 \times 3.9 = 23.4$, so Manuel needs to find another 1.6 m.

12 **a** 1482 kg **b** 936 kg **c** 2418 kg **d** 5 vans
e \$2762.10

EXERCISE 11E

1 **a** 0.8 **b** 1.5 **c** 0.42 **d** 0.51 **e** 3.02 **f** 0.41
g 0.08 **h** 20.4

2 **a** €8.50 **b** 2.15 kg **c** 0.7 m **d** 12 bags **e** £16.08

3 **a** 2.65 **b** 1.22 **c** 0.85 **d** 0.425 **e** 3.25
f 1.475 **g** 1.205 **h** 1.264

4 **a** $1.0\overline{3}$ **b** $1.1\overline{6}$ **c** $0.4\overline{5}$ **d** $0.82\overline{3}$ **e** $2.71\overline{6}$
f $1.5\overline{1}$ **g** $0.3\overline{714285}$ **h** $0.87\overline{857142}$

EXERCISE 11F.1

1 **a** 0.7 **b** 0.5 **c** 0.4 **d** 0.3 **e** 0.8 **f** 0.25
g 0.16 **h** 0.75 **i** 0.125 **j** 0.625 **k** 0.35 **l** 0.24

2 **a** 0.6 **b** 1.8 **c** 0.375 **d** 1.125 **e** 2.75
f 5.8 **g** 4.875 **h** 5.375

EXERCISE 11F.2

1 **a** $0.\overline{3}$ **b** $0.\overline{6}$ **c** $0.1\overline{6}$ **d** $0.\overline{142857}$ **e** $0.\overline{285714}$
f $0.08\overline{3}$ **g** $0.\overline{2}$ **h** $0.8\overline{3}$ **i** $0.\overline{27}$ **j** $0.58\overline{3}$

2 **a** $0.\overline{1}, 0.\overline{2}, 0.\overline{3}, 0.\overline{4}, 0.\overline{5}, 0.\overline{6}, 0.\overline{7}, 0.\overline{8}, 0.\overline{9}$ **b** $0.\overline{9} = 1$

3 **a** 0.718 75 **b** 0.6875 **c** 0.2125 **d** 0.44
e 1.1875 **f** $0.2\overline{142857}$ **g** $0.1\overline{3}$ **h** $0.\overline{81}$
i $2.2\overline{3}$ **j** 1.94 **k** $0.\overline{461538}$
l 0.306 25 **m** $3.41\overline{6}$ **n** $0.\overline{25203}$ **o** $0.5\overline{1}$

EXERCISE 11G

1 **a** $\frac{5}{17}, \frac{3}{10}, \frac{7}{22}, \frac{1}{3}, \frac{7}{20}$ **b** $\frac{3}{8}, \frac{5}{12}, \frac{7}{16}, \frac{5}{9}, \frac{4}{7}$
c $\frac{10}{13}, \frac{9}{11}, \frac{7}{8}, \frac{8}{9}, \frac{11}{12}$ **d** $\frac{12}{23}, \frac{10}{19}, \frac{8}{15}, \frac{6}{11}, \frac{11}{20}$

2 **a** $\frac{2}{3}, \frac{15}{23}, \frac{11}{17}, \frac{7}{11}, \frac{5}{8}$ **b** $\frac{5}{13}, \frac{8}{21}, \frac{3}{8}, \frac{4}{11}, \frac{6}{17}$
c $\frac{9}{25}, \frac{7}{20}, \frac{8}{23}, \frac{1}{3}, \frac{5}{16}$ **d** $\frac{20}{23}, \frac{17}{20}, \frac{16}{19}, \frac{14}{17}, \frac{3}{4}$

3 3.165 m **4** **a** 11.4 m **b** 1.425 m

5 **a** 17 **b** 0.28 m **6** 150 **7** \$18.55

REVIEW SET 11A

1 **a** 28.754 **b** 147.05 **c** 5.04 **d** 0.0768

2 42.238 **3** **a** \$109.90 **b** €1177.05 **c** \$28.35

4 1937.88 tonnes **5** **a** 5 kg **b** 35 kg

6 **a** 62 **b** 215.8 **c** 0.56 **d** 0.042

7 **a** 13.78 **b** 0.1378 **8** **a** $0.\overline{142857}$ **b** $0.\overline{5}$ **c** $1.1\overline{6}$

9 **a** $\square = 100$ **b** $\square = 1000$ **c** $\square = 100$

10 **a** €11 500 **b** €12 500

REVIEW SET 11B

1 **a** 1.899 **b** 2.574 **c** 8.884 **d** 1.54

2 37.314 **3** **a** 57.05 sec **b** 57.21 sec **4** 218.793 kg

5 **a** 63 **b** 5980 **c** 0.076 **d** 0.009 42

6 **a** $\square = 100$ **b** $\square = 1000$

7 **a** 272.6 **b** 2.726 **c** 27.26

8 **a** 6.16 **b** **i** 0.96 **ii** 0.015 **c** 14.8 m **d** \$20.60

9 **a** 14.065 km **b** 28.13 km

10 **a** 0.56 **b** $0.\overline{72}$ **c** 4.475

EXERCISE 12A

1 **a** kilograms **b** kilometres **c** metres
d milligrams **e** metres **f** kilograms
g centimetres **h** tonnes

EXERCISE 12B

1 **a** 24 cm **b** 13 cm **c** 10.2 cm **d** 16.8 cm
e 25.6 cm **f** 18.5 cm

2 **a** 35°C **b** 37.4°C **c** 38.3°C **d** 35.7°C

3 **a** $\frac{3}{4}$ full **b** $\frac{1}{4}$ full **c** $\frac{9}{16}$ full

4 **a** 120 km per hour **b** 95 km per hour
c 65 km per hour

5 **a** 45.2 kg **b** 71.6 kg **c** 63.63 kg

6 **a** 700 mL **b** 350 mL **c** 650 mL

EXERCISE 12C

1 **a** 400 **b** 3400 **c** 250 **d** 1560 **e** 245 **f** 46

2 **a** 3000 **b** 45 000 **c** 3600 **d** 16 200 **e** 5460
f 90

3 **a** 50 **b** 230 **c** 27 **d** 125 **e** 57.8 **f** 2.5

4 **a** 2 **b** 30 **c** 0.35 **d** 9.505 **e** 284.92 **f** 0.004

5 **a** 2 **b** 40 **c** 45 **d** 4.56 **e** 750 **f** 0.03

6 **a** 3000 **b** 75 000 **c** 6500 **d** 2 000 000
e 78 200 **f** 200

7 **a** 2 **b** 35 **c** 0.2345 **d** 34.567 **e** 3.9 **f** 0.0024

8 **a** 9.2 **b** 6.43 **c** 47.53 **d** 5 **e** 9.743
f 13.5 **g** 6200 **h** 13 500

9 **a** 72 000 **b** 1380 **c** 630 **d** 13.4 **e** 8.5
f 132.8 **g** 520 000 **h** 43 000

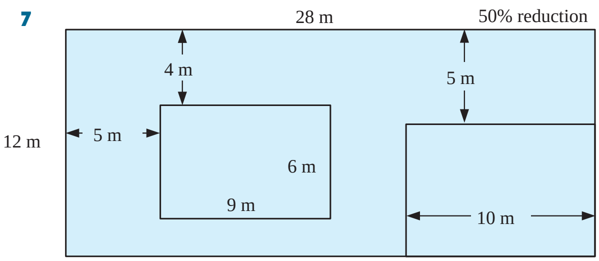
10 **a** 7000 **b** 34 **c** 780 **d** 460
11 **a** 4.562 **b** 17.458 **c** 6.53 **d** 0.164
12 **a** 49 mm **b** 63 mm **c** 132 mm **d** 82 mm **e** 151 mm **f** 235 mm **g** 116 mm **h** 205 mm **i** 102 mm **j** 101 mm
13 **a** 3110.32 **b** 72 043.486 **c** 155.218 **d** 15 348.727 **e** 23.808 **f** 23 079.906
14 **a** 37 mm, 40 mm **b** 750 cm, 780 cm, 800 cm **c** 1.25 km, 1.3 km **d** 4.85 m, 5 m, 5.2 m **e** 3.47 m, 3.5 m, 3.6 m **f** 128 m, 130 m, 134 m **g** 4.82 m, 4.9 m, 5.12 m **h** 71.5 m, 71.8 m, 72 m

EXERCISE 12D

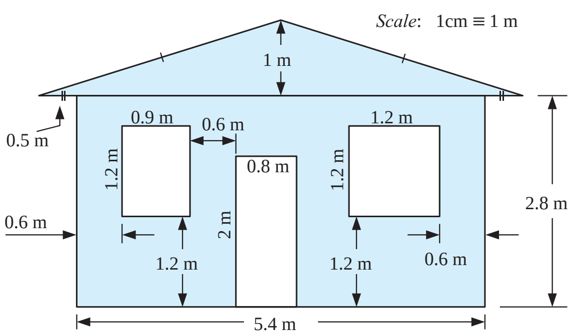
1 **a** 59 cm **b** 41 m **c** 12.6 km
2 **a** 48 cm **b** 28.6 cm **c** 26.6 km **d** 60 cm **e** 72 cm **f** 38 km **g** 96 cm **h** 42 cm **i** 60 cm
3 **a** 11.5 cm **b** 10.0 cm **c** 11.8 cm
4 **a** 8.9 cm **b** 8.6 cm **c** 10.4 cm
5 760 m **6** 7.05 km **7** $4350
8 **a** 106.5 mm **b** 27 cm **c** 19 m **d** 18 m **e** 112 cm
9 15 cm

EXERCISE 12E

1 **a** **i** 200 m **ii** 290 m **iii** 120 m **iv** 630 m **b** **i** 10 cm **ii** 35 mm **iii** 4 mm **iv** 2.16 cm
2 **a** **i** 6 m **ii** 9 m **iii** 16.4 m **iv** 1.6 m **b** **i** 1 m **ii** 9 cm **iii** 2.8 cm **iv** 6.1 cm
3 **a** 4.5 m **b** 1.5 m **c** 4.7 m
4 **a** 12 m **b** 5 m **c** 2 m by 4 m **d** 2.4 m by 2.4 m and 3.6 m by 2.4 m
5 **a** 5.3 m **b** 2.3 m
6 **a** 5 km **b** 40 cm **c** **i** 21 km **ii** 9.5 km **iii** 10.5 km
7



8



9 **a** 1 represents 100 **b** 1 represents 125
10 **a** 30 mm **b** 30 mm represents 5 mm ∴ 6 mm represents 1 mm **c** **i** 2.5 mm **ii** 2.2 mm **iii** 1.5 mm **iv** 1 mm

EXERCISE 12F

1 **a** kg **b** tonnes **c** mg **d** g **e** g **f** kg **g** mg **h** tonnes **i** g **j** kg **k** kg **l** g **m** g **n** tonnes **o** tonnes **p** g **q** g **r** kg **s** mg **t** kg
2 **a** C **b** None of these devices is suitable. **c** A **d** B **e** B **f** D **g** A **h** D **i** B **j** B **k** D **l** B **m** A **n** D **o** D **p** A **q** B **r** C **s** A **t** D
3 **a** 2000 **b** 34 000 **c** 350 000 **d** 4500 **e** 300
4 **a** 4000 **b** 25 000 **c** 3600 **d** 294 000 **e** 400
5 **a** 6000 **b** 34 000 **c** 2500 **d** 256 000 **e** 600
6 **a** 3 **b** 2.5 **c** 45 **d** 0.0675 **e** 0.0095
7 **a** 4 **b** 95 **c** 4.534 **d** 0.0456 **e** 0.0008
8 **a** 8000 **b** 3200 **c** 14 200 **d** 0.38 **e** 4.25 **f** 75.42 **g** 6 800 000 **h** 560 000
9 **a** 13.87 **b** 3400 **c** 0.786 **d** 0.003 496
10 **a** 24 kg **b** 200 nails **c** 33 tonnes

EXERCISE 12G

1 **a** 9.6 m **b** €44.64 **2** 40 kg
3 **a** 3900 m **b** $9360 **4** 7.65 m **5** 7.8 kg
6 **a** 924 kg **b** less than 1 tonne
7 **a** 84 m **b** **i** 42 sleepers **ii** 1680 kg
8 **a** 1800 bricks **b** 4.5 tonne
9 **a** 87.5 m, $393.75 **b** $210 **c** $630.75
10 228 L **11** **a** 7.92 m **b** $35.64 **12** 164 cm

REVIEW SET 12A

1 **a** 3560 mm **b** 3.2 kg **c** 0.45 km **d** 83 t **e** 7630 mm **f** 6.3 m
2 **a** 46 cm **b** 19 m
3 **a** 110 km per hour **b** $\frac{7}{8}$ full **c** 600 mL **d** 650 g
4 **a** **i** 19 km **ii** 32 km **iii** 61 km **b** **i** 10 cm **ii** 4.4 cm **iii** 26 cm
5 7.9 km **6** 270 kg **7** 30 truckloads **8** 124 m
9 **a** 72 + 4 corner tiles **b** 54 + 4 corner tiles

REVIEW SET 12B

1 **a** 3.48 kg **b** 8.623 m **c** 4600 mg **d** 540 cm **e** 13 200 kg **f** 13 300 m
2 **a** 51.1 cm **b** 35.6 km
3 **a** $\frac{1}{8}$ full **b** 75 km per hour **c** 3.2 kg **d** 3.5 L
4 2000 bricks
5 **a** **i** 120 km **ii** 17.5 km **b** **i** 4.8 cm **ii** 3.92 cm
6 **a** 66 m **b** €2560.80 **7** 1 represents 20 000 **8** 6.08 m

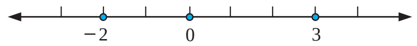
EXERCISE 13A

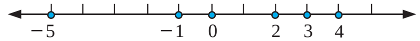
1

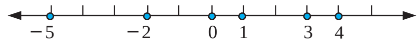
	Statement	Directed number	Opposite to statement	Directed number
a	20 m above sea level	+20	20 m below sea level	−20
b	45 km south of the city	−45	45 km north of the city	+45
c	a loss of 2 kg in weight	−2	a gain of 2 kg in weight	+2
d	a clock is 2 minutes fast	+2	a clock is 2 minutes slow	−2
e	she arrives 5 minutes early	−5	she arrives 5 minutes late	+5
f	a profit of $4000	+4000	a loss of $4000	−4000
g	2 floors above ground level	+2	2 floors below ground level	−2
h	10°C below zero	−10	10°C above zero	+10
i	an increase of €400	+400	a decrease of €400	−400
j	winning by 34 points	+34	losing by 34 points	−34

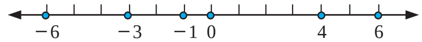
2 lift +1, car −3, parking attendant −2, rubbish skip −5
3 A −2, B −6, C +5, D +3, E 0
4 **a** +11 **b** −6 **c** −8 **d** +29 **e** −14
5 **a** −30 **b** +200 **c** −431 **d** −751 **e** +809 **f** +39 000
6 **a** +7 **b** −15 **c** −115 **d** +362 **e** −19.6
7 **a** +6 **b** −3 **c** +29 **d** −7 **e** −4
8 **a** −7 **b** +5 **c** −12 **d** +9 **e** −23
9 **a** deposit of $3 **b** £13 withdrawal **c** 5°C rise **d** 5° fall **e** 1 km east **f** remain in same position **g** 1 floor down **h** 2 kg loss
10 **a** Day 1: −28 g Day 2: −15 g Day 3: −13 g Day 4: +17 g Day 5: +29 g
b 3399 g
11 11 m away from the jetty **12** €124
13 **a** 2L **b** 4L **c** 5R **d** 9L
14 **a** 2R **b** 1L **c** 11L
15 **a** 3↓ **b** 1↑ **c** 13↓ **16** **a** 3↓ **b** 0 **c** 2↓
17 **a** A 35°C, B 5°C, C −10°C, D 25°C, E 10°C, F −5°C
b **i** 15°C **ii** 20°C **iii** 30°C **iv** 35°C
c **i** 45°C **ii** 20°C **iii** 5°C **iv** 15°C
d **i** 30°C **ii** 15°C **iii** 20°C **iv** 5°C **v** 10°C **vi** 30°C

EXERCISE 13B

1 **a** −8 **b** 5 **c** 0 **d** −11 **e** 2 **f** −6.4 **g** $3\frac{1}{2}$ **h** −56 **i** 23 **j** 23.6
2 **a** 5 **b** 2 **c** 3 **d** −4 **e** −1 **f** −1 **g** −3 **h** −4 **i** 2
3 **a** +10 **b** +6 **c** +4 **d** +7 **e** −2 **f** −5
4 **a** 12 **b** −2 **c** −3 **d** −9 **e** −2 **f** −6.5
5 **a** false **b** true **c** true **d** false **e** true **f** false **g** false **h** true **i** true
6 **a** 4 > −1 **b** −4 > −11 **c** 8 > −8 **d** −1 > −11 **e** −6 > −8 **f** −9 > −13 **g** 0 > −8 **h** −6 < 0 **i** −7 < −5.5
7 **a**


b


c


d


8 **a** {−4, −3, −1, 0, 4} **b** {5, 2, 0, −1, −2}
9 Rachel $852, Joey $311, Ross −$312, Monica −$592
10 Moscow −7°C, New York −3°C, London 0°C, Sydney 12°C, Mexico City 15°C
11 **a** −5, −2, 8 **b** −4, −3, 0, 4 **c** −3.1, −1.2, 2.5, 4 **d** −10, −9.7, −9.5, −8.9 **e** $-2\frac{1}{4}$, $-1\frac{1}{5}$, 1, $3\frac{1}{2}$ **f** $-\frac{7}{8}$, $-\frac{5}{8}$, $-\frac{3}{8}$, $-\frac{1}{8}$, $\frac{5}{8}$
12 **a** **i** 15 **ii** −1 **iii** −20 **b** **i** 5 **ii** 6 **iii** 32
13 A 4, B 1, C 0, D −3, E −4 **a** true **b** false **c** true **d** false **e** true **f** true **g** false **h** true
14 **a** 6 **b** 10 **c** 8 **d** 6 **e** −2 **f** 0 **g** −4 **h** −1

EXERCISE 13C

1 **a** 8 **b** −2 **c** 6 **d** −2 **e** 0 **f** 0 **g** 0 **h** 0 **i** 4 **j** 2 **k** 1 **l** 6 **m** 3 **n** 6 **o** 8 **p** 5
2 **a** 3 **b** −11 **c** −3 **d** −13 **e** −6 **f** −3 **g** −4 **h** −10 **i** −3 **j** 3 **k** 1 **l** −1 **m** −4 **n** −7 **o** −2 **p** −6
3 **a** 5 **b** 2 **c** −2 **d** 0 **e** −4 **f** −5 **g** 0 **h** −3 **i** 0 **j** −5 **k** −4 **l** 4 **m** 4 **n** 0 **o** −3 **p** −9
4 **a** 7 m below sea level **b** 7°C **c** 2 m below sea level **d** −5°C
5 **a** −2 **b** −6 **c** 2 **d** −3 **e** −2 **f** −2 **g** −8 **h** −7 **i** −3
6 $23 **7** **a** 3 **b** 0 **c** −7 **d** −2 **e** −5 **f** −1

EXERCISE 13D

1 **a** 4 **b** 10 **c** −10 **d** −10 **e** −4 **f** 10 **g** 4 **h** −4 **i** −6 **j** 6 **k** 16 **l** 16 **m** −16 **n** 6 **o** −16 **p** −6 **q** −7 **r** −6 **s** 4 **t** −8 **u** −4 **v** −10 **w** −19 **x** −20
2 8th floor
3 **a** −7 **b** 10 **c** −8 **d** 12 **e** 7 **f** 18 **g** −3 **h** −3 **i** −15 **j** −2 **k** −9 **l** 9 **m** −14 **n** 68 **o** 24 **p** −5
4 **a** 10 **b** 16 **c** −4 **d** −4 **e** 0 **f** −12 **g** 9 **h** 4 **i** 4 **j** 1 **k** −2 **l** −20
5 −1°C

EXERCISE 13E

1 **a** 6 **b** -6 **c** -6 **d** 6 **e** -16 **f** 16
g -16 **h** 16 **i** 77 **j** 77 **k** -77 **l** -77
m 0 **n** 0 **o** 18 **p** 25

2 **a** 8 **b** -8 **c** 2 **d** -2 **e** -3 **f** -3
g -5 **h** 5 **i** -5 **j** 5 **k** -3 **l** 3

3 **a** \$56 **b** 6000 m

4 **a** -30 **b** -6 **c** 12 **d** 49 **e** -1 **f** 20
g 40 **h** 28 **i** -8 **j** -50 **k** -18 **l** -24

5 $(-2)^2 = 4$, $-2^2 = -4$, no

6 **a** 1 **b** -1 **c** 1 **d** -1 **e** 1 **f** -1
-1 raised to even power equals 1,
-1 raised to odd power equals -1

EXERCISE 13F

1 **a** 2 **b** -2 **c** -2 **d** 2 **e** 6 **f** 6 **g** -6
h -6 **i** 1 **j** -1 **k** -1 **l** 1 **m** 6 **n** -6
o 6 **p** -6

2 **a** 4 **b** -4 **c** -4 **d** 4 **e** 11 **f** -11
g -11 **h** 11 **i** 2 **j** -2 **k** 2 **l** -2

3 **a** -6 **b** 6 **c** -2 **d** 2 **e** 9 **f** -9
g -35 **h** 35 **i** 16 **j** -16 **k** -15 **l** -15
m 16 **n** -16 **o** -1 **p** -1
q any integer, except 0 **r** no solution

4 **a** \$80 000 **b** -3^oC per hour

EXERCISE 13G

1 **a** -11 **b** 10 **c** -10 **d** 10 **e** -18 **f** -2
g -19 **h** 56 **i** -35 **j** -7

2 €40 000 profit **3** **a** \$4554 profit **b** \$759 **4** -2^oC

5 **a** B **b** C **c** 36 m **d** £67 800

6 **a** -1 **b** -1 **c** -2 **d** 2 **e** -3 **f** 3 **g** -9
h 3

EXERCISE 13H

1 **a** -14 **b** 70 **c** -48 **d** 17 **e** -425
f -70 **g** -24 **h** -8 **i** 100

2 **a** 2 m below **b** €245 **c** £2550 **d** RM 23 600

REVIEW SET 13A

1 **a** -2 **b** $7 > -12$ **c** -1 **d** $\{-5, -3, -2, 0, 1, 3, 4\}$
e borrowing 4 books **f** -3 **g** -12 **h** 22 **i** 3

2 **a** -27 **b** 4 **c** -7 **d** positive **e** -2 **f** -5
g -4 **h** $-6 < 0$ **i** -1

3 **a** \$633 **b** moving 63 m below to 33 m above (96 m)

REVIEW SET 13B

1 **a** going down 4 flights of stairs **b** -9 **c** $-4, -3, -2$
d -3 **e** 2 **f** -4 **g** $\{-6, -4, -2, 0, 5\}$

2 **a** 7 **b** -11 **c** 5 **d** -9 **e** 2 **f** -6

3 **a** depositing €26 **b** **i** 1 **ii** 10 **iii** -15 **c** £610
d 7 kg

EXERCISE 14A

1 **a** **i** $\frac{60}{100}$ **ii** 60% **b** **i** $\frac{46}{100}$ **ii** 46%
c **i** $\frac{40}{100}$ **ii** 40%

2 **a** M 11, C 17, L 10, X 35, V 27
b M $\frac{11}{100}$, C $\frac{17}{100}$, L $\frac{10}{100}$, X $\frac{35}{100}$, V $\frac{27}{100}$
c M 11%, C 17%, L 10%, X 35%, V 27%

3 **a** 50% **b** 20% **c** 25% **d** 10% **e** 20%
f 9% **g** 25% **h** 74%

EXERCISE 14B

1 **a** 14% **b** 38% **c** 67% **d** 95%

2 **a** 50% **b** 86% **c** 25%

3 **a** $\frac{3}{5} = \frac{60}{100} = 60\%$ **b** $\frac{1}{4} = \frac{25}{100} = 25\%$
c $\frac{7}{25} = \frac{28}{100} = 28\%$

4 **a** 31% **b** 3% **c** 37% **d** 54% **e** 79%
f 50% **g** 100% **h** 85% **i** 6.6% **j** 34.5%
k 7.5% **l** 35.6%

5 **a** 70% **b** 10% **c** 90% **d** 50% **e** 25%
f 75% **g** 60% **h** 80% **i** 35% **j** 55%
k 28% **l** 76% **m** 46% **n** 94% **o** 100%

6 **a** Fourteen percent means fourteen out of every hundred.
b If 53% of the students in a school are girls, 53% means the fraction $\frac{53}{100}$.

7

	Number	*Fraction*	*Denom. of 100*	*%*
a	4	$\frac{4}{20}$	$\frac{20}{100}$	20%
b	9	$\frac{9}{20}$	$\frac{45}{100}$	45%
c	9	$\frac{9}{20}$	$\frac{45}{100}$	45%
d	3	$\frac{3}{20}$	$\frac{15}{100}$	15%
e	1	$\frac{1}{20}$	$\frac{5}{100}$	5%
f	10	$\frac{10}{20}$	$\frac{50}{100}$	50%
g	20	$\frac{20}{20}$	$\frac{100}{100}$	100%

8 52% **9** 60%

10 **a** **i** 24% **ii** 52% **iii** 24% **iv** 32%
b **i** 28% **ii** 36% **iii** 36%
c **i** 50% **ii** 25% **iii** 50%

EXERCISE 14C

1 **a** $\frac{43}{100}$ **b** $\frac{37}{100}$ **c** $\frac{1}{2}$ **d** $\frac{3}{10}$ **e** $\frac{9}{10}$ **f** $\frac{1}{5}$
g $\frac{2}{5}$ **h** $\frac{1}{4}$ **i** $\frac{3}{4}$ **j** $\frac{19}{20}$ **k** 1 **l** $\frac{3}{100}$
m $\frac{1}{20}$ **n** $\frac{11}{25}$ **o** $\frac{37}{100}$ **p** $\frac{4}{5}$ **q** $\frac{99}{100}$ **r** $\frac{21}{100}$
s $\frac{8}{25}$ **t** $\frac{3}{20}$ **u** 2 **v** $3\frac{1}{2}$ **w** $1\frac{1}{4}$ **x** 8

2 **a** $\frac{1}{8}$ **b** $\frac{3}{40}$ **c** $\frac{1}{200}$ **d** $\frac{173}{1000}$ **e** $\frac{39}{40}$
f $\frac{1}{500}$ **g** $\frac{1}{2000}$ **h** $\frac{1}{5000}$

EXERCISE 14D

1 **a** 37% **b** 89% **c** 15% **d** 49% **e** 73%
f 5% **g** 102% **h** 117%

2 **a** 20% **b** 70% **c** 90% **d** 40% **e** 7.4%
f 73.9% **g** 0.67% **h** 0.18%

3 **a** 10% **b** 80% **c** 40% **d** 60% **e** 40% **f** 50% **g** 15% **h** 25% **i** 95% **j** 6% **k** 78% **l** 68% **m** $37\frac{1}{2}$% **n** 100% **o** 11% **p** $87\frac{1}{2}$% **q** $33\frac{1}{3}$% **r** $66\frac{2}{3}$%

4 **a** $\frac{1}{4}$ is 25%, $\frac{1}{8}$ is $12\frac{1}{2}$%, $\frac{1}{16}$ is $6\frac{1}{4}$%
b $\frac{2}{5}$ is 40%, $\frac{3}{5}$ is 60%, $\frac{4}{5}$ is 80%, $\frac{5}{5}$ is 100%
c $\frac{2}{3}$ is $66\frac{2}{3}$%, $\frac{3}{3}$ is 100%
d $\frac{1}{4}$ is 25%, $\frac{1}{2}$ is 50%, $\frac{3}{4}$ is 75%, $\frac{4}{4}$ is 100%

EXERCISE 14E

1 **a** 0.5 **b** 0.3 **c** 0.25 **d** 0.6 **e** 0.85 **f** 0.05 **g** 0.45 **h** 0.42 **i** 0.15 **j** 1 **k** 0.67 **l** 1.25

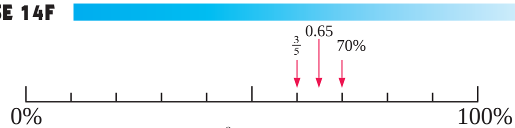
2 **a** 0.075 **b** 0.183 **c** 0.172 **d** 1.067 **e** 0.0015 **f** 0.0863 **g** 0.375 **h** 0.065 **i** 0.005 **j** 0.015 **k** 0.0075 **l** 0.0425

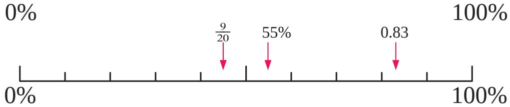
3

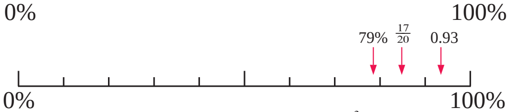
	%	*Fraction*	*Decimal*
a	20%	$\frac{1}{5}$	0.2
b	40%	$\frac{2}{5}$	0.4
c	50%	$\frac{1}{2}$	0.5
d	75%	$\frac{3}{4}$	0.75
e	85%	$\frac{17}{20}$	0.85
f	8%	$\frac{2}{25}$	0.08
g	35%	$\frac{7}{20}$	0.35
h	12.5%	$\frac{1}{8}$	0.125
i	62.5%	$\frac{5}{8}$	0.625
j	100%	1	1.00
k	15%	$\frac{3}{20}$	0.15
l	37.5%	$\frac{3}{8}$	0.375

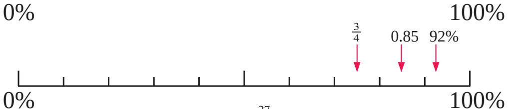
4 **a** $45\% = \frac{9}{20} = 0.45$ **b** $\frac{7}{25} = 0.28 = 28\%$
c $\frac{1}{5}\% = 0.2\% = 0.002 = \frac{1}{500}$ **d** $250\% = 2.5 = 2\frac{1}{2}$

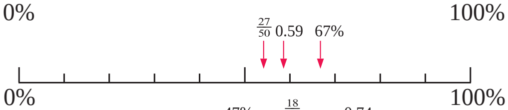
EXERCISE 14F

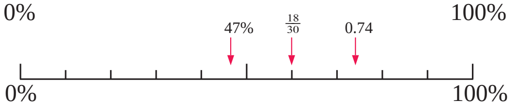
1 **a**


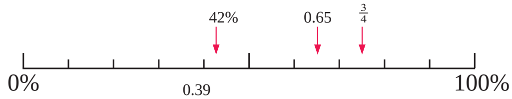
b


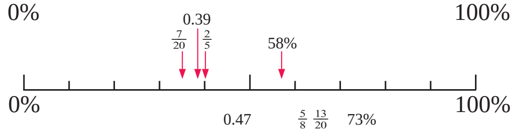
c


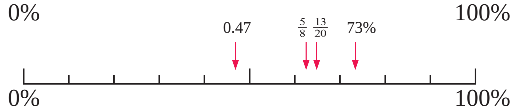
d


e


f


g


h


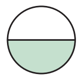
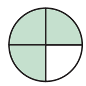
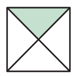
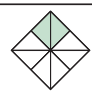
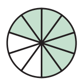
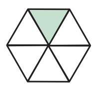
i


2 **a** $\frac{28}{100} = 0.28 = 28\%$, $\frac{45}{100} = 0.45 = 45\%$, $\frac{68}{100} = 0.68 = 68\%$
b $\frac{58}{100} = 0.58 = 58\%$, $\frac{74}{100} = 0.74 = 74\%$, $\frac{89}{100} = 0.89 = 89\%$
c $\frac{22}{100} = 0.22 = 22\%$, $\frac{37}{100} = 0.37 = 37\%$, $\frac{55}{100} = 0.55 = 55\%$

3 $25\% = \frac{1}{4}$ which is *smaller* than $\frac{1}{2}$.
($\frac{1}{4}$ of an icecream is less than $\frac{1}{2}$ of it.)

EXERCISE 14G

1

	Figure	*Frac.*	*% shaded*	*% unshaded*
a		$\frac{1}{2}$	50%	50%
b		$\frac{3}{4}$	75%	25%
c		$\frac{1}{4}$	25%	75%
d		$\frac{1}{4}$	25%	75%
e		$\frac{7}{10}$	70%	30%
f		$\frac{1}{6}$	$16\frac{2}{3}$%	$83\frac{1}{3}$%

2 **a** 35 squares **b** $\frac{13}{20}$

3 **a** 18% **b** 14% **c** 28% **d** 32%

4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{10}$ **d** $\frac{7}{20}$ **e** $\frac{3}{5}$ **f** 1

5 **a** **i** 20% **ii** 20% **b** $\frac{2}{5}$

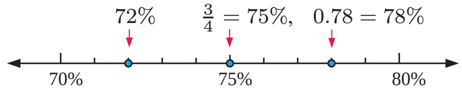
REVIEW SET 14A

1 **a** $\frac{40}{100}$ **b** 40% **2** 8% **3** 80%

4 **a** 75% **b** $\frac{33}{50}$ **c** 12.5% **d** 0.82

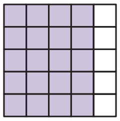
5 **a** $\frac{28}{100}$ **b** $33\frac{1}{3}$% **c** 215%

6 65% **7** **a** 0.81 **b** 0.108 **c** 0.085

8 72% $\frac{3}{4} = 75\%$, $0.78 = 78\%$


9 **a** $\frac{10}{100} = 0.1 = 10\%$ **b** $\frac{35}{100} = 0.35 = 35\%$
c $\frac{62}{100} = 0.62 = 62\%$

10 **a** 20
b $\frac{1}{5}$



REVIEW SET 14B

1 **a** 34 Xs, 66 Vs **b** $\frac{34}{100}$ Xs, $\frac{66}{100}$ Vs **c** 34% Xs, 66% Vs

2 19% **3** $\frac{4}{10} = \frac{40}{100} = 40\%$

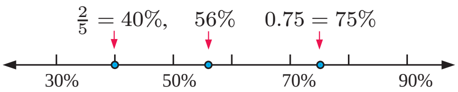
4 **a** 47% **b** $\frac{2}{5}$ **c** $66\frac{2}{3}\%$ **d** 0.125

5 **a** 27% **b** 3.5% **c** 72% **d** 65% **6** 60%

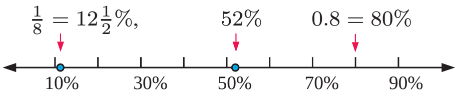
7 **a** $\frac{4}{25}$ **b** $2\frac{1}{2}$ **c** $\frac{17}{200}$ **d** $\frac{1}{10\,000}$

8 **a** 45% **b** 0.0579

9 **a**



b



10 **a** $37\frac{1}{2}\%$ **b** $\frac{1}{4}$

EXERCISE 15A

1 **a** 125AD **b** 1800 years **c** Japanese Script

2 **a** Elizabeth II **b** 15 years **c** 8 years

3 **a** Zhou Warlords **b** 450 years **c** 450 years

EXERCISE 15B

1 **a** 444 min **b** 4663 min **c** 18 216 min **d** 24 977 min

2 **a** 2438 s **b** 12 927 s **c** 51 163 s **d** 82 331 s

3 **a** 1440 min **b** 10 080 min **c** 525 600 min

4 **a** 86 400 s **b** 1 209 600 s **c** 31 536 000 s

5 **a** 1461 days **b** 35 064 h **c** 2 103 840 min

6 **a** 8 h 30 min **b** 13 h 17 min **c** 4 h 17 min
d 19 h 23 min **e** 4 h 49 min **f** 8 h 22 min

7 **a** 5 days 4 h **b** 23 days **c** 36 days 9 h **d** 90 days 7 h

8 **a** 47 days **b** 22 days 37 min

EXERCISE 15C

1 **a** Wei joined the club on the 17th Dec. 2007.
b Jon arrived on the 13th March 2006.
c Piri is departing for Malaysia on the 30th July 2010.
d Sam will turn 21 on the 28th May 2014.

2 **a** 27 days **b** 43 days **c** 117 days **d** 111 days
e 68 days **f** 179 days **g** 119 days **h** 99 days

3 **a** 45 days **b** \$6.20

4 **a** 195 days **b** €3510 **c** Yes, €1939

5 21st October 2002

6 **a** 7:00 am **b** 1:00 am **c** 6:49 am **d** 8:06 pm
e 10:32 pm **f** 4:09 pm **g** 11:05 am **h** 6:42 am
i 11:43 am **j** 10:44 pm Sunday

7 **a** 8 h 19 min **b** 3 h 19 min **c** 7 h 17 min
d 12 h 52 min **e** 20 h 9 min **f** 9 h 37 min
g 26 h 48 min **h** 87 h 54 min

8 3 round trips **9** 25 **10** 480 s = 8 min **11** 171 s

12 7:45 am, 2:05 pm, 8:25 pm (Mon); 2:45 am, 9:05 am, 3:25 pm, 9:45 pm (Tue); 4:05 am (Wed)

13 1440 times

EXERCISE 15D

1 **a** 0313 h **b** 1117 h **c** 0000 h **d** 1247 h
e 1741 h **f** 1200 h **g** 2019 h **h** 2359 h

2 **a** 3:00 am **b** 6:30 am **c** 6:00 pm **d** 12:00 noon
e 6:15 am **f** 3:45 pm **g** 8:17 pm **h** 11:48 pm

3 **a** 0930 h **b** 1240 h **c** 1915 h

4 **a** More than 60 minutes is not possible. **b** 0713 h is correct.
c Greater than 24 hours in a day is not possible.

5 **a** 2:50 pm, 3:50 pm, 4:25 pm, 4:45 pm, 5:15 pm
b 4:25 pm **c** **i** 5:25 pm **ii** 6:20 pm

EXERCISE 15E

1 **a** 7:21 am **b** 9:08 pm **c** 0.9 m, 3:20 am
d 1.2 m, 1:46 pm

2 **a** 6 **b** 8:45 am **c** 5:00 pm
d **i** 1 h 55 min **ii** 40 min **e** $9\frac{1}{2}$ h **f** bus A or bus B
g bus B

3 **a** **i** arrival time **ii** departure time **b** 4:50 pm
c 5:27 pm **d** 6:20 pm
e **i** 4:11 pm **ii** 4:36 pm **iii** 5 min
f **i** 45 min **ii** 51 min **iii** 44 min
There would be more trains on the track and more passengers for the 5:23 pm train (peak hour).

EXERCISE 15F

1 **a** 3 pm **b** 8 pm **c** 9 pm **d** 7 am

2 **a** 2:00 am Tuesday **b** 5:00 am Tuesday
c 9:00 am Tuesday **d** 12:00 midnight Monday

3 **a** 9:00 am Tuesday **b** 6:00 am Tuesday
c 5:00 pm Tuesday **d** 4:00 pm Tuesday

4 **a** 9:45 am Saturday **b** 10:45 pm Saturday
c 12:45 pm Saturday **d** 5:45 am Sunday

5 **a** 2:00 pm Friday **b** 8:00 pm Friday
c noon Friday **d** 4:00 am Friday

EXERCISE 15G

1 **a** 90 km per hour **b** 70 km per hour
c 83 km per hour **d** 94 km per hour

2 **a** 630 km **b** 450 km **c** 900 km **d** 315 km
e 1026 km

3 **a** 255 km **b** 880 km **c** 441 km **d** 171 km

4 **a** 3 h **b** 6 h **c** $5\frac{1}{2}$ h **d** 8 h 20 min **e** 3 h 15 min

EXERCISE 15H

1 **a** 122°F **b** 176°F **c** 68°F **d** 23°F

2 **a** 38°C **b** 10°C **c** 27°C **d** −18°C

4 **a** 100°F ≈ 37.8°C, 50°F = 10°C, 80°F ≈ 26.7°C, 0°F ≈ −17.8°C
b ≈ 32.2°C

REVIEW SET 15A

1 **a** **i** 1968 **ii** 1970 **iii** 1978 **iv** 1984
b **i** 25 years **ii** 8 years
2 **a** 49 days **b** 720 s **c** 555 min **d** 1000 years
3 **a** 20 h 41 min **b** 3 h 38 min **4** May 31st
5 **a** 176 days **b** $2640 **c** $360
6 **a** **i** 6:45 am **ii** 0645 h **b** **i** 12:15 am **ii** 0015 h
c **i** 9:30 pm **ii** 2130 h
7 **a** 3:45 pm **b** **i** 1 h 35 min **ii** 2 h 15 min **c** 1:45 pm
8 **a** **i** 2:00 pm Saturday **ii** 6:00 am Saturday
b **i** 3:00 am Wednesday **ii** 6:00 am Wednesday
9 54 km **10** 94 km per hour

REVIEW SET 15B

1 **a** 1194 min **b** 19 days 19 h
2 **a** 107 days **b** 61 h **c** 81 min **d** 300 s
3 **a** 13 h 30 min **b** 6 days 1 h 3 min **c** 1 h 18 min
4 **a** 80 days **b** 730 days **c** 3652 days **5** 53
6 **a** 15 min 1.07 s, 15 min 5.42 s **b** 3 hrs 1 min 1 s
c 14 h 19 min
7 **a** 4:15 am **b** 1:00 pm **c** 11:35 pm
8 **a** two minutes to three in the afternoon **b** 2:58 pm
c 1458 h
9 **a** **i** 1:00 pm **ii** 2:00 am
b **i** 11:05 pm Wed **ii** 11:05 am Wed
10 348 km **11** 1 h 15 min **12** **a** 60°C **b** −5°C
13 **a** 41°F **b** 104°F

EXERCISE 16A

1 **a** 20% **b** 20% **c** 75% **d** 25% **e** 5%
f 50% **g** $16\frac{2}{3}$% **h** 12.5% **i** 5% **j** 4.8%
k $33\frac{1}{3}$% **l** 6.25% **m** 50% **n** 40% **o** 60%
p 10% **q** 0.25% **r** 12.5%
2 **a** 85% **b** 44% **c** 72.5% **d** 90% **e** 74%
f 69%
3 **a** 85.4% **b** 32.5% **c** 68.5% **d** 76%
4 **a** 70% **b** 85% **c** 22% **d** 5% **e** 17.5%
f 40% **g** 42% **h** $66\frac{2}{3}$% **i** 174%

EXERCISE 16B

1 **a** 72 ha **b** 1050 m^2 **c** 45 cm **d** 160 t
e 18 min **f** 640 cm **g** 600 kg **h** 480 mm
i 108 min **j** 187.5 kL
2 388 students **3** 360 kg **4** 1215 tonnes
5 3 h 48 min **6** 1.584 m
7 **a** 40% of a litre **b** $\frac{1}{4}$ of a metre **c** $\frac{1}{3}$ of 1000 **d** 315 g
8 202.5 g **9** 3.2 L **10** 1680 acres

EXERCISE 16C

1 **a** $4 **b** €54 **c** £147 **d** €2.20
e $30 **f** RM 4365 **g** £597.60 **h** $162
i €388 **j** 354 rupees **k** £259 **l** €1350
m RMB 700 **n** £2.72 **o** ¥5450
2 **a** 25% **b** 10% **c** 15% **d** 5% **e** 20% **f** $6\frac{2}{3}$%
g 5% **h** 25% **i** $3\frac{1}{3}$% **j** $6\frac{2}{3}$% **k** 1% **l** 10%

EXERCISE 16D

1 **a** £100 **b** 25% loss **2** **a** €200 **b** 4% profit
3 **a** $15 000 **b** 18.75% profit
4 **a** RM 100 **b** 40% loss
5 **a** €100 **b** €350 **6** **a** £75 **b** £200
7 **a** ¥17 000 **b** ¥83 000 **8** **a** €5400 **b** €12 600

EXERCISE 16E

1 **a** $272 **b** €345 **c** $127.40 **d** £528
2 **a** $5.60 **b** $27 **c** £128.80 **d** RM 49.28
e €16.15 **f** $455 **g** $91.84 **h** £340
i €32.90 **j** €376 **k** $47.10 **l** ¥37 187.50

EXERCISE 16F

1 **a** $2 **b** €80 **c** £5.60
2 **a** €15 **b** $1.50 **c** £2.40 **d** RMB 48
3 **a** $118 **b** €56.64 **c** $2360 **d** $755.20
4 **a** £24 **b** £184 **5** **a** $131.25 **b** $881.25
6 **a** €31.25 **b** €281.25

EXERCISE 16G

1 **a** $150 **b** £700 **c** $1600 **d** €3600 **e** $14 000
2 **a** $2800 **b** €8450 **c** $12 320 **d** ¥200 000

REVIEW SET 16A

1 5% **2** €226.80 **3** 30 students **4** 70 households
5 $4.50 **6** **a** 75 students **b** 275 students **7** 25%
8 81 kg **9** **a** $840 **b** $4340
10 **a** £420 **b** £1680 **11** **a** $15 **b** 37.5%

REVIEW SET 16B

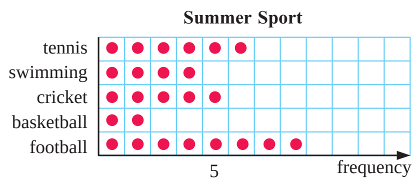
1 **a** 52% **b** 29% **2** 76% **3** 32.5% **4** €3
5 **a** €126 **b** €714 **6** $189 **7** **a** $2800 **b** $9800
8 **a** 60 students **b** 140 students **9** 30%
10 **a** $27 **b** $297 **11** **a** £60 **b** £340

EXERCISE 17A

1 **a** Choose 400 names from the electoral roll.
b Select every 100th bottle, for example.
c Choose 30 names from a list of students at the school.
d Select a random page of a dictionary, and randomly select a word on that page.
2 **a** Place the tickets in a hat, select one at random.
b Toss a coin, if it lands heads select A, if it lands tails select B.
c Roll a die, and select the number that appears.
d Shuffle the pack, and select the card on top.
3 **a** 10 000 ants **b** 300 ants **c** 12% **d** 1200 ants
4 **a** 750 people **b** 50 people **c** 34% **d** 255 people

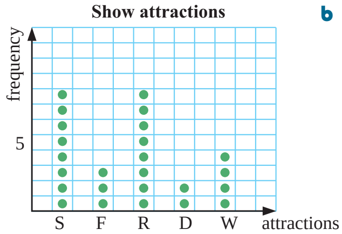
EXERCISE 17B.1

1 **a** 50 students **b** 22 students **c** 74%
2 **a** 12 students **b** 28 students **c** violin
3 **a**



b football

4 **a**



b side shows and ring events

EXERCISE 17B.2

1 **a**

Colour	*Tally*	*Frequency*
Brown	卌 卌 |	11
Blue	卌 ||	7
Green	卌 |	6
Grey	||||	4
	Total	28

b brown

2 **a**

Result	*Tally*	*Frequency*
A	|||	3
B	卌	5
C	卌 卌 卌 |	16
D	|||	3
E	|	1
	Total	28

b **i** 16 **ii** $\frac{5}{28}$ **c** C

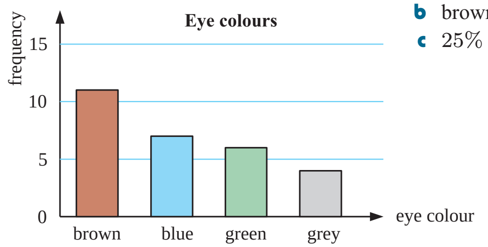
3 **a**

Rating	*Tally*	*Frequency*
Excellent	||||	4
Good	卌 卌 |	11
Satisfactory	卌 ||	7
Unsatisfactory	|||	3
	Total	25

b good **c** to determine if service needed to improve

EXERCISE 17C.1

1 **a**



b brown **c** 25%

2 **a** Mercedes: 20, Audi: 28, Volkswagon: 35, BMW: 25, Other: 12
b Volkswagon **c** 23.3%

3 **a** 2003, 2004, 2005, 2008 **b** broke even
c $7 million profit

EXERCISE 17C.2

1 **a** garden **b** cleaning
c **i** 156 kilolitres **ii** 20 kilolitres

2 **a** size 14 **b** 30 women

3 **a** Shows how city council funding is spent.
b It is easier to compare the actual amounts spent on each area.
c **i** 8% **ii** 4% **d** **i** €26 m **ii** €69 m
e water and sewerage **f** €1095 m

EXERCISE 17D.1

1

6	3 4 6 7	
7	0 1 2 2 2 5 8	
8	0 1 4 4 6 6 7 7 9 9	
9	0 1 3	unit = 1 kg

2

1	3 7 8 9	
2	0 0 3 5 7 7 8 9 9	
3	0 0 0 2 3 4 6 7 7 8 8 9	
4	3 4 7	
5	0 1	unit = 1 kg

3

0	7 9	
1	2 3 8	
2	4 4 5 7 8	
3	0 2 6	
4	1	unit = 1 hour

4

0	2 5 7 8 9	
1	0 2 3 8	
2	4 4 7	
3	0 6	unit = 0.1 hour

EXERCISE 17D.2

1 **a**

Children	*Tally*	*Frequency*
0	||||	4
1	||||	4
2	卌 ||||	9
3	||||	4
4	卌 |	6
5	||	2
6	|	1
	Total	30

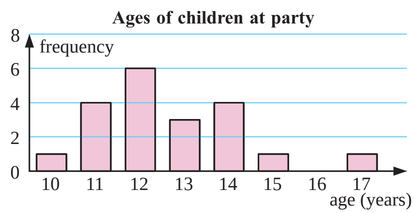
b **i** 9 families **ii** $\frac{2}{15}$

2 **a**

Age	*Tally*	*Frequency*
10	|	1
11	||||	4
12	卌 |	6
13	|||	3
14	||||	4
15	|	1
16		0
17	|	1
	Total	20

b 20 children
c 9 children
d 45%

e



3 **a** 7 points **b** 10 **c** ≈ 18.3%

4 **a**

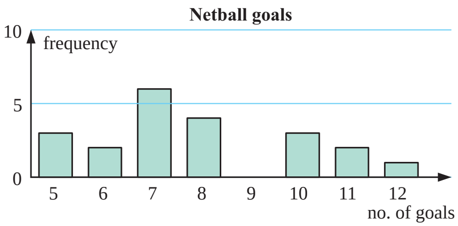
Goals	*Tally*	*Frequency*
0	|||	3
1	||||	4
2	卌 |	6
3	卌 |	6
4	||	2
5	|	1
	Total	22

b **i** 6 games
ii 9 games

5 **a**

Goals	Tally	Frequency
5	|||	3
6	||	2
7	卌 |	6
8	||||	4
9		0
10	|||	3
11	||	2
12	|	1
	Total	21

b



c **i** 4 games **ii** 10 games

6 **a**

Matches	Tally	Frequency
48	卌 |	6
49	卌 |||	8
50	卌 卌 |	11
51	卌 ||	7
52	卌 |	6
53	||	2
	Total	40

b 11 boxes
c 26 boxes
d $\frac{7}{20}$

e Yes, most of the boxes contain at least 50 matches.

EXERCISE 17E

1 4 **2** 4 **3** 66 g **4** 113 m **5** 34.75 points

6 **a** Sean: 128 km per hour; Rick: 129 km per hour **b** Rick

7 **a** Group X: 6.5; Group Y: 7.64 **b** false **c** Group Y

8 **a**

Name	Mean
Sally Brown	26.4
Jan Simmons	23.5
Jane Haren	25.3
Peta Piper	31.3
Lee Wong	25.9
Polly Lynch	28.4
Sam Crawley	32.8

b Sam Crawley

REVIEW SET 17A

1 **a**

Month	Tally	Frequency
January	||	2
February		0
March	|||	3
April	||||	4
May	||||	4
June	||	2
July	卌	5
August	|	1
September	||||	4
October	||	2
November	|	1
December	||	2
	Total	30

b **i** 30
ii 16
iii $\frac{2}{15}$
iv 10%

2 **a** 8.91 **b** 8.92

3 **a** **i** air conditioners **ii** hot weather, sale on air conditioners
b \$215 000 **c** \$50

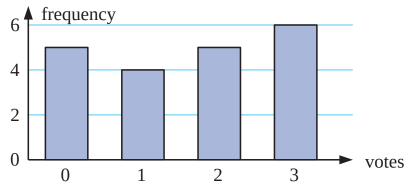
4 **a** lung cancer
b lung cancer and chronic bronchitis/emphysema
c **i** 4800 **ii** 5400 **iii** 1600

REVIEW SET 17B

1 **a**

Votes	Tally	Frequency
0	卌	5
1	||||	4
2	卌	5
3	卌 |	6
	Total	20

b



c 5 games **d** 32 votes **e** 75% **f** 1.6 votes

2 **a** football **b** €26 000 **c** **i** €100 000 **ii** €130 000

3 **a** 20 pets **b** dog **c** 20% **4** 15.9

5 **a**

Stem	Leaf
0	8 9
1	3 5 5 7 8
2	0 1 2 4 8 8
3	1 5

unit = 1 minute

b 20.3 minutes

EXERCISE 18A

1 **a**

b

Figure number (n)	1	2	3	4	5
Matches needed (M)	1	3	5	7	9

c M increases by 2

d

$2 \times n$	2	4	6	8	10
M	1	3	5	7	9

e $M = 2 \times n - 1$

2 **a**

b

Figure number (n)	1	2	3	4	5
Matches needed (M)	4	7	10	13	16

c M increases by 3

d

$3 \times n$	3	6	9	12	15
M	4	7	10	13	16

e $M = 3 \times n + 1$

3 **a**

b

Figure number (n)	1	2	3	4	5
Matches needed (M)	5	8	11	14	17

c M increases by 3

d

$3 \times n$	3	6	9	12	15
M	5	8	11	14	17

e $M = 3 \times n + 2$

4 **a** $M = 4 \times n + 1$ **b** $M = 4 \times n - 1$ **c** $M = 5 \times n - 1$

EXERCISE 18B

1 **a** cd **b** cd **c** abc **d** $5a$ **e** $2mn$ **f** $3ab$
g $3t+2$ **h** $7n-4$ **i** $7ab$ **j** $10ac$ **k** $5+3s$
l $6-pt$ **m** $11+pq$ **n** $3r-6$ **o** $ab+ac$ **p** $3c+2d$

2 **a** $M=3n$ **b** $s=n+2$ **c** $M=5n+3$
d $N=2n-4$ **e** $M=3n+2$ **f** $N=2(n+1)$

EXERCISE 18C

1 **a** $m+n$ **b** $a+3$ **c** $b-c$ **d** $\frac{g}{3}$ **e** $2n$ **f** $3y$
g $a+4$ **h** $d-2$ **i** $a+d$ **j** $r-q$ **k** $4n$ **l** $2n+5$

2 **a** $\frac{m+n}{2}$ **b** $\frac{x+y}{4}$ **c** $\frac{5}{r+s}$

3 **a** $24,\ 3m,\ am$ **b** $4,\ d-5,\ d-c$ **c** $4,\ a-2,\ a-x$
d $12,\ 7+t,\ r+t$ **e** $300,\ 100D$ **f** $4,\ \frac{c}{100}$

4 **a** $11-x$ **b** $86+y$ **c** $4xy$ **d** $\frac{7c}{100}$ dollars

5 **a** $n-12$ **b** $n+6$ **c** $2n$ **d** €$(40x+15y)$

EXERCISE 18D

1 **a** **i** 17 **ii** -3 **iii** -18
b **i** -1 **ii** -15 **iii** 4 **c** **i** 8 **ii** 3 **iii** $6\frac{1}{2}$
d **i** 2 **ii** 8 **iii** $4\frac{2}{3}$ **e** **i** 1 **ii** -1 **iii** $1\frac{2}{3}$
f **i** 3 **ii** $1\frac{2}{5}$ **iii** $-1\frac{2}{5}$

2 **a** $10,\ 16,\ 1$ **b** $-7,\ 18,\ 38$ **c** $0,\ -1\frac{2}{3},\ -3\frac{1}{3}$
d $0,\ 6,\ 4\frac{1}{6}$ **e** $3,\ -3,\ 1\frac{1}{2}$ **f** $20,\ -2\frac{1}{2},\ 1\frac{1}{2}$

EXERCISE 18E

1 **a**

n	1	2	3	4	5
S	4	5	6	7	8

b

b	1	3	5	7	9
L	4	12	20	28	36

c

d	1	4	8	10	15
C	9	15	23	27	37

d

t	1	2	5	9	15
P	-1	2	11	23	41

2 **a** **i** $y=23$ **ii** $y=33$ **iii** $y=78$
b **i** $y=9$ **ii** $y=16$ **iii** $y=65$
c **i** $y=21$ **ii** $y=42$ **iii** $y=66$
d **i** $y=30$ **ii** $y=18$ **iii** $y=10$
e **i** $y=36$ **ii** $y=22$ **iii** $y=12$
f **i** $y=13$ **ii** $y=29$ **iii** $y=97$

3 **a** £170 **b** £320 **c** £720

4 **a** 12.5 mL **b** 20 mL **c** 25 mL

5 **a**

Figure number (n)	1	2	3	4	5	6
Matchsticks needed (M)	4	6	8	10	12	14

b $M=2n+2$ **c** 128 matchsticks

6 **a**

Figure number (n)	1	2	3	4	5	6
Matchsticks needed (M)	3	5	7	9	11	13

b $M=2n+1$ **c** 151 matchsticks

7 **a**

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)	4	10	16	22	28

b $M=6n-2$ **c** 340 matchsticks

8 **a**

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)	7	12	17	22	27

b $M=5n+2$ **c** 402 matchsticks

9 **a**

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)	16	19	22	25	28

b $M=3n+13$ **c** 100 matchsticks

EXERCISE 18F

1 **a** $30h$ dollars **b** $C=40+30h$
c **i** \$70 **ii** \$137.50 **iii** \$166

2 **a** $42m$ euro **b** $C=75+42m$ **c** **i** €369 **ii** €2343

3 **a** 26 cumecs **b** $F=8+2h$
c

Time (h)	0	1	2	3	4	5	6	7	8	9
Flow (cumecs)	8	10	12	14	16	18	20	22	24	26

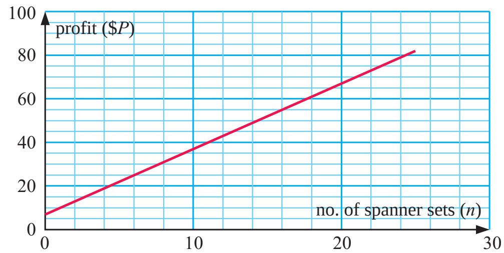
4 **a** $C=30+40h$ dollars **b** **i** \$950 **ii** \$730

5 **a** $T=4.8+0.2n$ min **b** **i** 6 min **ii** 7 min 24 s
c 88 min 12 s

EXERCISE 18G

1 **a** **i** £35 **ii** £56 **iii** £67.10
b The graph does not show values of d greater than 20.

2



a **i** \$37 **ii** \$61 **iii** \$73 **b** **iii** \$112

3 **a** **i** 8 cm **ii** 14 cm **iii** 32 cm
b **i** 8 weeks **ii** 12 weeks **iii** 17 weeks

REVIEW SET 18A

1 **a** $2xy$ **b** $M=3n+d$ **c** $ab+3c$ **d** $\frac{n}{3}$ **e** $\frac{a+b}{c}$
f $\frac{100}{x-3}$

2 **a** $2c$ **b** $3(a+6)$ **c** $t+5$ **d** $n-d$ **3** £(xy)

4 $(2x+y+5z)$ dollars **5** **a** $y=31$ **b** $y=11$

6 **a**

b

Figure number (n)	1	2	3	4	5
Matchsticks needed (M)	4	7	10	13	16

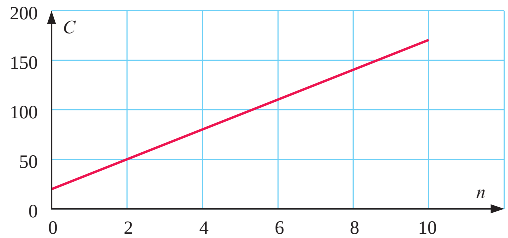
c $M=3n+1$ **d** **i** 22 **ii** 304

7

n	0	1	2	3	4	5
C	20	35	50	65	80	95

a $C=20+15n$ **b** \$425

c



REVIEW SET 18B

1 **a** $5x$ **b** $N = 5g - 6$ **c** $\frac{m}{10}$ **d** $4ac$ **e** $a + \frac{b}{c}$
f $\frac{a+b}{m}$

2 **a** $a + 2b$ **b** $2(a + b)$ **c** $3 + d$ **d** $3c - n$

3 €$(8x)$ **4** **a** $M = 3$ **b** $5\frac{1}{2}$ **5** £$(100 - nd)$

6 **a**

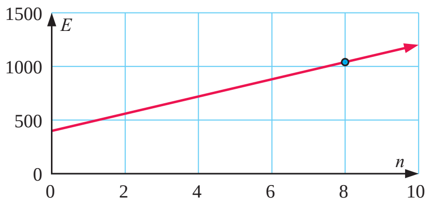
Figure number (n)	1	2	3	4	5	6
Matchsticks needed (M)	6	11	16	21	26	31

b $M = 5n + 1$
c The number of matchsticks needed is five times the figure number, plus one.
d 401

7 **a**

n	0	1	2	3	4	5
E	400	480	560	640	720	800

b $E = 400 + 80n$ **c** $1040
d



EXERCISE 19A.1

1 **a** 10 u^2 **b** 20 u^2 **c** 40 u^2 **d** 28 u^2 **e** 20 u^2
f 28 u^2

2 **a** all have area 16 u^2
b **i** 16 u **ii** 20 u **iii** 18 u **iv** 26 u **v** 34 u
vi 24 u **vii** 26 u **viii** 22 u
c Shapes with the same area do not necessarily have the same perimeter.

EXERCISE 19A.2

1 **a** m^2 **b** cm^2 **c** ha **d** cm^2 **e** mm^2
f km^2 **g** mm^2 **h** cm^2 **i** cm^2 **j** cm^2

2 **a** **i** 132 **ii** 302 **b** 17.36 m^2 **c** €953.06

3 **a** **i** 540 **ii** 280 **b** 16.4 m^2 **c** $506.76

4 **a** 6 **b** 5 **c** $\frac{1}{4}$ **d** $8\frac{3}{4}$ **e** $8\frac{3}{4}$

EXERCISE 19B

1 **a** multiply by 100 **b** multiply by 10 000
c multiply by 10 000 **d** multiply by 100
e multiply by 1 000 000 **f** multiply by 1 000 000
g divide by 100 **h** divide by 10 000
i divide by 10 000 **j** divide by 100
k divide by 1 000 000 **l** divide by 1 000 000

2 **a** 4.52 cm^2 **b** 75 000 cm^2 **c** 58 000 m^2
d 0.3579 m^2 **e** 630 ha **f** 36 500 000 mm^2
g 0.55 m^2 **h** 520 mm^2 **i** 0.68 ha
j 44 cm^2 **k** 6000 m^2 **l** 2 km^2
m 70 mm^2 **n** 4.8 km^2 **o** 2500 mm^2
p 8000 cm^2 **q** 88 cm^2 **r** 0.66 m^2
s 50 ha **t** 5.5 km^2 **u** 0.001 m^2

EXERCISE 19C

1 **a** 720 cm^2 **b** 504 mm^2 **c** 48 km^2
d 225 m^2 **e** 70.56 cm^2 **f** 4 ha

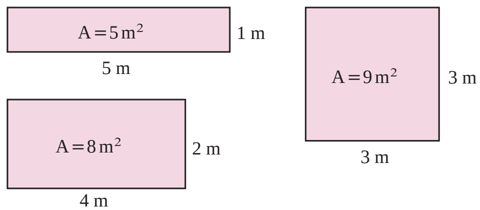
2 **a** 200 cm^2 **b** 64 cm^2 **c** 28 m^2
d 92 m^2 **e** 150 m^2 **f** 400 cm^2

3 65 m^2 **4** **a** 27 ha **b** $4860

5 **a** 625 cm^2 **b** 17.5 m^2 **c** 280 **d** £980

6 **a** 3 units by 2 units, perimeter 10 units
6 units by 1 unit, perimeter 14 units
b 4 units by 2 units, perimeter 12 units
8 units by 1 unit, perimeter 18 units
c 4 units by 3 units, perimeter 14 units
6 units by 2 units, perimeter 16 units
12 units by 1 unit, perimeter 26 units
d 4 units by 4 units, perimeter 16 units
8 units by 2 units, perimeter 20 units
16 units by 1 unit, perimeter 34 units

7 **a**



b 9 m^2, 16 m^2, 21 m^2, 24 m^2, 25 m^2
c 17 km^2, 32 km^2, 45 km^2, 56 km^2, 65 km^2, 72 km^2, 77 km^2, 80 km^2, 81 km^2

EXERCISE 19D

1 **a** 42 m^2 **b** 20 cm^2 **c** 38.5 cm^2 **d** 37.1 m^2
e 6 m^2 **f** 12.48 m^2

2 **a** 78 cm^2 **b** 89 m^2 **c** 11 m^2

3 **a** 48 ha **b** €17 280 **4** **a** 24 m^2 **b** £333.60

5 **a** 2625 cm^2 **b** 240 cm^2

EXERCISE 19E.1

1 **a** 0.008 cm^3 **b** 60 000 cm^3 **c** 11 800 mm^3
d 640 mm^3 **e** 3 000 000 000 mm^3 **f** 7 500 000 mm^3

2 **a** 0.5 cm^3 **b** 7 cm^3 **c** 5 m^3 **d** 0.45 m^3
e 0.002 m^3 **f** 5.4 m^3

EXERCISE 19E.2

1 **a** 63 u^3 **b** 64 u^3 **c** 96 u^3 **d** 51 u^3
e 40 u^3 **f** 61 u^3

2 **B**, **C**, **A**, **D** **3** **a** 60 cm^3 **b** 420 m^3 **c** 110 cm^3

4 **a** 50 m^3 **b** 48.44 cm^3 **c** 315.4 m^3 **d** 1332 cm^3
e 73.44 m^3 **f** 108.63 m^3

5 **a** 36 cm^3 **b** 140 m^3 **c** 80.64 cm^3

6 4 cm × 3 cm × 3 cm, 4 cm × 9 cm × 1 cm,
6 cm × 6 cm × 1 cm, 6 cm × 3 cm × 2 cm,
9 cm × 2 cm × 2 cm, 12 cm × 3 cm × 1 cm,
18 cm × 2 cm × 1 cm, 36 cm × 1 cm × 1 cm

7 **a** 162 cm^3 **b** 198 cm^2

EXERCISE 19F

1 **a** mL **b** L **c** kL **d** L **e** mL **f** kL **g** mL **h** ML **i** ML **j** ML **k** kL **l** mL

2 **a** 5600 L **b** 3.54 L **c** 0.76 ML **d** 7.2 L **e** 6.3 m^3 **f** 12 400 000 mL **g** 62.5 mL **h** 400 mL **i** 3500 kL

3 **a** 2.4 L **b** 5 L **c** 120 kL

4 **a** 210 mL **b** 600 mL **c** 216 mL

5 72 L **6** 66 times **7** 510 **8** 210 kL

EXERCISE 19G

1 **a** 66 m^2 **b** €4422 **2** **a** 4400 m^2 **b** 88

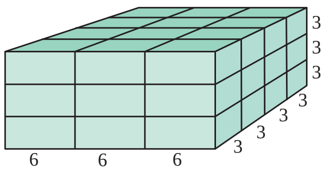
3 7.875 m^2

4 **a** 100 m **b** 10 m **c** 40 m **d** 5 m **e** 12.5 m **f** 8 m **g** 20 m **h** 800 m

5 18 m^2 **6** 1350 cm^3 **7** 30 000 **8** 180 000 m^3

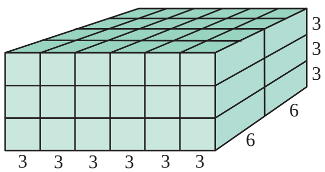
9 **a** 630 L **b** 78.75 buckets

10



i.e., 36 can be packed

or



REVIEW SET 19A

1 **a** 35 600 m^2 **b** 0.357 m^2 **c** 7200 mm^3

2 **a** 24.8 m^2 **b** 57 m^2

3 **a** 120 u^3 **b** 54 m^3 **c** 199.52 cm^3

4 **a** 0.38 L **b** 5.4 m^3 **c** 7.528 L

5 **a** 47 m^2 **b** 3.13 L **6** **a** 9 posters **b** 0.54 m^2

7 **a** 100 stamps **b** £22.50 **8** 67.5 kL

REVIEW SET 19B

1 **a** 0.34 ha **b** 320 mm^2 **c** 7 200 000 mm^2

2 **a** 92 m^2 **b** 329 m^2

3 **a** 45 kL **b** 8.9 cm^3 **c** 4600 L

4 **a** 36 u^3 **b** 0.36 m^3 **c** 100 m^3 **5** 1000 containers

6 4 prisms **7** 1731 kL **8** £99.75 **9** 37 m^2

EXERCISE 20A

1 **a** equation **b** expression **c** expression **d** equation **e** equation **f** expression

2 **a** $\square = 10$ **b** $\square = 13$ **c** $\square = 13$ **d** $\square = 19$ **e** $\square = 6$ **f** $\square = 24$ **g** $\square = 5$ **h** $\square = 6$ **i** $\square = 5$

3 **a** $x + 7 = 10$ **b** $x - 5 = 11$ **c** $4x = 12$ **d** $\frac{x}{10} = 2$

EXERCISE 20B

1 **a** $a = 8$ **b** $p = 8$ **c** $n = 7$ **d** $t = 40$ **e** $* = 52$ **f** $d = 6$ **g** $n = 7$ **h** $a = 25$ **i** $b = 63$ **j** $t = 0$ **k** $m = 12$ **l** $t = -5$ **m** $x = 5$ **n** $\square = 3$ **o** $y = -3$ **p** $x = 0$ **q** $x = -4$ **r** $t = -3$ **s** $x = 20$ **t** $x = -3$ **u** $x = 13$ **v** $n = 3$ **w** $x = 48$ **x** $t = 5$

2 **a** $x = 5$ **b** $x = 4\frac{1}{2}$ **c** $x = 7$ **d** $x = 6$ **e** $x = -1$ **f** $x = -2$

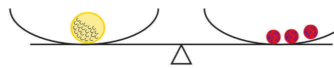
3 **a** $x = 7$ **b** $x = 10$ **c** $x = 9$ **d** $x = 2\frac{1}{2}$ **e** $x = 1\frac{1}{4}$ **f** $x = 4$

EXERCISE 20C.1

1 **a** $O = 2V$ **b** $2C = 3O$ **c** $X = 3V + 3O$ **d** $y = 2O$ **e** $g = y + O$ **f** $t = O$ **g** $e = w$ **h** $B = 4b$ **i** $M = 5$

2 **a** take three strawberries **b** 8 strawberries **c** 4 strawberries

3 **a** take 6 marbles **b** remove 1 golf ball **c**



d 3 marbles

EXERCISE 20C.2

1 **a** $x + 3 = 7$ **b** $x + 12 = 10$ **c** $x = 13$ **d** $2x = 10$

2 **a** $x - 2 = 6$ **b** $x = -7$ **c** $-x = 4$ **d** $3x = -7$

3 **a** $2x = 12$ **b** $6x = 3$ **c** $x = 10$ **d** $7(x + 1) = 63$ **e** $x + 1 = -2$ **f** $1 - x = 12$

4 **a** $x = 3$ **b** $x + 2 = 2$ **c** $x + 3 = 0$ **d** $x + 3 = 5$ **e** $x = \frac{14}{3}$ **f** $x - 1 = 3$ **g** $x - 4 = -1$ **h** $x + 2 = 3$

EXERCISE 20D

1 **a** $\div 3$ **b** -5 **c** $+4$ **d** $\times 7$ **e** $+\frac{3}{4}$ **f** $\div \frac{2}{3}$ **g** -10 **h** $\times \frac{1}{3}$

2 **a** x **b** x **c** x **d** x **e** x **f** x **g** x **h** $2x$

3 **a** $x = 3$ **b** $x = -9$ **c** $x = -3$ **d** $x = -15$ **e** $x = 2$ **f** $x = 6$

4 **a** $y = 11$ **b** $y = 2$ **c** $y = 5$ **d** $y = 43$ **e** $y = 0$ **f** $y = -17$

5 **a** $t = 2$ **b** $t = 5$ **c** $t = 2$ **d** $t = 5$ **e** $t = 4$ **f** $t = -3$ **g** $t = -8$ **h** $t = 8$ **i** $t = -7$

6 **a** $d = 6$ **b** $d = 28$ **c** $d = 16$ **d** $d = 30$ **e** $d = -12$ **f** $d = -7$

7 **a** $x = -7$ **b** $x = 11$ **c** $d = -10$ **d** $p = 14$ **e** $g = 5$ **f** $x = 32$ **g** $m = 4$ **h** $y = 8$ **i** $k = -8$ **j** $s = -4$ **k** $t = 4$ **l** $t = -9$ **m** $p = 38$ **n** $y = -4$ **o** $k = -14$ **p** $n = -8$ **q** $e = 13$ **r** $n = -9$ **s** $d = -72$ **t** $w = 13$ **u** $y = 49$

EXERCISE 20E.1

1 **a** $x \xrightarrow{\times 3} 3x \xrightarrow{-5} 3x-5$

b $x \xrightarrow{-5} x-5 \xrightarrow{\times 3} 3(x-5)$

c $x \xrightarrow{-4} x-4 \xrightarrow{\div 2} \dfrac{x-4}{2}$

d $x \xrightarrow{\div 2} \dfrac{x}{2} \xrightarrow{-4} \dfrac{x}{2}-4$

2 **a** $x \xrightarrow{\times 4} 4x \xrightarrow{-7} 4x-7$

b $x \xrightarrow{+5} x+5 \xrightarrow{\div 3} \dfrac{x+5}{3}$

c $x \xrightarrow{\div 6} \dfrac{x}{6} \xrightarrow{+1} \dfrac{x}{6}+1$

d $x \xrightarrow{-2} x-2 \xrightarrow{\times 8} 8(x-2)$

3 **a** $x \xrightarrow{\times 3} 3x \xrightarrow{+4} 3x+4$, $3x+4 \xrightarrow{-4} 3x \xrightarrow{\div 3} x$

b $x \xrightarrow{\times 2} 2x \xrightarrow{-5} 2x-5$, $2x-5 \xrightarrow{+5} 2x \xrightarrow{\div 2} x$

c $x \xrightarrow{\times 7} 7x \xrightarrow{+11} 7x+11$, $7x+11 \xrightarrow{-11} 7x \xrightarrow{\div 7} x$

d $x \xrightarrow{\times 8} 8x \xrightarrow{-15} 8x-15$, $8x-15 \xrightarrow{+15} 8x \xrightarrow{\div 8} x$

e $x \xrightarrow{\times 12} 12x \xrightarrow{+5} 12x+5$, $12x+5 \xrightarrow{-5} 12x \xrightarrow{\div 12} x$

f $x \xrightarrow{\times 23} 23x \xrightarrow{+10} 23x+10$, $23x+10 \xrightarrow{-10} 23x \xrightarrow{\div 23} x$

g $x \xrightarrow{\div 2} \dfrac{x}{2} \xrightarrow{+1} \dfrac{x}{2}+1$, $\dfrac{x}{2}+1 \xrightarrow{-1} \dfrac{x}{2} \xrightarrow{\times 2} x$

h $x \xrightarrow{+1} x+1 \xrightarrow{\div 2} \dfrac{x+1}{2}$, $\dfrac{x+1}{2} \xrightarrow{\times 2} x+1 \xrightarrow{-1} x$

i $x \xrightarrow{\div 3} \dfrac{x}{3} \xrightarrow{-2} \dfrac{x}{3}-2$, $\dfrac{x}{3}-2 \xrightarrow{+2} \dfrac{x}{3} \xrightarrow{\times 3} x$

j $x \xrightarrow{-2} x-2 \xrightarrow{\div 3} \dfrac{x-2}{3}$, $\dfrac{x-2}{3} \xrightarrow{\times 3} x-2 \xrightarrow{+2} x$

k $x \xrightarrow{\div 4} \dfrac{x}{4} \xrightarrow{+5} \dfrac{x}{4}+5$, $\dfrac{x}{4}+5 \xrightarrow{-5} \dfrac{x}{4} \xrightarrow{\times 4} x$

l $x \xrightarrow{+5} x+5 \xrightarrow{\div 4} \dfrac{x+5}{4}$, $\dfrac{x+5}{4} \xrightarrow{\times 4} x+5 \xrightarrow{-5} x$

m $x \xrightarrow{\times 2} 2x \xrightarrow{+5} 2x+5$, $2x+5 \xrightarrow{-5} 2x \xrightarrow{\div 2} x$

n $x \xrightarrow{+5} x+5 \xrightarrow{\times 2} 2(x+5)$, $2(x+5) \xrightarrow{\div 2} x+5 \xrightarrow{-5} x$

o $x \xrightarrow{\times 3} 3x \xrightarrow{-1} 3x-1$, $3x-1 \xrightarrow{+1} 3x \xrightarrow{\div 3} x$

p $x \xrightarrow{-1} x-1 \xrightarrow{\times 3} 3(x-1)$, $3(x-1) \xrightarrow{\div 3} x-1 \xrightarrow{+1} x$

EXERCISE 20E.2

1 **a** $x=3$ **b** $x=-6$ **c** $x=1$ **d** $x=-3$ **e** $x=2\frac{1}{2}$ **f** $x=-\frac{1}{2}$ **g** $x=2$ **h** $x=0$ **i** $x=4$ **j** $x=-5$ **k** $x=1$ **l** $x=-\frac{19}{7}$

2 **a** $x=10$ **b** $x=15$ **c** $x=-25$ **d** $x=-42$ **e** $x=42$ **f** $x=50$

3 **a** $x=37$ **b** $x=-54$ **c** $x=-4$ **d** $x=-10$ **e** $x=7$ **f** $x=-22$ **g** $x=76$ **h** $x=-13$ **i** $x=-49$

4 **a** $x=2$ **b** $x=3$ **c** $x=9$ **d** $x=-4$ **e** $x=13$ **f** $x=7$ **g** $x=5$ **h** $x=3$ **i** $x=-6$ **j** $x=1$ **k** $x=-9$ **l** $x=15\frac{1}{3}$

5 **a** $x=2$ **b** $x=-\frac{20}{3}$ **c** $x=3$ **d** $x=\frac{7}{2}$ **e** $x=-12$ **f** $x=24$ **g** $x=-20$ **h** $x=40$ **i** $x=22$ **j** $x=31$ **k** $x=-14$ **l** $x=-16$ **m** $x=-38$ **n** $x=8$ **o** $x=-3$ **p** $x=0$ **q** $x=4\frac{3}{4}$ **r** $x=6$

EXERCISE 20F

1 42 lollies **2** 7 cats **3** \$250 **4** 4 cartons **5** £15 **6** 19 **7** 8 **8** 19 boys **9** €300 **10** 15 cakes **11** 8 km

REVIEW SET 20A

1 6 **2** **a** equation **b** $\div 6$ **c** $3x=5$ **d** $x=-2$

3 $x=5$ **4** **a** 2✚ = 3● **b** $2t=a+v$

5 **a** $x=6$ **b** $x=6$ **c** $x=-8$ **d** $x=-3$

6 **a** $x \xrightarrow{\times 3} 3x \xrightarrow{+8} 3x+8$

b $x \xrightarrow{-3} x-3 \xrightarrow{\div 4} \dfrac{x-3}{4}$

7 **a** $x \xrightarrow{+4} x+4 \xrightarrow{\div 6} \dfrac{x+4}{6}$

b $x \xrightarrow{\times 4} 4x \xrightarrow{-5} 4x-5$

8 **a** $\dfrac{x}{5}+8 \xrightarrow{-8} \dfrac{x}{5} \xrightarrow{\times 5} x$

b $3(x-9) \xrightarrow{\div 3} x-9 \xrightarrow{+9} x$

9 **a** $x=\frac{7}{4}$ **b** $x=-12$ **c** $x=36$ **d** $x=\frac{8}{11}$ **e** $x=7$ **f** $x=6$

10 €6

REVIEW SET 20B

1 **a** $\square=45$ **b** $\square=4$

2 **a** $a=42$ **b** $3x=17$ **c** multiplying by 7 **d** $x=-24$

3 $x=3$ **4** $3-x=10$ **5** **a** $+5$ **b** $\div\frac{1}{2}$ **c** $\times 6$

6 **a** $t=-4$ **b** $t=6$ **c** $t=5$ **d** $t=-24$

7 **a** $x \xrightarrow{\div 4} \dfrac{x}{4} \xrightarrow{-7} \dfrac{x}{4}-7$

b $x \xrightarrow{+6} x+6 \xrightarrow{\times 5} 5(x+6)$

8 **a** $x \xrightarrow{\times 3} 3x \xrightarrow{-7} 3x-7 \xrightarrow{\times 2} 2(3x-7)$

b $x \xrightarrow{\times 2} 2x \xrightarrow{+3} 2x+3 \xrightarrow{\div 6} \dfrac{2x+3}{6}$

9 **a** $\dfrac{5x-3}{4} \xrightarrow{\times 4} 5x-3 \xrightarrow{+3} 5x \xrightarrow{\div 5} x$

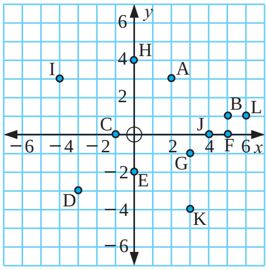
b $6(2x+1) \xrightarrow{\div 6} 2x+1 \xrightarrow{-1} 2x \xrightarrow{\div 2} x$

10 **a** $x = 9$ **b** $x = \frac{3}{2}$ **c** $x = 39$ **d** $x = -10$
e $x = 3$ **f** $x = 21$

11 19 truffles

EXERCISE 21A

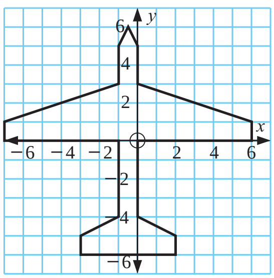
1



2 **a** A 3, D 3, E 2, G 0, H −4, J −2, L −2, O 0
b B 1, C 0, F −2, G −2, I 2, K 2, L 0, O 0
c A(3, 4), B(2, 1), C(1, 0), D(3, −2), E(2, −3), F(−2, −2), G(0, −2), H(−4, 1), I(−3, 2), J(−2, 4), K(0, 2), L(−2, 0)

3 **a** A, B **b** H, I, J **c** F **d** D, E **e** C, L **f** G, K

4



5 **a** first **b** third **c** second **d** fourth

EXERCISE 21B

1 **a** (1, 0) **b** (3, 2) **c** (0, −1) **d** (−2, −3)
e $(5\frac{1}{2}, 4\frac{1}{2})$

2 **a** (3, 3) **b** (0, −3) **c** (−1, −5) **d** $(\frac{1}{2}, -2)$
e (1.7, 0.4)

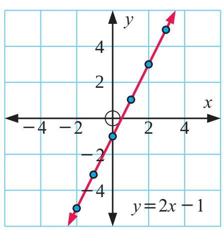
3 **a** (4, 5) **b** (−2, 2) **c** (100, 53) **d** $(5, 5\frac{1}{2})$
e (2.6, 4.3)

4 **a** (0, 5) **b** (−4, 13) **c** (3, −1) **d** $(-\frac{1}{2}, 6)$
e $(\frac{3}{5}, 3\frac{4}{5})$

EXERCISE 21C

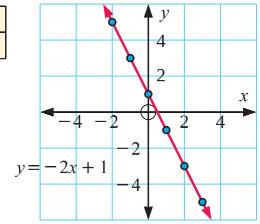
1

x	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5



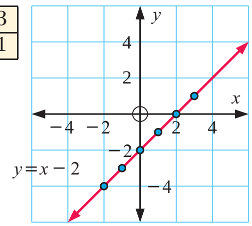
2

x	−2	−1	0	1	2	3
y	5	3	1	−1	−3	−5



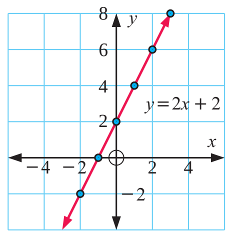
3 **a**

x	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1



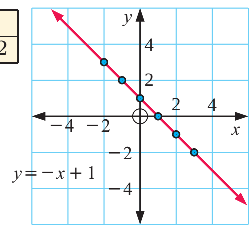
b

x	−2	−1	0	1	2	3
y	−2	0	2	4	6	8



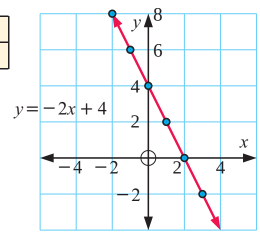
c

x	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2



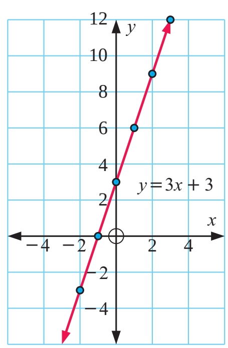
d

x	−2	−1	0	1	2	3
y	8	6	4	2	0	−2



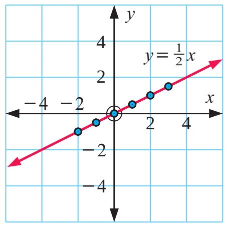
e

x	−2	−1	0	1	2	3
y	−3	0	3	6	9	12



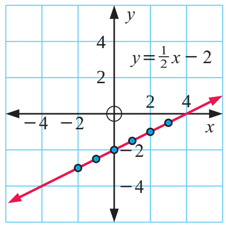
f

x	-2	-1	0	1	2	3
y	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$



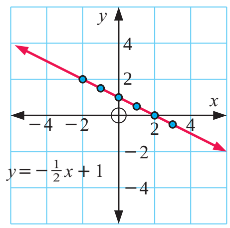
g

x	-2	-1	0	1	2	3
y	-3	$-2\frac{1}{2}$	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$



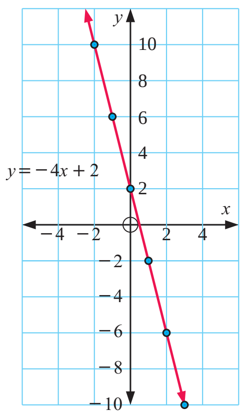
h

x	-2	-1	0	1	2	3
y	2	$1\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$



i

x	-2	-1	0	1	2	3
y	10	6	2	-2	-6	-10



4 **a** x-intercept 2, y-intercept -2
b x-intercept -1, y-intercept 2
c x-intercept 1, y-intercept 1
d x-intercept 2, y-intercept 4
e x-intercept -1, y-intercept 3
f x-intercept 0, y-intercept 0
g x-intercept 4, y-intercept -2
h x-intercept 2, y-intercept 1
i x-intercept $\frac{1}{2}$, y-intercept 2

5 **a**

x	-2	-1	0	1	2	3
y	5	7	9	11	13	15

b

x	-2	-1	0	1	2	3
y	8	6	4	2	0	-2

c

x	-1	0	1	2	3	4
y	8	6	4	2	0	-2

d

x	-3	-2	-1	0	1	2
y	-2	1	4	7	10	13

e

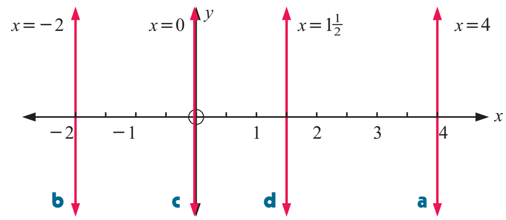
x	-2	-1	0	1	3	5
y	4	2	0	-2	-6	-10

f

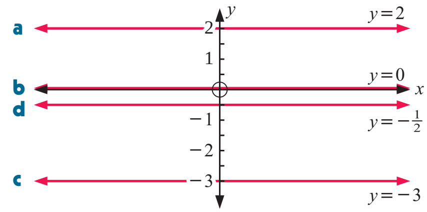
x	-3	-1	1	2	3	4
y	-7	-3	1	3	5	7

EXERCISE 21D

1



2



3 **a**

x	-4	-3	-2	-1	0	1	2	3	4
y	-4	-3	-2	-1	0	1	2	3	4

b $y = x$ **c** 1.4

4 **a**

x	-4	-3	-2	-1	0	1	2	3	4
y	4	3	2	1	0	-1	-2	-3	-4

b $y = -x$ **c** 2.31

5 **a** $x = k$ **b** $y = k$ **c** $y = x$ **d** $y = -x$

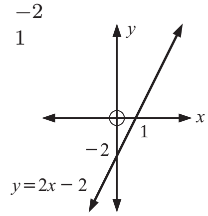
EXERCISE 21E

1 **a** -4 **b** -5 **c** 6 **d** -9 **e** -1 **f** 1
g 3 **h** 7 **i** -10 **j** 5 **k** -2 **l** $\frac{3}{2}$

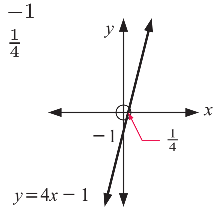
2 c

3 **a** 2 **b** 5 **c** -3 **d** 3 **e** $\frac{1}{2}$ **f** $-\frac{1}{2}$
g $\frac{3}{2}$ **h** 7 **i** $\frac{10}{7}$ **j** -10 **k** -4 **l** $-\frac{9}{2}$

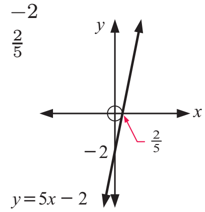
4 **a** **i** -2 **ii** 1 **iii**



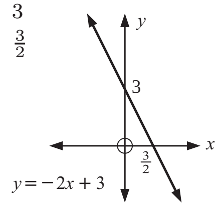
b **i** -1 **ii** $\frac{1}{4}$ **iii**



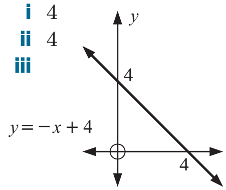
c **i** -2 **ii** $\frac{2}{5}$ **iii**



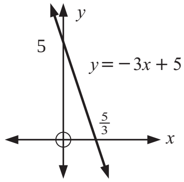
d **i** 3 **ii** $\frac{3}{2}$ **iii**



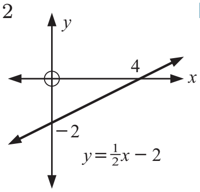
e **i** 4 **ii** 4 **iii**



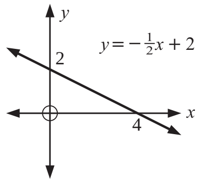
f **i** 5 **ii** $\frac{5}{3}$ **iii**



g **i** −2 **ii** 4 **iii**

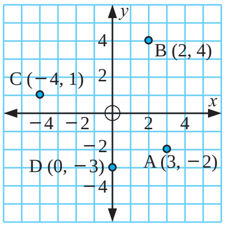


h **i** 2 **ii** 4 **iii**



REVIEW SET 21A

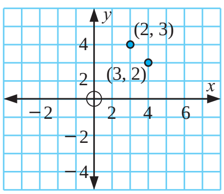
1



2 **a** A 2, D −3 **b** B 1, C −2
c A(2, 2), B(−2, 1), E(0, −1), F(2, 0)

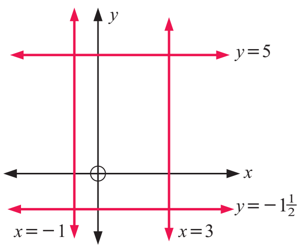
3 **a** second **b** first **c** lies on the y-axis

4 Not the same point.



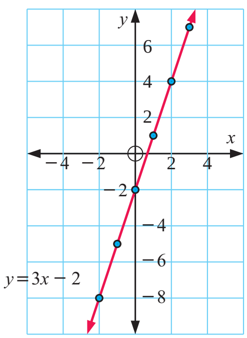
5 $y = x - 1$

6



7

x	−2	−1	0	1	2	3
y	−8	−5	−2	1	4	7

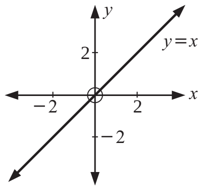


8 **a** (0, −1) **b** (5, 14) **c** (−2, −7) **d** $(\frac{3}{5}, \frac{4}{5})$

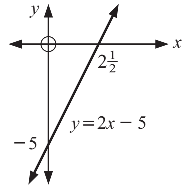
9

x	−2	−1	0	1	2	3
y	2	$\frac{1}{2}$	−1	$-2\frac{1}{2}$	−4	$-5\frac{1}{2}$

10

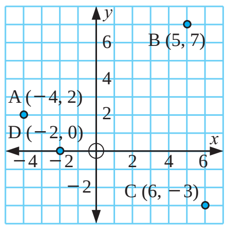


11 **a** −5 **b** $2\frac{1}{2}$
c



REVIEW SET 21B

1

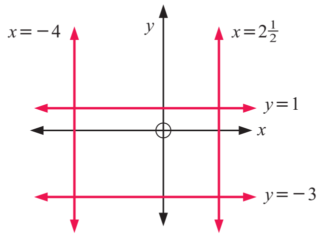


2 **a** D −4, F −4 **b** C −2, E 2
c A(1, 3), B(5, 1), C(2, −2), D(−4, 0)

3 one point (4, 5) **4** **a** first and fourth **b** third **5** $y = 2$

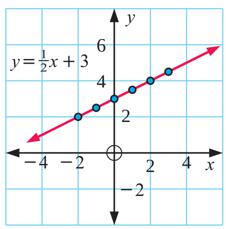
6 **a** −3 **b** −5 **c** −7

7



8

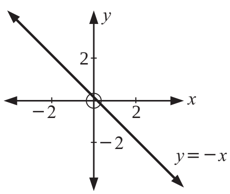
x	−2	−1	0	1	2	3
y	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$



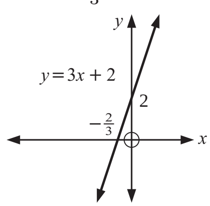
9

x	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1

10

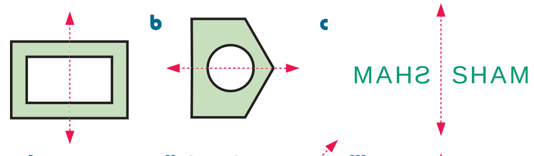


11 **a** 2 **b** $-\frac{2}{3}$
c

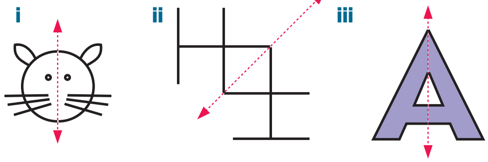


EXERCISE 22A.1

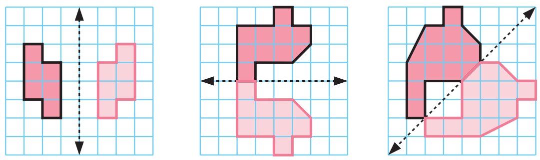
1 a b c



2 a i ii iii

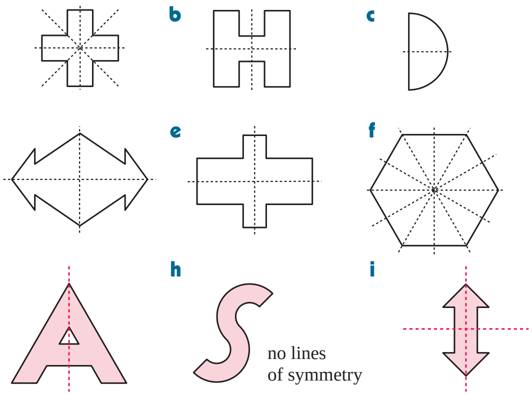


3 a b c

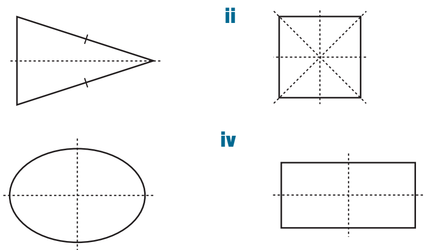


EXERCISE 22A.2

1 a b c d e f g h i



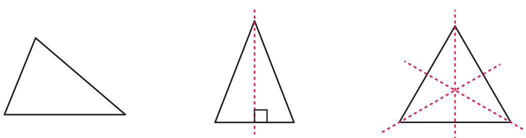
2 a i ii iii iv



b ii

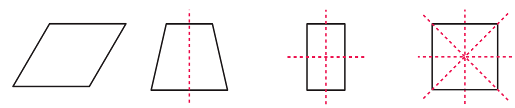
3 a 1 (vertical) b 1 (horizontal)

4 a



0 lines of symmetry | 1 line of symmetry | 3 lines of symmetry

b

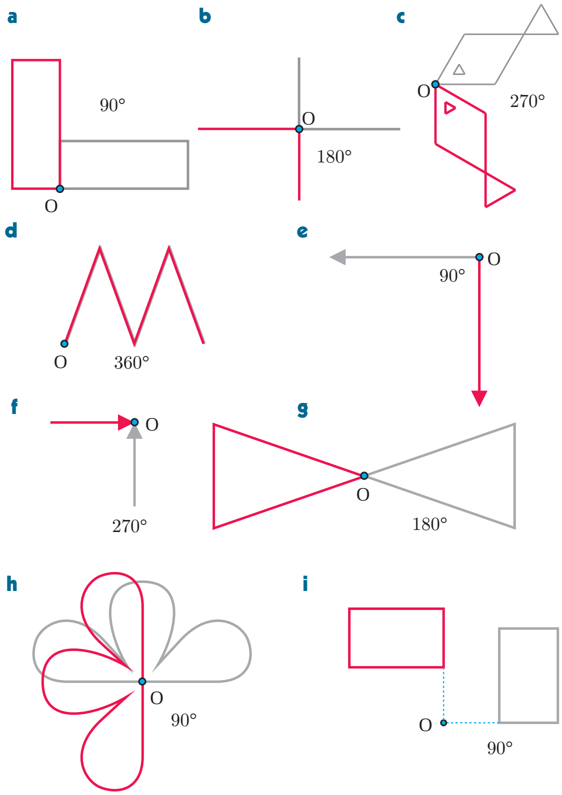


0 lines of symmetry | 1 line of symmetry | 2 lines of symmetry | 4 lines of symmetry

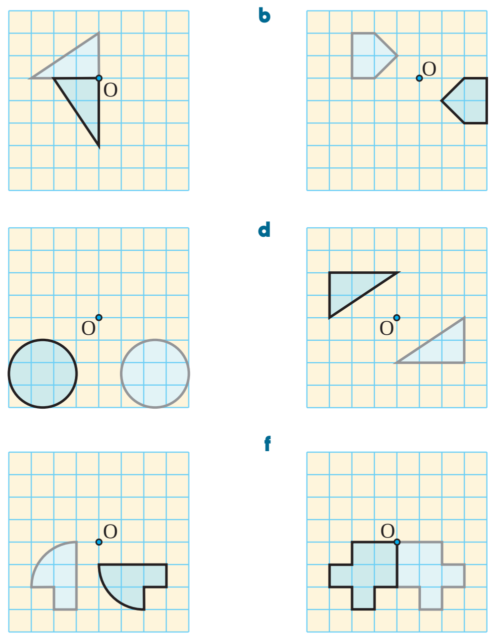
EXERCISE 22B.1

1 a B b D c A d C

2 a b c d e f g h i

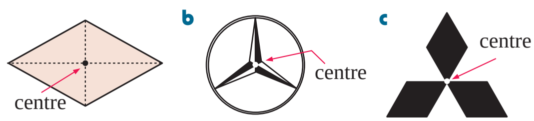


3 a b c d e f



EXERCISE 22B.2

1 a b c



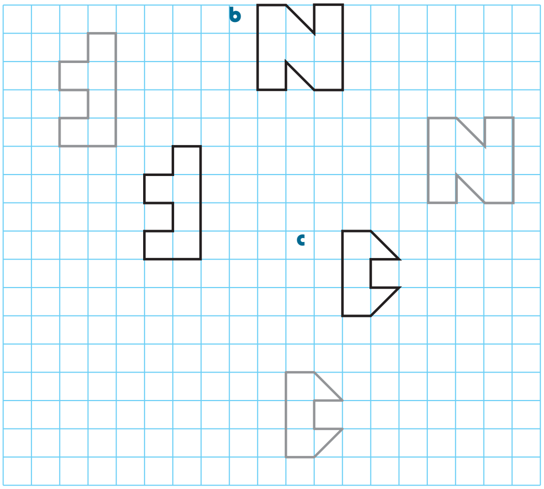
2 **a** 2 **b** 4 **c** 2 **d** 2 **e** 4 **f** 2 **g** 2 **h** 6

EXERCISE 22C

1 **a** translation 3 units right, 2 units down
b translation 4 units right, 3 units up
c translation 4 units down
d translation 4 units left
e translation 3 units left, 2 units down

2 **a** 7 units left, 1 unit up **b** 7 units right, 1 unit down
c 3 units right, 4 units down **d** 3 units left, 4 units up
e 4 units left, 3 units down **f** 4 units right, 3 units up

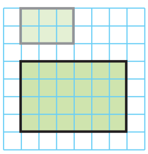
3 **a**



EXERCISE 22D

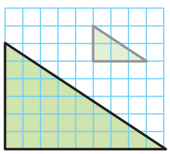
1 **a** 4 **b** 2 **c** 2 **2** **a** $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$

3 **a**



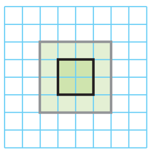
scale factor 2

b



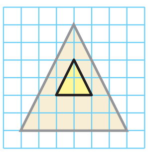
scale factor 3

c



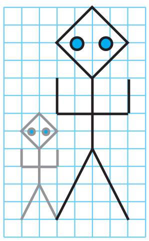
scale factor $\frac{1}{2}$

d



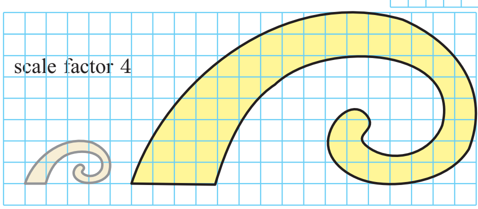
scale factor $\frac{1}{3}$

f



scale factor 2

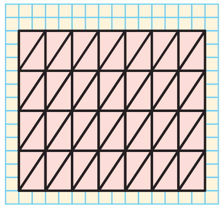
e



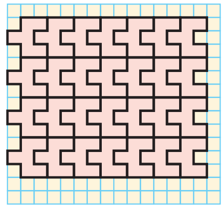
4 **a** 3 **b** 2 **c** $\frac{1}{2}$ **5** **a** $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** 2

EXERCISE 22E

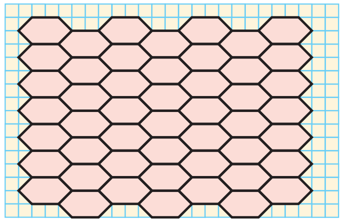
1 **a**



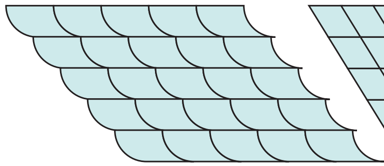
b



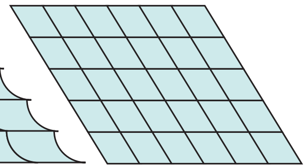
c



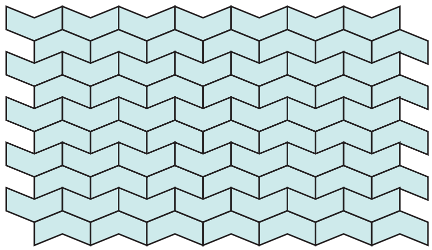
2 **a**



b

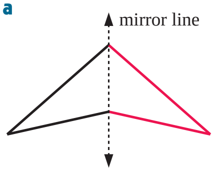


c

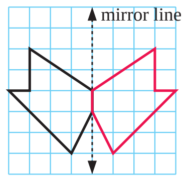


REVIEW SET 22A

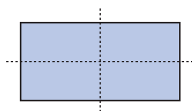
1 **a**



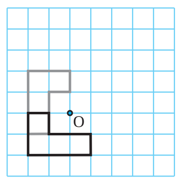
b



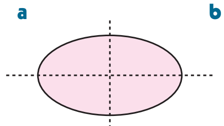
2



3

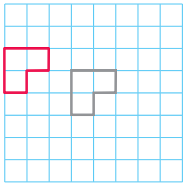


4 **a**

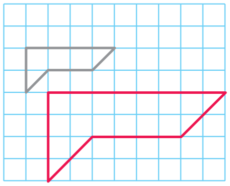


b 2

5 **a**

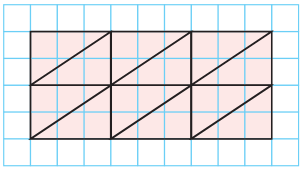


b

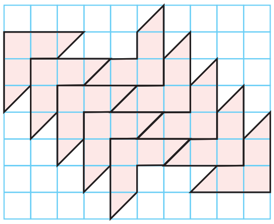


6 $\frac{1}{2}$

7 **a**

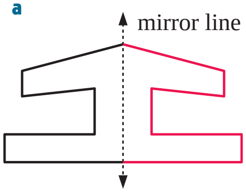


b

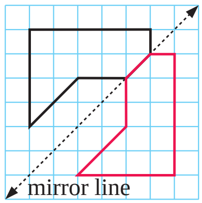


REVIEW SET 22B

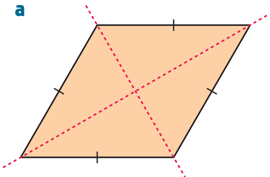
1 **a**



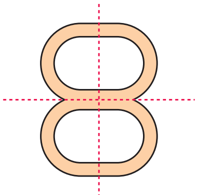
b



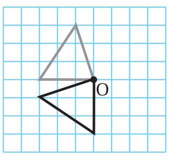
2 **a**



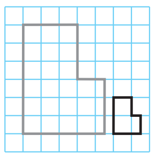
b



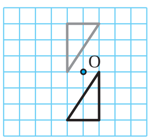
3 **a**



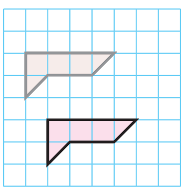
b



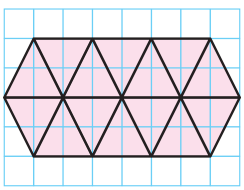
4 **a** 6 **b**



5



6



EXERCISE 23A.1

1 **a** $\{2, 4, 6, 8, 10, 12\}$ **b** $\{17, 19, 21, 23, 25, 27, 29\}$
c $\{2, 3, 5, 7, 11, 13, 17, 19\}$ **d** $\{2\}$ **e** $\{2, 3\}$
f $\{7, 14, 21, 28, 35, 42, 49\}$ **g** $\{10, 20, 30\}$
h $\{25, 36, 49, 64, 81\}$

2 **a** true **b** true **c** false **d** true **e** true
f false **g** true **h** true

3 **a** $P = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, $n(P) = 9$
b $M = \{21, 28, 35, 42, 49, 56\}$, $n(M) = 6$
c $D = \{1, 2, 4, 8, 16, 32\}$, $n(D) = 6$

4 **a** $P = \{2, 3, 5, 7, 11, 13\}$, $Q = \{1, 3, 9, 27\}$, $R = \{3, 6, 9, 12, 15\}$
b **i** $x = 3$ **ii** $x = 3$ or 9 **iii** $x = 6, 12$ or 15
iv $x = 2, 5, 7, 11$ or 13 **v** $x = 3$
vi $x = 6, 9, 12$ or 15

5 **a** $x = 2, 5, 8, 11$ **b** $x = 2, 5, 8$ **c** $x = 5, 8, 11, 14$

EXERCISE 23A.2

1 $x = 5$ **2** yes **3** $x = 5,\ y = 9$ or $x = 9,\ y = 5$

4 **a** true **b** false

EXERCISE 23A.3

1 **a** true **b** true **c** false **d** true **e** true

2 **a** $\{3\}, \{4\}, \{5\}$ **b** $\{3, 4\}, \{3, 5\}, \{4, 5\}$ **c** $\{3, 4, 5\}$

3 **a** $\{1, 2\},\ \{1, 3\},\ \{1, 4\},\ \{2, 3\},\ \{2, 4\},\ \{3, 4\}$
b $\{1, 2, 3\},\ \{1, 2, 4\},\ \{1, 3, 4\},\ \{2, 3, 4\}$

4 $A = B$ if $A \subseteq B$ and $B \subseteq A$

EXERCISE 23B

1 $A \cap B = \{a, e\}$

2 **a** $M = \{a, p, r, t, m, e, n\}$, $N = \{p, r, o, s, e, c, t\}$
b $M \cap N = \{p, r, t, e\}$

3 **a** $M = \{4, 8, 12\}$, $F = \{1, 2, 4, 8, 16\}$
b $M \cap F = \{4, 8\}$, $n(M \cap F) = 2$

4 **a** $P = \{2, 3, 5, 7, 11, 13, 17\}$, $F = \{1, 5, 7, 35\}$
b $P \cap F = \{5, 7\}$, $n(P \cap F) = 2$

5 **a** $\varnothing$ **b** $\{1, 2, 4\}$ **c** $\{15, 30, 45, 60, 75, 90\}$

EXERCISE 23C

1 **a** $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
b $A \cup B = \{a, b, c, d, e, f, g, m\}$
c $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
d $A \cup B = \{*, \#, !, \times, :, 5, +\}$

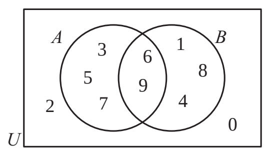
2 **a** $A \cup B = \{1, 2, 4, 5, 6, 8, 9\}$ **b** $A \cap B = \{1, 5\}$

3 **a** $P \cup Q = \{2, 3, 5, 6, 7, 9, 11, 13\}$ **b** $P \cap Q = \{7\}$

4 **a** $R \cup S = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$ **b** $R \cap S = \varnothing$

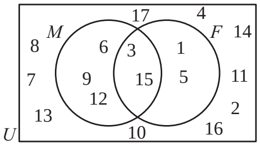
EXERCISE 23D

1 **a** **i** $A \cap B = \{6, 9\}$ **ii** $A \cup B = \{1, 3, 4, 5, 6, 7, 8, 9\}$
b



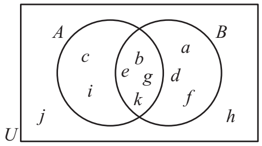
2 **a** $P = \{2, 3, 6, 7\}$ **b** $Q = \{1, 3, 5, 7\}$
c $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ **d** $P \cap Q = \{3, 7\}$
e $P \cup Q = \{1, 2, 3, 5, 6, 7\}$

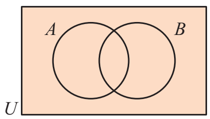
3 **a** $M = \{3, 6, 9, 12, 15\}$, $F = \{1, 3, 5, 15\}$
b **i** $M \cap F = \{3, 15\}$
ii $M \cup F = \{1, 3, 5, 6, 9, 12, 15\}$
iii $n(M \cap F) = 2$ **iv** $n(M \cup F) = 7$

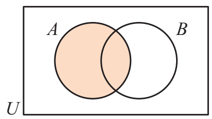
c 

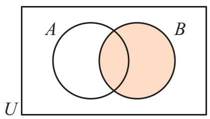
4 **a** **i** $A \cap B = \{b, e, g, k\}$

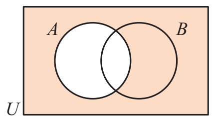
ii $A \cup B = \{a, b, c, d, e, f, g, i, k\}$

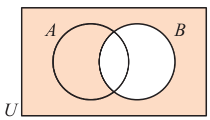
b 

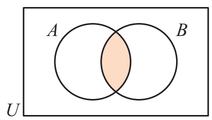
5 **a** 

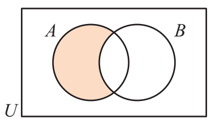
b 

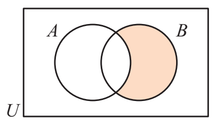
c 

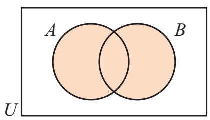
d 

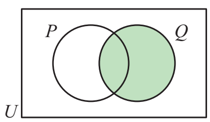
e 

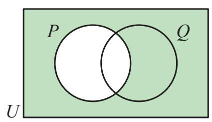
f 

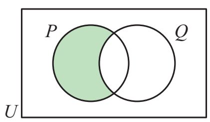
g 

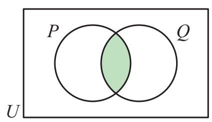
h 

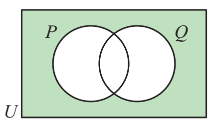
i 

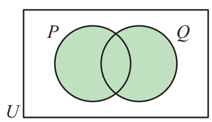
6 **a** 

b 

c 

d 

e 

f 

7 **a** The elements in set S.

b The elements that are not in both set R and set S.

c The elements in both set R and set S.

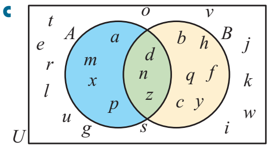
d The elements in neither set R nor set S.

e The elements not in set R. **f** The elements not in set S.

g The elements in either set R or set S.

h The elements in set R but not set S.

i The elements in set S but not set R.

8 **a - c** 

d $A \cap B = \{d, n, z\}$

e The elements in neither set A nor set B.

9 **a** 70 **b** **i** 40 **ii** 31 **iii** 11 **iv** 29 **v** 10 **vi** 60

10 **a** $x = 13$ **b** **i** 17 **ii** 25 **iii** 5 **iv** 12 **v** 2

REVIEW SET 23A

1 **a** $\{8, 16, 24, 32, 40, 48\}$ **b** $\{29, 31, 37\}$

2 **a** $A = \{26, 28, 30, 32, 34\}$ **b** $n(A) = 5$

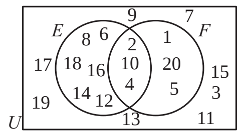
3 **a** $x = 6$ **b** $x = 6$ **c** $x = 6$

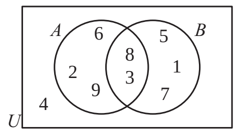
4 $x = 4,\ y = 6$ **5** **a** true **b** false

6 **a** **i** $E = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

ii $F = \{1, 2, 4, 5, 10, 20\}$ **iii** $E \cap F = \{2, 4, 10\}$

iv $E \cup F = \{1, 2, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20\}$

b 

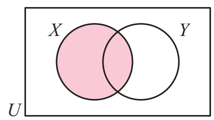
7 **a** 

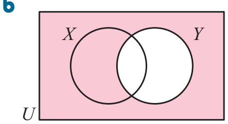
b **i** $A \cap B = \{3, 8\}$

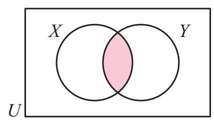
ii $A \cup B = \{1, 2, 3, 5, 6, 7, 8, 9\}$

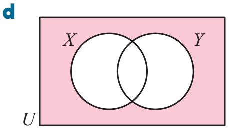
iii $n(A \cap B) = 2$

iv $n(A \cup B) = 8$

8 **a** 

b 

c 

d 

9 **a** 44 **b** **i** 25 **ii** 18 **iii** 7 **iv** 8 **v** 11

REVIEW SET 23B

1 **a** $\{16, 25, 36, 49\}$ **b** $\{1, 2, 3, 6, 9, 18\}$

2 **a** false **b** false **c** true

3 **a** $M = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39\}$

$N = \{4, 8, 12, 16, 20, 24, 28, 32, 36\}$

b $M \cap N = \{12, 24, 36\}$

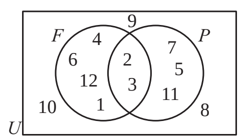
c {multiples of 3} $\cap$ {multiples of 4} $=$ {multiples of 12}

4 **a** $\{1, 5\}$, $\{1, 7\}$, $\{1, 9\}$, $\{5, 7\}$, $\{5, 9\}$, $\{7, 9\}$

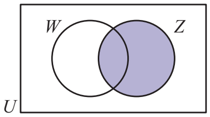
b $\{1, 5, 7\}$, $\{1, 5, 9\}$, $\{1, 7, 9\}$, $\{5, 7, 9\}$

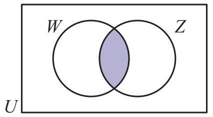
5 **a** $n(A) = 8$ **b** $n(B) = 9$ **c** $n(A \cap B) = 3$

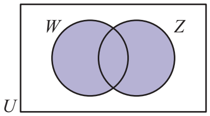
d $n(A \cup B) = 14$ **e** $n(U) = 21$

6 

7 **a** The elements in both set M and set N.
b The elements in set N, but not set M.
c The elements in neither set M nor set N.

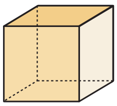
8 **a**


b


c


9 **a** $x = 8$
b **i** 19 **ii** 24 **iii** 8 **iv** 11 **v** 35 **vi** 27

EXERCISE 24A

1 **a**

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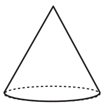
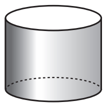

c

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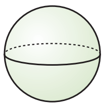
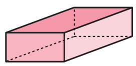
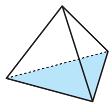

e

f


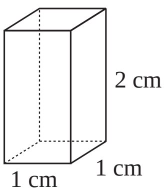
2 **a** sphere **b** cone **c** cylinder **d** cube
e rectangular prism **f** cylinder

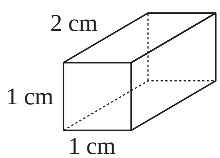
3 **a** a cube **b** a cone **c** a rectangular prism
d a sphere **e** a triangular prism
f a tetrahedron *or* triangular-based pyramid
g a cylinder **h** a square-based pyramid
i a hexagonal prism **j** a pentagonal-based pyramid

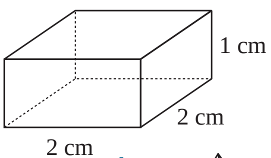
4 **a** A, B, C, D, E, F, G, H
b ABCD, BCFE, CDGF, ADGH, ABEH, EFGH
c [AB], [DC], [GF], [HE], [BC], [AD], [EF], [HG], [AH], [BE], [CF], [DG]

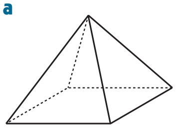
5 **a** rectangles **b** triangles **6** **a** 5 **b** 5 **c** 8

EXERCISE 24B

1 **a**


b


c


2 **a**

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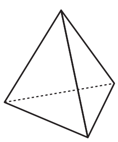
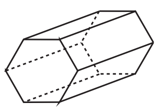

c

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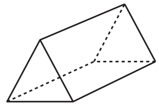
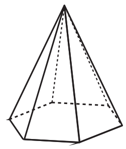
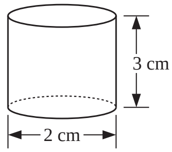
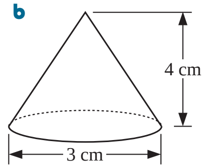

e


3 **a**


b


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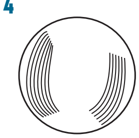
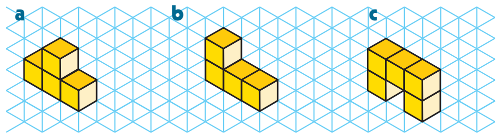
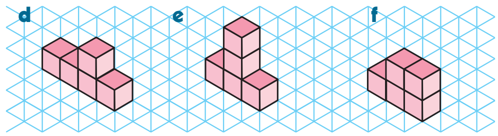


EXERCISE 24C

1




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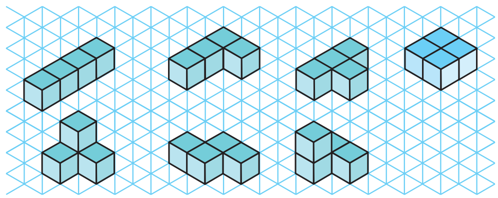
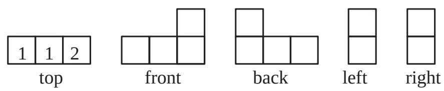
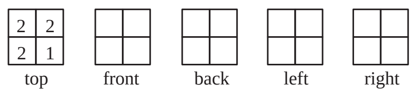
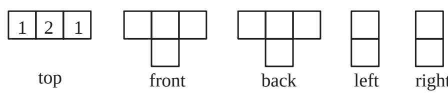
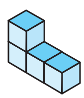
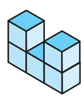


EXERCISE 24D

1 **a**


b


c


2 **a**

b


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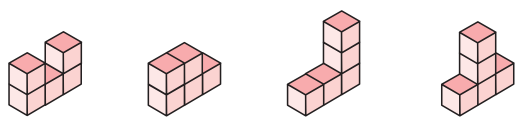
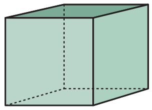


EXERCISE 24E

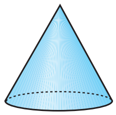
1 **a** **C**, (2) **b** **A**, (3) **c** **B**, (4) **d** **D**, (1)
3 Yes! Make it!

REVIEW SET 24A

1 **a**

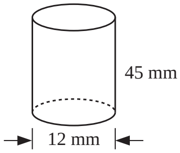


b

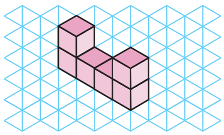


2 **a** U, V, W, X, Q, R, S, T
b UVWX, QRST, QTXU, RSWV, QRVU, TSWX
c [QR], [RS], [ST], [TQ], [QU], [RV], [SW], [TX], [UV], [VW], [WX], [XU]

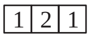
3



4



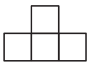
5 **a**



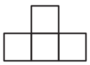
b

c

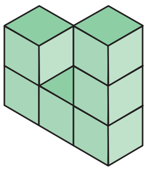
d



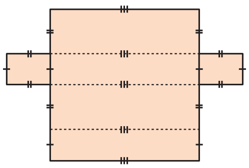
e



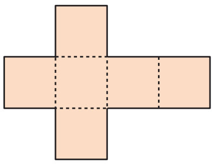
6



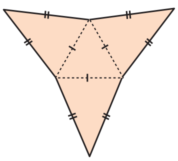
7 **a**



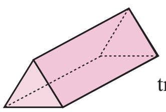
b



c



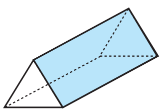
8



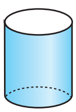
triangular prism

REVIEW SET 24B

1 **a**

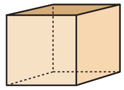


b

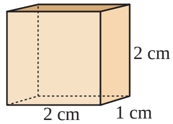


2 **a** A, B, C, D, E, F
b [AB], [AC], [BC], [AE], [BF], [CD], [DF], [DE], [EF]
c ABC, EFD, ABFE, ACDE, BCDF

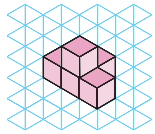
3 **a**



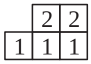
b



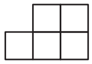
4



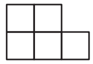
5



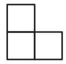
top



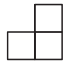
front



back

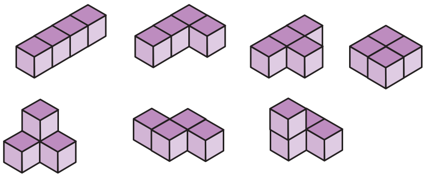


left

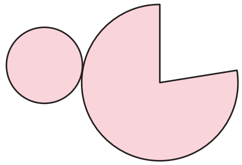


right

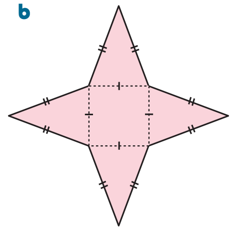
6 Any 5 of these:



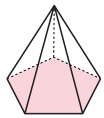
7 **a**



b



c



pentagonal-based pyramid

INDEX